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A NEW OPERATION AND RANKING ON PENTAGON FUZZY NUMBERS

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Abstract: The objective of this paper is to introduce a new operation for addition, subtraction and multiplication of pentagon fuzzy numbers on the basis of alpha cut sets of fuzzy numbers and a new approach for ranking of pentagon fuzzy numbers using incentre of the centroids.

keywords: Fuzzy arithmetic, pentagon fuzzy numbers, ranking .

2010 Mathematics Subject Classification: 03E72

INTRODUCTION: In real world, we frequently deal with vague or imprecise information available is sometimes vague, sometimes inexact sometimes insufficient. The concept of fuzzy sets was introduced by zadeh [11] in 1965. The usual arithmetic operation real numbers can be extended to the ones defined on fuzzy numbers by means of zadeh's extention principle[12,13] then some of the note worthy contributions on fuzzy numbers and its applications have been made by Dubois and Prade[4,5],Kaufmann[7],Mizumoto and Tanaka[9],internal arithmetic was first suggested by Dwyer[6]in 1951. Various operations on fuzzy numbers were also available in the literature which includes a new operation on trapezoidal fuzzy number. the ranking of fuzzy numbers has been a concern in fuzzy multiple criteria attribute decision making since its inception. More than 25 fuzzy ranking methods have been proposed since 1976.various techniques are applied in the literature that compares fuzzy numbers. In 1988 Lee and Li[8] ranked fuzzy numbers based on two different criteria, namely, the fuzzy mean and fuzzy spread of fuzzy numbers. Cheng[2] proposed the co-efficient of variance (i,e) $cv=\sigma/\mu, \mu\neq 0, \sigma>0$where σ −standard error and μ- mean ,to improve Lee and Li's ranking method. Chu and Tsao[3] pointed out the short comings of Cheng's method and suggested to rank fuzzy numbers with the area-based method. . Yager[10] was the researcher who contributed the centroid concept in the ranking method and used the horizontal co-ordinates as x and the vertical y co-ordinates of the centroid point as the ranking index. Abbasby and Asady[1] suggested a sign distance method for ranking fuzzy numbers in 2006.in this paper, section 2 deals with the preliminary definitions. In section 3 new operations on pentagon fuzzy number is discussed. In section 4 we proposed a new method for ranking pentagon fuzzy number which is based on incentre of the centroids.

2.PRELIMINARIES:

2.1 DEFINITION:

FUZZY SET: A fuzzy set is characterized by a membership function mapping the elements of domain, space or universe of discourse x to the unit interval [0,1].

A fuzzy set $\widetilde{A}$ is set of ordered pairs $\{x, \mu_A(x)/x\in R\}$ where $\mu_A(x): R\rightarrow[0,1]$ is upper semi- continuous function $\mu_A(x)$ is called membership function of the fuzzy set.

2.2 DEFINITION: A fuzzy number f in the real line R is a fuzzy set $f: R\rightarrow[0,1]$that satisfies the following properties.

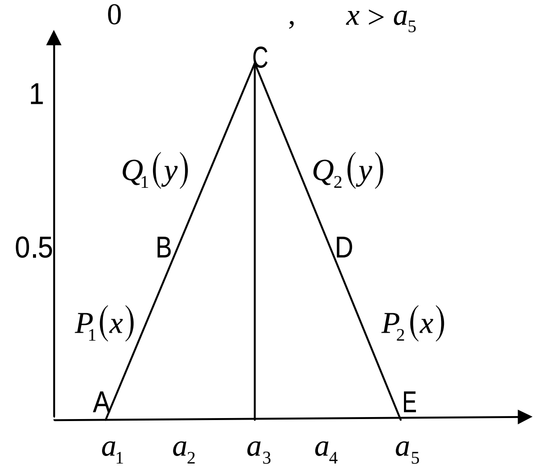
(i) f is piecewise continuous.

(ii) There exists an $x\in$ R such that $f(x)=1$.

(iii) f is convex,(i.e).,if $x_1, x_2 \in R$ and $a\in$[0,1] then $f(\lambda x_1+(1-\lambda)x_2) \geq f(x_1) \wedge f(x_2)$.

