

# On a unified integral formula involving the product of multivariable

## Aleph-function and multivariable I-function with applications I

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### ABSTRACT

In this paper we first evaluate a new unified finite integral involving products of multivariable Aleph-function, multivariable I-function, general class of multivariable polynomials and the generalized hypergeometric function. Next, we make use of the results given by Orr and Bailey in establishing three theorems. On account of most general nature of the functions and their arguments occurring in our main findings, several new results follow as their simple special cases. The present study thus provides interesting unifications and extensions of a number of integrals.

Keywords: Multivariable Aleph-function, class of multivariable polynomials, generalized hypergeometric function, finite integral, multivariable I-function, multivariable H-function..

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### 1. Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [2] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

$$\text{We define : } \aleph(z_1, \dots, z_r) = \aleph_{P_i, Q_i, \tau_i; R: P_i(1), Q_i(1), \tau_i(1); R(1); \dots; P_i(r), Q_i(r), \tau_i(r); R(r)}^{0, N: M_1, N_1, \dots, M_r, N_r} \left( \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix} \right)$$

$$[(a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)})_{1, N_1}], [\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \dots, \alpha_{ji}^{(r)})_{N_1+1, P_i}] :$$

$$\dots, [\tau_i(b_{ji}; \beta_{ji}^{(1)}, \dots, \beta_{ji}^{(r)})_{M_i+1, Q_i}] :$$

$$[(c_j^{(1)}); \gamma_j^{(1)}]_{1, N_1}, [\tau_i(c_{ji}^{(1)}; \gamma_{ji}^{(1)})_{N_1+1, P_i(1)}]; \dots; [(c_j^{(r)}); \gamma_j^{(r)}]_{1, N_r}, [\tau_i(c_{ji}^{(r)}; \gamma_{ji}^{(r)})_{N_r+1, P_i(r)}]$$

$$[(d_j^{(1)}); \delta_j^{(1)}]_{1, M_1}, [\tau_i(d_{ji}^{(1)}; \delta_{ji}^{(1)})_{M_1+1, Q_i(1)}]; \dots; [(d_j^{(r)}); \delta_j^{(r)}]_{1, M_r}, [\tau_i(d_{ji}^{(r)}; \delta_{ji}^{(r)})_{M_r+1, Q_i(r)}]$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \psi(s_1, \dots, s_r) \prod_{k=1}^r \theta_k(s_k) y_k^{s_k} ds_1 \dots ds_r \quad (1.1)$$

with  $\omega = \sqrt{-1}$

$$\psi(s_1, \dots, s_r) = \frac{\prod_{j=1}^N \Gamma(1 - a_j + \sum_{k=1}^r \alpha_j^{(k)} s_k)}{\sum_{i=1}^R [\tau_i \prod_{j=N+1}^{P_i} \Gamma(a_{ji} - \sum_{k=1}^r \alpha_{ji}^{(k)} s_k) \prod_{j=1}^{Q_i} \Gamma(1 - b_{ji} + \sum_{k=1}^r \beta_{ji}^{(k)} s_k)]} \quad (1.2)$$

$$\text{and } \theta_k(s_k) = \frac{\prod_{j=1}^{M_k} \Gamma(d_j^{(k)} - \delta_j^{(k)} s_k) \prod_{j=1}^{N_k} \Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}} [\tau_{i^{(k)}} \prod_{j=M_k+1}^{Q_{i^{(k)}}} \Gamma(1 - d_{ji^{(k)}}^{(k)} + \delta_{ji^{(k)}}^{(k)} s_k) \prod_{j=N_k+1}^{P_{i^{(k)}}} \Gamma(c_{ji^{(k)}}^{(k)} - \gamma_{ji^{(k)}}^{(k)} s_k)]} \quad (1.3)$$

