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BASIS TO DETECT THE OBJECT BOUNDRIES OF FETAL BIOMETRY BY SUITABLE WAVELET

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Abstract

Analysis and understanding of fetal ultrasound images have importance to measure fetal gestational age (GA) which is of extreme importance for estimating the date of confinement, estimating the expected delivery date, assessing of fetal size and monitoring of fetal growth.This paper contains a method to find the object boundries of fetal biometry(BPD). One of the most fundamental issues is the detection of object boundries or singularities, Which is often the basis for further processes such as mesurement of anatomical and physiological parameters. The focus of this work involved taking a correlation based approach towards edge detection ,by exploiting some of desirable properties of wavelet analysis.This leads to the possibility of constructing a bank of detectors,consisting of multiple wavelet basis functions of different scales which are optimal for specific types of edges, in order to optimally detect all the edges in an ultrasound(US) image. This work involved developing a set of wavelet function which matches the shape of the ramp and pulse edges.The matching algorithm used focuses on matching the edges in the frequency domain. It was proved that this technique could create matching wavelets applicable at all scales.

Keywords: Wavelets, Signal Processing, Filter Banks, Edge detection, Matching Algorithm,Fetal Boundries.

1 Introduction

Many authors had proposed numerous edge detection algorithm over the years [1][2][3][4][5]. However, which wavelet to be use on fetal biometry remains an open question. Wavelets are especially well suited to edge detection due to their ability to localize transient components in signals and images in a multiscale fashion, such as step, ramp and pulse edges in medical images. Earlier efforts have concluded that the decomposition of a signal in the presence of noise using a wavelet that is matched to the signal would produce a sharper or taller peak in time-scale spaces as compared to standard non-matched wavelets. This paper provides a solution to the problem of finding the appropriate wavelets for the detection of different types of edges. In section 2 we give a general idea of DWT & continous wavelet Transform and Multiresolution analysis. In section 3 we provide the methodology to get the suitable wavelets. Sections 4 & 5 include result and conclusion respectively. The matching algoritm was originally proposed by [6]. His algorithm generates the associated scaling function which in turn satisfies the orthonormal multiresolution analysis criteria.The MRA property had been proved to be robust against noise.

2 Background

This section contains the basics of wavelets like Continous wavelet transform,Discrete Wavelet Transform and multiresolution analysis followed by brief discussion on maximum modulus method for edge detection.

2.1 Continous Wavelet Transform (CWT)

Wavelets are functions that are used to keep track of both time and frequency information about a signal.They can be used to zoom in on short bursts continous signal or zoom out to detect long,slow oscillations.Let $\psi(t)$ denotes the mother wavelet and

$$\psi_{b,a}(t) = a^{\frac{-1}{2}}\psi\left(\frac{t-b}{a}\right) \text{ and } \psi_a(t) = a^{\frac{-1}{2}}\psi\left(\frac{t}{a}\right) \tag{1}$$

Then the 1-D CWT can be defined as:

$$X_{CWT}(b,a) = \langle x(t)\tilde{\psi}_{b,a}(t)\rangle = \int_t x(t)\tilde{\psi}_a^*(t)dt \tag{2}$$

Where t(time),b(traslation) and a(scale) are all continous parameters. Equation(2) can be interpreted as a correlation between the signal $f(t)$ and the mother wavelet.The wavelet coefficient $X_{CWT}(b,a)$ is a function of the translation b,and the scale a. The traslation parameter indicates the location of the wavelet window along the time axis.The scale can be interpreted as the inverse of frequency and it defines how much the mother wavelet would shrink and dilate.According to the equation(1),if the scale increses ,it will cause $\psi_{b,a}(t)$ to dilate.Hence,when a transform takes place ,the signal is correlated with a streched version of $\psi_{b,a}(t)$ and so it will be less effective in picking out sharp spikes in the signal versus when the signal is correlated with a contracted version of $\psi_{b,a}(t)$. Thus ,when a increses, the wavelet has less time resolution.According to Parseval's Theorem, this will translate to more frequency resolution .On the other hand ,if a decreases, the wavelet has higher time resolution, thus lower frequency resolution. Beside its ability to change scale,the freedom to choose the appropriate mother wavelet(or basis function)in the transform allows one to analyze a particular signal with the appropriate basis function.This is one of wavelet's main advantage compared to fourier based analysis, since the basis function is restricted to be an exponential function $e^{-j\omega}$. However,only functions that satisfy the admissibility condition can be considered to be wavelets.Let $\Psi(j\Omega)$ denotes the Fourier transform of $\psi(t)$, the admissibility condition is defined as :

$$C_\psi = \int_\Omega \frac{|\Psi(j\Omega)|^2}{|\Omega|} d\Omega < \infty \tag{3}$$

Equation(3) implies that $\Psi(0) = 0$ in order to make C_ψ a finite number, which implies $\Psi(j\Omega)$ behaves like a bandpass filter.It also implies that all wavelets must oscillate.A wavelet also has to have finite support,which means its oscillation quickly goes down to zero in the time domain.