2.3 PENTAGON FUZZY NUMBERS: A pentagon fuzzy number $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ where a_1, a_2, a_3, a_4, a_5 are real numbers and its membership is given below.

$$\mu_{\tilde{A}_P}(x) = \begin{cases} 0 & , \quad x < a_1 \\ \frac{1}{2}\left[\frac{x-a_1}{a_2-a_1}\right] & , \quad a_1 \leq x \leq a_2 \\ \frac{1}{2}+\frac{1}{2}\left[\frac{y-a_2}{a_3-a_2}\right] & , \quad a_2 \leq x \leq a_3 \\ 1-\frac{1}{2}\left[\frac{a_4-y}{a_4-a_3}\right] & , \quad a_3 \leq x \leq a_4 \\ \frac{1}{2}\left[\frac{a_5-x}{a_5-a_4}\right] & , \quad a_4 \leq x \leq a_5 \\ 0 & , \quad x > a_5 \end{cases}$$



Graphical representation of a normal pentagon fuzzy number for $x \in$ [0,1]

2.4 DEFINITION: $\tilde{A}_P$ is called a Canonical pentagon fuzzy number if it is a closed and bounded pentagon fuzzy number and its membership function is strictly increasing on the interval $[a_2, a_3]$ and strictly decreasing on the interval $[a_3, a_4]$.

REMARK: Pentagon fuzzy number $\tilde{A}_P$ is the ordered quadruple $(P_1(x), Q_1(y), Q_2(y), P_2(x))$ for $x \in [0,0.5]$ and y$\in$[0.5,1] where,

$$P_1(x) = \frac{1}{2}\left[\frac{x-a_1}{a_2-a_1}\right] \quad , \quad P_2(x) = \frac{1}{2}\left[\frac{a_5-x}{a_5-a_4}\right]$$

$$Q_1(y) = \frac{1}{2}+\frac{1}{2}\left[\frac{y-a_2}{a_3-a_2}\right] \text{ , } Q_2(y) = 1-\frac{1}{2}\left[\frac{a_4-y}{a_4-a_3}\right]$$

2.5 DEFINITION: A positive pentagon fuzzy number $\tilde{A}_P$ is denoted as $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ where a_i's > 0 for all i=1,2,3,4,5 (e.g): $\tilde{A}_P$ = (3,5,7,9,11).

2.6 DEFINITION: A negative pentagon fuzzy number $\tilde{A}_P$ is denoted as $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ where a_i's < 0 for all i=1,2,3,4,5 (e.g): $\tilde{A}_P$ = (-5,-4,-3,-2,-1).

2.7 DEFINITION: Let $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ be two pentagon fuzzy number if $\tilde{A}_P$ is identically equal to $\tilde{B}_P$ only if $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$, $a_4 = b_4$, $a_5 = b_5$.

3. ALPHA CUT: The classical alpha-cut set is the set of elements whose degree of membership in $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ is no less than α it is defined as

$$A_\alpha = \{x \in X / \mu_{\tilde{A}_p}(x) \geq \alpha\}$$

$$= \begin{cases} [P_1(\alpha), P_2(\alpha)] & \text{for } \alpha \in [0,0.5] \\ [Q_1(\alpha), Q_2(\alpha)] & \text{for } \alpha \in [0.5,1] \end{cases}$$

3.1 α -cut OPERATIONS: If we get crisp interval by α -cut operations interval A_α shall be obtained as follows, for all $\alpha \in$ [0,1].

Consider, Q_1(y) = α

$$\frac{1}{2} + \frac{1}{2}\left[\frac{y - a_2}{a_3 - a_2}\right] = \alpha$$

Hence, $Q_1(\alpha) = 2\alpha(a_3 - a_2) + 2a_2 - a_3$

Similarly, Q_2(y) = α

$$Q_2(\alpha) = 2\alpha(a_4 - a_3) - a_4 + 2a_3$$

$$P_1(\alpha) = 2\alpha(a_2 - a_1) + a_1$$

$$P_2(\alpha) = -2\alpha(a_5 - a_4) + a_5$$

Hence,

$$A_\alpha = \begin{cases} [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_5 - a_4) + a_5] & \text{for } \alpha \in [0,0.5] \\ [2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) - a_4 + 2a_3] & \text{for } \alpha \in [0.5,1] \end{cases}$$