Suppose , as usual , that the parameters

$$b_j, j = 1, \dots, Q; a_j, j = 1, \dots, P;$$

$$c_{ji^{(k)}}^{(k)}, j = n_k + 1, \dots, P_{i^{(k)}}; c_j^{(k)}, j = 1, \dots, N_k;$$

$$d_{ji^{(k)}}^{(k)}, j = M_k + 1, \dots, Q_{i^{(k)}}; d_j^{(k)}, j = 1, \dots, M_k;$$

with  $k = 1 \dots, r, i = 1, \dots, R, i^{(k)} = 1, \dots, R^{(k)}$

are complex numbers , and the  $\alpha' s, \beta' s, \gamma' s$  and  $\delta' s$  are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^N \alpha_j^{(k)} + \tau_i \sum_{j=N+1}^{P_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{N_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{P_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{Q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{M_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leq 0 \quad (1.4)$$

The reals numbers  $\tau_i$  are positives for  $i = 1$  to  $R$  ,  $\tau_{i^{(k)}}$  are positives for  $i^{(k)} = 1$  to  $R^{(k)}$

The contour  $L_k$  is in the  $s_k$ -p lane and run from  $\sigma - i\infty$  to  $\sigma + i\infty$  where  $\sigma$  is a real number with loop , if necessary , ensure that the poles of  $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$  with  $j = 1$  to  $m_k$  are separated from those of  $\Gamma(1 - a_j + \sum_{i=1}^r \alpha_j^{(k)} s_k)$  with  $j = 1$  to  $N$  and  $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$  with  $j = 1$  to  $N_k$  to the left of the contour  $L_k$  . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi, \text{ where}$$

$$A_i^{(k)} = \sum_{j=1}^N \alpha_j^{(k)} - \tau_i \sum_{j=N+1}^{P_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{Q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{N_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} + \sum_{j=1}^{M_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1, \dots, r, i = 1, \dots, R, i^{(k)} = 1, \dots, R^{(k)} \quad (1.5)$$

The complex numbers  $z_i$  are not zero. Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \dots, z_r) = O(|z_1|^{\alpha_1}, \dots, |z_r|^{\alpha_r}), \max(|z_1|, \dots, |z_r|) \rightarrow 0$$

$$\aleph(z_1, \dots, z_r) = O(|z_1|^{\beta_1}, \dots, |z_r|^{\beta_r}), \min(|z_1|, \dots, |z_r|) \rightarrow \infty$$

where, with  $k = 1, \dots, r : \alpha_k = \min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \dots, M_k$  and

$$\beta_k = \max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \dots, N_k$$

Serie representation of Aleph-function of several variables is given by

$$\begin{aligned} \aleph(y_1, \dots, y_r) = & \sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{M_1} \dots \sum_{g_r=0}^{M_r} \frac{(-)^{G_1+\dots+G_r}}{\delta_{g_1} G_1! \dots \delta_{g_r} G_r!} \psi(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) \\ & \times \theta_1(\eta_{G_1, g_1}) \dots \theta_r(\eta_{G_r, g_r}) y_1^{-\eta_{G_1, g_1}} \dots y_r^{-\eta_{G_r, g_r}} \end{aligned} \quad (1.6)$$

Where  $\psi(\cdot, \dots, \cdot), \theta_i(\cdot), i = 1, \dots, r$  are given respectively in (1.2), (1.3) and

$$\eta_{G_1, g_1} = \frac{d_{g_1}^{(1)} + G_1}{\delta_{g_1}^{(1)}}, \dots, \eta_{G_r, g_r} = \frac{d_{g_r}^{(r)} + G_r}{\delta_{g_r}^{(r)}}$$

$$\text{which is valid under the conditions } \delta_{g_i}^{(i)}[d_j^i + p_i] \neq \delta_j^{(i)}[d_{g_i}^i + G_i] \quad (1.7)$$

$$\text{for } j \neq M_i, M_i = 1, \dots, \eta_{G_i, g_i}; P_i, N_i = 0, 1, 2, \dots; y_i \neq 0, i = 1, \dots, r \quad (1.8)$$