2.2 Discrete wavelet transform(DWT) and multiresolution analysis

In DWT,there exists a scaling function ϕ and a wavelet function ψ which are used to break up and reconstruct a signal.If the translation and scale parameters are discrete, the scale and translation are 2^{-s} (known as dyadic scale) and $k2^{-s}$, respectively,with k being an integer. It was shown by Mallat [7] that the wavelet transform are both complete and have perfect reconstruction. This allows the transform to perform multiresolution analysis at a fast pace by filter bank implementation.

In multiresolution or multiscale analysis,a signal f(t) belonging to the square integrable space $L^2(\Re)$ can be approximated by a series of functions $f_s(t)$ with increasing accuracy toward the original signal $f(t)$ as s becomes larger.This can be interpreted as a projection of $f(t)$ onto a series of closed subspace $\{\mathbb{V}_s\}_{s\epsilon\mathbb{Z}}$ satisfying the following:

- $0 \leftarrow \cdots \subset V_{-1} \subset V_0 \subset V_1 \subset \cdots \rightarrow L^2$
- if $f(t)\,\epsilon\, V_s \rightarrow f(2t)\,\epsilon\, V_{s+1}$
- $V_{s+1} = W_s \oplus V_s = W_s \oplus W_{s-1} \oplus V_s - 1 = W_s \oplus W_{s-1} \oplus W_{s-2} \oplus \cdots \oplus V_0$ where V_s is called the scaling subspace and W_s is called the wavelet subspace. The above

conditions imply the following:
Let $\phi_{sm} = 2^{s/2}\,\phi(2^s t - m)$ and $\psi_{sm} = 2^{s/2}\psi(2^s t - m)$, then:

- There exists basis function ψ_{sm} of W_s, togather with the basis function ϕ_{sm} of V_s ,forming a basis for V_{s+1}.

- Every basis function ψ_{sm} of W_s is orthogonal to every basis function ϕ_{sm} of V_s under the chosen inner product.

- Time resolustion becomes finer (coarser) in the space V_s as s increses(decreases).

The above conditions can be formulated by the following refinement equations:

$$\begin{aligned} \phi(t) &= 2\textstyle\sum_k h_0[k]\phi(2t-k) \\ \psi(t) &= 2\textstyle\sum_k h_1[k]\phi(2t-k) \end{aligned} \tag{4}$$

from this we get:

$$\begin{aligned} c_s[m] &= \tfrac{\sqrt{2}}{2}\left(h_0[-m] * c_{s+1}[m]\right)_{\downarrow 2} \\ d_s[m] &= \tfrac{\sqrt{2}}{2}\left(h_1[-m] * c_{s+1}[m]\right)_{\downarrow 2} \end{aligned} \tag{5}$$

Where $c_s[m]$ is the scaling coefficient,$d_s[m]$ is the wavelet coefficient, both at scale s, and $h_k[n](k = 0,1)$ are linear filters. As s gets smaller, the frequency resolution becomes finer.The DWT(analysis) corresponds to the repeated application of this two-channel analysis filter bank on the lowpass branch. One of the perfect reconstruction(PR) results, known as the power symmetric conditions states that:

$$|H_o(e^{j\omega})|^2 + |H_o(e^{j(\omega-\pi)})|^2 = 1 \tag{6}$$

It can be shown in a two-channel filter bank such as this one ,$h_1[n]$ is equal to:

$$h_1[n] = -c(-1)^n h_0^*[N-n] \leftrightarrow H_1(z) \;=\; cz^{-N}\,\tilde{H}(-z) \tag{7}$$

Where c is a constant, N is the order of the filter (N+l is the length) and it is odd, $h_o^*[n]$ is the complex conjugate of $h_o[n]$ and $\tilde{H}(z)$ is the paraconjugate of $H(z)$ defined as $\tilde{H}(z) \;=\; H^*\left(\frac{1}{z}^*\right)$ for any z.Since $h_1[n]$ is a function of $h_o[n]$,the PR condition only depends on $H_o(z)$.