3.2 OPERATIONS OF PENTAGON FUZZY NUMBERS: Following are the three operations that can be performed on pentagon fuzzy numbers, suppose $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ are the two pentagon fuzzy numbers then

- Addition: $\tilde{A}_p \oplus \tilde{B}_P = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$
- Subtraction: $\tilde{A}_p (-) \tilde{B}_P = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1)$
- Multiplication: $\tilde{A}_p (*) \tilde{B}_P = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5)$
- Division: $A_{\tilde{P}} / B_{\tilde{P}} = (a_1/b_5, a_2/b_4, a_3/b_3, a_4/b_2, a_5/b_1)$

3.3 A NEW OPERATION FOR ADDITION, SUBTRACTION AND MULTIPLICATION ON CANONICAL PENTAGON FUZZY NUMBER:

ADDITION OF TWO CANONICAL PENTAGON FUZZY NUMBER:

Let $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ be two Canonical pentagon fuzzy numbers for all $\alpha \in [0,1]$. Let us add alpha cuts $\tilde{A}_\alpha$ and $\tilde{B}_\alpha$ of $\tilde{A}_p$ and $\tilde{B}_P$ using internal arithmetic.

$$A_\alpha + B_\alpha =$$

$$\begin{cases} [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_5 - a_4) + a_5] + [2\alpha(b_2 - b_1) + b_1, -2\alpha(b_5 - b_4) + b_5] & \text{for } \alpha \in [0.0.5] \\ [2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) - a_4 + 2a_3] + [2\alpha(b_3 - b_2) + 2b_2 - b_3, 2\alpha(b_4 - b_3) - b_4 + 2b_3] & \text{for } \alpha \in [0.5,1] \end{cases}$$

Let $\tilde{A}_p$ =(1,3,5,7,9) and $\tilde{B}_P$ =(2,4,6,8,10)

$$A_\alpha + B_\alpha = \begin{cases} 8\alpha + 3, -8\alpha + 19 & \alpha \in [0,0.5] \\ 8\alpha + 3, 8\alpha + 7 & \alpha \in [0.5,1] \end{cases}$$

When $\alpha = 0$, $A_0 + B_0 = [3, 19]$

When $\alpha = 0.5$, $A_{0.5} + B_{0.5} = [7, 15]$

When $\alpha = 0.5$, $A_{0.5} + B_{0.5} = [7, 11]$

When $\alpha = 1$, $A_1 + B_1 = [11, 15]$

Hence,

$A_\alpha + B_\alpha = [3,7,11,15,19]$ All the points coincides with the sum of the two Canonical pentagon fuzzy number. Therefore addition of two α -cuts lies within the interval.

3.4 SUBTRACTION OF TWO CANONICAL PENTAGON FUZZY NUMBERS:

$$A_\alpha - B_\alpha = \begin{cases} [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_5 - a_4) + a_5] - [2\alpha(b_2 - b_1) + b_1, -2\alpha(b_5 - b_4) + b_5] \text{ for } \alpha \in [0.0.5] \\ [2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) - a_4 + 2a_3] - [2\alpha(b_3 - b_2) + 2b_2 - b_3, 2\alpha(b_4 - b_3) - b_4 + 2b_3] \text{ for } \alpha \in [0.5,1] \end{cases}$$

Consider $\tilde{A}_p$ =(1,2,3,5,6) and $\tilde{B}_P$ =(2,4,6,10,12)

$$A_\alpha - B_\alpha = \begin{cases} [-2\alpha - 1, 2\alpha - 6] & \text{for } \alpha \in [0,0.5] \\ [-2\alpha - 1, -4\alpha - 1] & \text{for } \alpha \in [0.5,1] \end{cases}$$

$A_0 - B_0 = [-1,-6]$, $A_{0.5} - B_{0.5} = [-2,-5]$

$A_{0.5} - B_{0.5} = [-2,-3]$, $A_1 - B_1 = [-3,-5]$

Hence,

$A_\alpha - B_\alpha$ = [-1,-2,-3,-5,-6] all the points coincides with the differences of two Canonical pentagon fuzzy numbers.