2.3 Maximum Modulus of Wavelet

In [7], edges in an image can be found by computing the maximum modulus of the wavelet coefficients:

$$M = \sqrt{(W_s^1 f(x,y))^2 \;+\; (W_s^2 f(x,y))^2} \tag{8}$$

Where the coefficients equal to :

$$\begin{bmatrix} W_s^1 f(x,y) \\ W_s^2 f(x,y) \end{bmatrix} \;=\; s \begin{bmatrix} \frac{\partial}{\partial x}(f * \theta_s)(x,y) \\ \frac{\partial}{\partial y}(f * \theta_s)(x,y) \end{bmatrix} \;=\; s\vec{\nabla}(f * \theta_s)(x,y) \tag{9}$$

$\theta_s(x,y)$ is a smoothing function whose integral over x and y is equal to 1 and converges to 0 at infinity. s is the scale parameter of the function and:

$$\psi^1(x,y) \;=\; \frac{\partial\theta(x,y)}{\partial x} and \psi^2(x,y) \;=\; \frac{\partial\theta(x,y)}{\partial y} \tag{10}$$

In the case where $\theta_s(x,y)$ is a Gaussian, the extrema detection corresponds to the Canny edge detection algorithm.The nonmaximum suppression scheme from Canny [1] can then be used to thin the edges in order to enhance the quality of the edge based image.

3 Methodology

There have been many efforts to construct wavelets that are matched to a particular signal.Tewfik find matching wavelet [8] by matching them to signal in the time-domain. The problem is the multiresolution analysis is only valid up to a certain scale. Others had tried to refine the methods in

[8] but the same scale restriction still holds. Chapa [6] had developed a matching wavelet algorithm that matches the signal and wavelet in the frequency domain. This was carried out by matching the energy of the spectrum magnitude and the group delay separately within the passband of the spectrum, under the minimum squared error criteria. To guarantee the end result would satisfy the OMRA criteria across all scales, the algorithm first matched the wavelet's spectrum to the signal's, then worked bachward through the refinement equation(4) and the power symmetric criteria(6) to derive the corresponding equation for the scaling function:

$$|\Phi(\omega)|^2 = \sum_{s=1}^{\infty} |\Psi(2^s\omega)|^2 \tag{11}$$

The discrete version of (11) is:

$$|(\Phi\left(\frac{\pi k}{2^l}\right)|^2 = \sum_{p=0}^{l} |\Psi\left(\frac{2\pi k}{2^p}\right)|^2\,,\ fork \neq 0 \tag{12}$$

where l is a constant chosen to be 4 and ω is sampled at $\frac{k\pi}{2^l}$.

3.1 Amplitude Matching

It was proved by Chapa that the wavelet (and scaling function) would have finite support in the frequency domain (finite support also in the time domain because the limit of the signal goes to 0 as time goes to infinity):

$$\begin{aligned} |\Phi(\omega)| &= 1, |\omega| < \pi - \alpha \\ |\Phi(\omega)|^2 + |\Phi(2\pi - \omega)|^2 &= 1,\ \pi - \alpha < |\omega| < \pi + \alpha \end{aligned} \tag{13}$$

$$|\Psi(\omega)| = \begin{cases} 0 & \text{for} \quad 0 \leq |\omega| < \pi - \alpha \\ |\Phi(2\pi - \omega)| & \text{for} \quad \pi - \alpha \leq |\omega| < \pi + \alpha \\ 1 & \text{for} \quad \pi + \alpha \leq |\omega| < 2\pi - 2\alpha \\ |\Phi(\frac{\omega}{2})| & \text{for} \quad 2\pi - 2\alpha \leq |\omega| < 2\pi + 2\alpha \\ 0 & \text{for} \quad 2\pi + 2\alpha \leq |\omega| \end{cases} \tag{14}$$

Where $0 \leq \alpha \leq \frac{\pi}{3}$.
The orthogonality condition of the scaling function, i.e.
$\langle \phi_{sk}(t), \phi_{sm}(t) \rangle = \delta(k-m)$,
gives the Poisson summation equation:

$$\sum_{m} |\Phi(\omega + 2\pi m)|^2 = 1 \tag{15}$$

If we let $Y(k) = |\Psi(k\Delta\omega)|^2$, $W = |F(k\Delta\omega)|^2, \Delta\omega = \frac{\pi}{2^l}$, and combining (12) and (15),we get:

$$\begin{gathered} \sum_{p=0}^{l} \sum_{m} Y\left(\frac{2^l}{2^p}(k + 2^{l+1}m)\right) = 1 \\ Where\, 2^{l-1}/3 < |\frac{2^l}{2^p}(k + 2^{l+1}m)| < 2^{l+2}/3 \end{gathered} \tag{16}$$

The matching criteria between the energy of the wavelet's spectrum magnitude and that of the signal is done by minimizing the square of their error:

$$E = \frac{(\mathbf{W} - a\mathbf{Y})^T(\mathbf{W} - \mathbf{Y})}{\mathbf{W}^T\mathbf{W}} \tag{17}$$

Minimizing E using the Lagrange multiplier, Y and a becomes:

$$\mathbf{Y} = \frac{1}{a}\mathbf{W} + \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}(\mathbf{1} - \frac{1}{a}\mathbf{A}\mathbf{W}) \tag{18}$$

$$a = \frac{\mathbf{1}^T(\mathbf{A}\mathbf{A}^T)^{-1}\mathbf{A}\mathbf{W}}{\mathbf{1}^T(\mathbf{A}\mathbf{A}^T)^{-1}\mathbf{1}} \tag{19}$$

$\mathbf{A}$ is a $Lx2^l$ matrix with values 0,1 or 2 and was used as a constraint ($\mathbf{AY} = \mathbf{l}$) to solve for $\mathbf{Y}$ and a in the optimization problem in (17).

3.2 Matching Phase

Similer to the magnitude, the phase of the matching wavelet was determined by minimizing the squared error between the phase of the signal and the matching wavelet within the passband of the spectrum. However, because of the discontinuity of the phase, it was more convenient to deal with the group delay, calculated as a 1^{st} order difference of the phase. This error was also weighted by the result from the magnitude:

$$\begin{aligned} \gamma &= \textstyle\sum_{n=-N/2}^{N/2-l}[\Omega(n)(\Gamma_F(n) - \Gamma_\psi(n))]^2 \\ \Omega(n) &= Y(n)/\sum Y(n) \end{aligned} \tag{20}$$

Where N is the number of DFT points, $\Gamma_F(n)$ and $\Gamma_\psi(n)$ are the group delay of the signal and matching wavelet, respectively.
To obtain the expression for $\Gamma_\psi(n)$, it is necessary to apply the PR conditions (7) for the h_1 filter.let:

$$h_1[n] = (-1)^{n+1}h[1-k] \Leftrightarrow H_1(\omega) = e^{-j\omega}H_0^*(\omega+\pi) \tag{21}$$

and take the Fourier transform of (4):

$$\begin{aligned} \Phi(\omega) &= H_0\left(\tfrac{\omega}{2}\right)\Phi\left(\tfrac{\omega}{2}\right) \\ \Psi(\omega) &= H_1\left(\tfrac{\omega}{2}\right)\Phi\left(\tfrac{\omega}{2}\right) \end{aligned} \tag{22}$$

combining (21) and (22), we obtain the expression for Φ and Ψ in terms of $H_0(\omega)$:

$$\begin{aligned} \Phi(\omega) &= \textstyle\prod_{s=1}^{\infty} H_0\left(\tfrac{\omega}{2^s}\right) \\ \Psi(\omega) &= e^{-j(\omega/2)}H_0^*\left(\tfrac{\omega}{2}+\pi\right)\textstyle\prod_{s=2}^{\infty} H_0\left(\tfrac{\omega}{2^s}\right) \end{aligned} \tag{23}$$

The phase θ can be obtained easily from (23). Taking the derivative of the phase(this is actually the negative group delay), we obtain the following expression for the group delay:

$$\Lambda_\Phi(\omega) = \frac{d\theta_\Phi}{d\omega} = \sum_{s=1}^{\infty} 2^{-s}\lambda\left(\frac{\omega}{2^s}\right) \tag{24}$$

$\Lambda_\Psi(\omega) = \frac{d\theta_\Psi}{d\omega} = -\frac{1}{2} - \frac{1}{2}\lambda\left(\frac{\omega}{2}+\pi\right) + \sum_{s=2}^{\infty} 2^{-s}\lambda\left(\frac{\omega}{2^s}\right)$
and
$\Gamma_\Psi(\omega) = \Lambda(\omega) + \frac{1}{2}$

with $\lambda(\omega) = \frac{d\theta_{H_0}}{d\omega}$.
In order to find both $\Lambda's$, there is a need to model λ because we do not have an expression for $H_0(\omega)$. since $H_0(\omega)$ was assumed to be real, its phase is odd, so the group delay is even. Because $H_0(\omega)$ is the frequency response of a digital filter, it has a period of 2π. Therefore, one period λ

can be modeled by a polynomial with even power with a "rect" function to filter out $|\omega| > \frac{\pi}{2}$. Equation(25) shows the model and (26) is the discrete form with $\Delta\omega = \frac{2\pi}{T}N$ = number of samples in $F(n\Delta\omega)$. $-N/2 \leq n < N/2$, and p=N/T.