3.5 MULTIPLI CATION OF TWO CANONICAL PENTAGON FUZZY NUMBER :

$$A_\alpha * B_\alpha = \begin{cases} [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_5 - a_4) + a_5] * [2\alpha(b_2 - b_1) + b_1, -2\alpha(b_5 - b_4) + b_5] \text{ for } \alpha \in [0.0.5] \\ [2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) - a_4 + 2a_3] * [2\alpha(b_3 - b_2) + 2b_2 - b_3, 2\alpha(b_4 - b_3) - b_4 + 2b_3] \text{ for } \alpha \in [0.5,1] \end{cases}$$

Consider $\tilde{A}_p$ =(1,2,3,5,6) and $\tilde{B}_P$ =(2,4,6,10,12)

$$A_\alpha * B_\alpha = \begin{cases} [2\alpha + 1, -2\alpha + 6] * [4\alpha + 2, -4\alpha + 12] & \text{for } \alpha \in [0,0.5] \\ [2\alpha + 1, 4\alpha + 1] * [4\alpha + 2, 8\alpha + 2] & \text{for } \alpha \in [0.5,1] \end{cases}$$

$A_0 * B_0 = [2,72]$

$A_{0.5} * B_{0.5} = [8,50]$

$A_{0.5} * B_{0.5} = [8,18]$

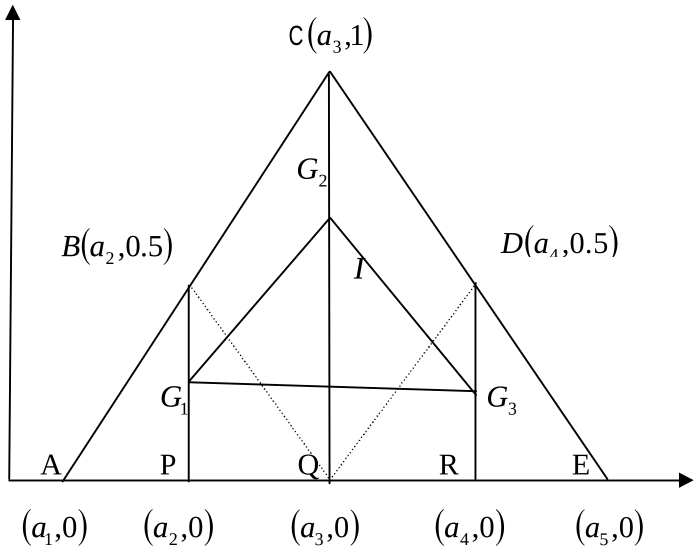
$A_1 * B_1 = [18,50]$

Hence,

$A_\alpha * B_\alpha$ = [2,8,18,50,72] all the points coincides with the product of two Canonical pentagon fuzzy number.

4. RANKING OF PENTAGON FUZZY NUMBERS: An efficient approach for comparing the fuzzy numbers is by the use of a ranking function R: F(R)→R where F(R) is a set of fuzzy numbers defined on set of real numbers which maps each fuzzy number into a real number,

PROPOSED SYSTEM:



(Fig:1) Normalized Pentagon Fuzzy Number.

The centroid of a pentagon fuzzy number is considered to be the balancing point of the pentagon (fig.1). Divide the pentagon into three plane figures. In these three plane first figure is a triangular ABQ, second figure is a square QBCD and third is again a triangle QDE. Let the centroid of the three plane figures be G_1, G_2, G_3 respectively. The Incentre of the centroid G_1, G_2, G_3 is taken as the point of reference to define the ranking of normalized pentagon fuzzy numbers.Consider a normalized pentagon fuzzy number $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ the centroid of the three plane figures are

$G_1 = \{[a_1 + a_2 + a_3]/3,\ 1/6\}$

$G_2 = \{[a_2 + 2a_3 + a_4]/4,\ 1/2\}$

$G_3 = \{[a_3 + a_4 + a_5]/3,\ 1/6]\}$ respectively.