$$\begin{aligned} \lambda(\omega) &= \textstyle\sum_k \sum_{r=0}^{R/2} C_r(\omega - 2\pi l)^{2r} \prod\left(\frac{\omega-2\pi k}{2\pi}\right) \\ \text{with } \textstyle\prod(\omega) &= \left\{ \begin{array}{ll} 1 & -1/2 \leq \omega < 1/2 \\ 0 & else \end{array} \right\} \end{aligned} \tag{25}$$

$$\begin{aligned} \lambda(n) &= Bc = \textstyle\sum_{r=0}^{R/2} c_r \sum_{k=-p/2}^{p/2-1}(n-kT)^{2r} \prod\left(\frac{n-kT}{T}\right) \\ \text{with } b_{n,r} &= \textstyle\sum_{k=-p/2}^{p/2-1}(n-kT)^{2r} \prod\left(\frac{n-kT}{T}\right) \end{aligned} \tag{26}$$

Combining (24-26), $\Gamma_\Psi(\omega)$ and $\Gamma_\Phi(\omega)$ becomes:

$$\begin{aligned} \Gamma_\Psi &= D_\Psi c = -1/2B_{\frac{q+T}{2}} + \textstyle\sum_{m=2}^{\infty} 2^{-m} B_{\frac{q}{2^m}} \\ \Gamma_\Phi &= D_\Phi c = \textstyle\sum_{m=1}^{\infty} 2^{-m} B_{\frac{q}{2^{-m}}} \end{aligned} \tag{27}$$

With all the elements in place, we used equation(20) to find an optimal value for C by solving $\Delta_c \gamma = 0$;

$$\hat{C} = (\bar{D}_\Psi^T \bar{D}_\Psi)^{-1} \bar{D}_\Psi^T \bar{\Gamma}_F \tag{28}$$

With $\bar{\Gamma}_F$ = nonzero values of $(\Omega(n)\Gamma_F(n))$ and $\bar{D}_F$ = corresponding nonzero values of $(\Omega(n)d_{n,r})$

3.3 Edge model

Our focus is to find matching wavelet for both ramp and edge models. According to [5], ramp edge could be modeled by :

$$\begin{aligned} e(x) &= 1 - \tfrac{1}{2}e^{-sx} \text{ for } x \geq 0 \\ e(x) &= \tfrac{1}{2}e^{sx} \text{ for } x \leq 0 \end{aligned} \tag{29}$$

where s was a positive constant used to control the scale. In this case, we used a signal of length 1024 with s set to 1. From equation(3), it was clear that the ramp edge cannot be categorized as a wavelet and modification needs to be made to force the model to reassemble a wavelet. This was done by applying a Hamming window to the ramp model, forcing both tail ends to decrease to zero.With this modification, the matching process could be carried out. A 256-point DFT was taken and the square of the spectrum magnitude would be matched to a particular wavelet using the algorithm outlined above.

The model for the pulse edge was created by convolving a box function with a Gaussian of standard deviation sigma. Figure(1) shows the pulse edge when σ is equal to 1. A signal of length 256 was used. A 256-point DFT was taken and the square of its spectrum magnitude was matched.

A whole set of ramp and pulse edge models was then be created with Gaussian and Poisson noise added to them. The same procedure was then carried out to generate the spectrum for the matching step.

The following parameters were set before the matching process could begin :l=4,M=10,R=16. The first one was used to control the magnitude matching while the phase was controlled by the last two. The parameter M, is the number of terms in the summation in equation (27). α was an adjustable parameter used to minimize the MSE.

A library of matching wavelets was then be created for the online edge detection process.This library would be searched to see which wavelet gives the highest modulus maxima when convolved with the edge signal. If this value is above some threshold, it will be labeled as an edge. Since the wavelets developed are all in 1-D, a Gaussian filter would be used in conjunction with the wavelet to act as a direction guide to locate the direction that is normal to the edge before they are processed by the wavelet.

4 Results

In order to test the matching wavelets' ability to detect edges, a 1-D horizontal profile of the BPD ultrasound image from Figure(1) was taken with its location illustrated by the horizontal line running across

Figure(1). The intensity of the pixels was plotted in Figure(1). Both the pulse edge was taken from this profile, Figure(1), for testing. The result is illustrated in Figure(2), which were obtained by convolving the pulse edge with its corresponding wavelet. According to [5], the edges can be detected by the maximum value of the convolution result, the pulse edge can clearly be detected,however the ramp edge can not be detected due to absentism of ramp in our signal. Since the peaks on the pulse edge are at 45 and at 210, after convolution,with our wavelet at scale 5 the peaks are at 46 and at 211.
matching wavelet algorithm can be used to generate 2-D multiscale filters with directional properties that can be used to detect the edges,thereby shunning the use of the Gaussian filter completely.

5 Conclusion

A wavelet based edge detection algorithm had been developed based on the theory of multiresolution.Through the use of matching wavelet algorithm, we can find the wavelet that is closely correlated with the edge signal. We strongly believed that a 2-D filter with directional and shift invarience properties could be developed using the existing algorithm.

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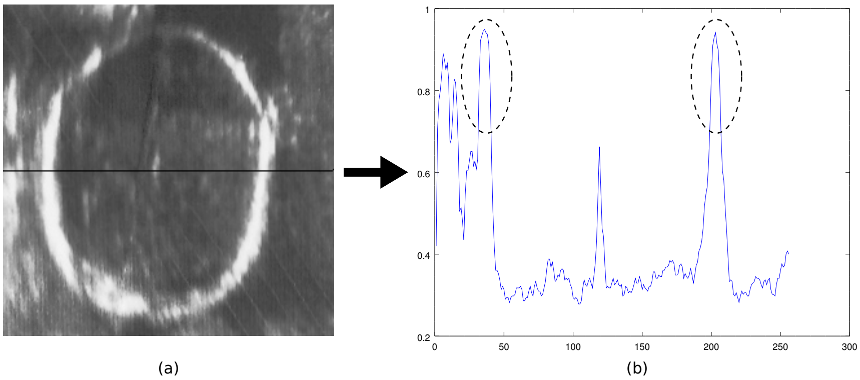

Figure 1: 1D Profile of fetal

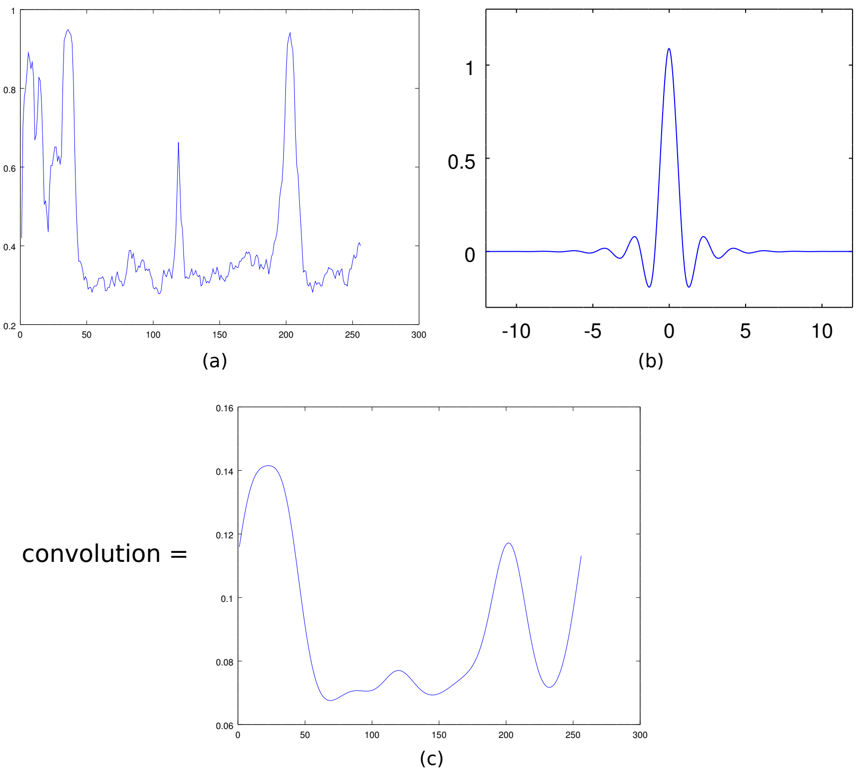

Figure 2: Convolution of signal with gauss wavelet

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