$$I_{\tilde{A}_P} = \left\{ \frac{\alpha_{\tilde{A}_P}\left[\frac{a_1 + a_2 + a_3}{3}\right] + \beta_{\tilde{A}_P}\left[\frac{a_2 + 2a_3 + a_4}{4}\right] + \gamma_{\tilde{A}_P}\left[\frac{a_3 + a_4 + a_5}{3}\right]}{\alpha_{\tilde{A}_P} + \beta_{\tilde{A}_P} + \gamma_{\tilde{A}_P}}, \frac{\alpha_{\tilde{A}_P}\left[\frac{1}{6}\right] + \beta_{\tilde{A}_P}\left[\frac{1}{2}\right] + \gamma_{\tilde{A}_P}\left[\frac{1}{6}\right]}{\alpha_{\tilde{A}_P} + \beta_{\tilde{A}_P} + \gamma_{\tilde{A}_P}} \right\}$$

Where,

$$\alpha_{\tilde{A}_P} = \frac{\sqrt{(a_4 + 4a_5 - 2a_3 - 3a_2)^2 + 8}}{12}, \quad \beta_{\tilde{A}_P} = \frac{\sqrt{(a_1 + a_2 - a_4 - a_5)^2}}{3} \text{ and}$$

$$\gamma_{\tilde{A}_P} = \frac{\sqrt{(4a_1 + a_2 - 2a_3 - 3a_4)^2 + 8}}{12}$$

The rank of two fuzzy numbers $\tilde{A}_p$ and $\tilde{B}_P$ based on the incentre of the centroids is given by the following steps. Let $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ be two normalized pentagon fuzzy numbers then,

STEP: 1 Find $\alpha_{\tilde{A}_P}, \beta_{\tilde{A}_P}, \gamma_{\tilde{A}_P}$ and $\alpha_{\tilde{B}_P}, \beta_{\tilde{B}_P}, \gamma_{\tilde{B}_P}$.

STEP: 2 Find $I_{\tilde{A}_P}(\bar{x}_0, \bar{y}_0)$ and $I_{\tilde{B}_P}(\bar{x}_0, \bar{y}_0)$.**STEP: 3** Find $R_{\tilde{A}_P} = \sqrt{\bar{x}_0^{\,2} + \bar{y}_0^{\,2}}$ and $R_{\tilde{B}_P} = \sqrt{\bar{x}_0^{\,2} + \bar{y}_0^{\,2}}$.

4.1 NUMERICAL EXAMPLE:

Let $\widetilde{A}_p$ =(1,3,5,7,9) and $\widetilde{B}_P$ =(2,4,6,8,10) be two normalized fuzzy numbers then

STEP: 1

$\alpha_{\widetilde{A}_P}$ =4.006, $\beta_{\widetilde{A}_P}$ =4, $\gamma_{\widetilde{A}_P}$ =2.01 and

$\alpha_{\widetilde{B}_P}$ =2.01, $\beta_{\widetilde{B}_P}$ =1.0, $\gamma_{\widetilde{B}_P}$ =2.013

STEP: 2

$I_{\widetilde{A}_P}$ = [4.60 , 0.29] and $I_{\widetilde{B}_P}$ = [6.01 , 0.23]

STEP: 3

$R(\widetilde{A}_p) = \sqrt{(4.60)^2 + (0.29)^2} = 4.6$

$R(\widetilde{B}_P) = \sqrt{(6.01)^2 + (0.23)^2} = 6.01$

So, $R(\widetilde{A}_p) < R(\widetilde{B}_P)$ then $\widetilde{A}_p < \widetilde{B}_P$ similarly, $R(\widetilde{A}_p) > R(\widetilde{B}_P)$ then $\widetilde{A}_p > \widetilde{B}_P$.

REMARK: A natural order exists, for any two pentagon fuzzy numbers $\widetilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\widetilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ we have the following comparison.

$(i)\widetilde{A}_p \approx \widetilde{B}_P \Leftrightarrow R(\widetilde{A}_p) = R(\widetilde{B}_P)$

$(ii)\widetilde{A}_p \geq \widetilde{B}_P \Leftrightarrow R(\widetilde{A}_p) \geq R(\widetilde{B}_P)$

$(iii)\widetilde{A}_p \leq \widetilde{B}_P \Leftrightarrow R(\widetilde{A}_p) \leq R(\widetilde{B}_P)$

CONCLUSION: In this paper pentagon fuzzy number has been newly introduced and α-cut operations using addition, subtraction and multiplication has been discussed. We also introduced a new ranking method for pentagon fuzzy number. The pentagon fuzzy number plays a vital role in solving the problems and also easy to apply in the real life problems.

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