In the document , we will note :

$$G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) = \phi(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) \theta_1(\eta_{G_1, g_1}) \dots \theta_r(\eta_{G_r, g_r}) \quad (1.9)$$

where  $\phi(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}), \theta_1(\eta_{G_1, g_1}), \dots, \theta_r(\eta_{G_r, g_r})$  are given respectively in (1.2) and (1.3)

$$\text{We will note the Aleph-function of r variables } \aleph_{u:w}^{0,N:v} \left( \begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_r \end{matrix} \right) \quad (1.10)$$

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$\begin{aligned} I(z_1, z_2, \dots, z_s) = & I_{p_2, q_2, p_3, q_3; \dots; p_s, q_s: p', q'; \dots; p^{(s)}, q^{(s)}}^{0, n_2; 0, n_3; \dots; 0, n_r: m', n'; \dots; m^{(s)}, n^{(s)}} \left( \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_s \end{matrix} \middle| \begin{matrix} (a_{2j}; \alpha'_{2j}, \alpha''_{2j})_{1, p_2}; \dots; \\ (b_{2j}; \beta'_{2j}, \beta''_{2j})_{1, q_2}; \dots; \end{matrix} \right. \\ & \left. (a_{sj}; \alpha'_{sj}, \dots, \alpha_{sj}^{(s)})_{1, p_s} : (a'_j, \alpha'_j)_{1, p'}; \dots; (a_j^{(s)}, \alpha_j^{(s)})_{1, p^{(s)}} \right) \\ & (b_{sj}; \beta'_{sj}, \dots, \beta_{sj}^{(s)})_{1, q_s} : (b'_j, \beta'_j)_{1, q'}; \dots; (b_j^{(s)}, \beta_j^{(s)})_{1, q^{(s)}} \end{aligned} \quad (1.11)$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L_1} \cdots \int_{L_s} \xi(t_1, \dots, t_s) \prod_{i=1}^s \phi_i(t_i) z_i^{t_i} dt_1 \cdots dt_s \quad (1.12)$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z_i| < \frac{1}{2} \Omega_i \pi, \text{ where}$$

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left( \sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) + \cdots +$$

$$\left( \sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left( \sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \quad (1.13)$$

where  $i = 1, \dots, s$

The complex numbers  $z_i$  are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the asymptotic expansion in the following convenient form :

$$I(z_1, \dots, z_s) = O(|z_1|^{\alpha'_1}, \dots, |z_s|^{\alpha'_s}), \max(|z_1|, \dots, |z_s|) \rightarrow 0$$

$$I(z_1, \dots, z_s) = O(|z_1|^{\beta'_1}, \dots, |z_s|^{\beta'_s}), \min(|z_1|, \dots, |z_s|) \rightarrow \infty$$

where, with  $k = 1, \dots, s : \alpha'_k = \min[Re(b_j^{(k)})/\beta_j^{(k)}], j = 1, \dots, m_k$  and

$$\beta'_k = \max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)}), j = 1, \dots, n_k]$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \dots; p_{s-1}, q_{s-1}; V = 0, n_2; 0, n_3; \dots; 0, n_{s-1} \quad (1.14)$$

$$W = (p', q'); \dots; (p^{(s)}, q^{(s)}); X = (m', n'); \dots; (m^{(s)}, n^{(s)}) \quad (1.15)$$

$$A = (a_{2k}, \alpha'_{2k}, \alpha''_{2k}); \dots; (a_{(s-1)k}, \alpha'_{(s-1)k}, \alpha''_{(s-1)k}, \dots, \alpha_{(s-1)k}^{(s-1)}) \quad (1.16)$$

$$B = (b_{2k}, \beta'_{2k}, \beta''_{2k}); \dots; (b_{(s-1)k}, \beta'_{(s-1)k}, \beta''_{(s-1)k}, \dots, \beta_{(s-1)k}^{(s-1)}) \quad (1.17)$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{sk}, \alpha''_{sk}, \dots, \alpha_{sk}^s) : \mathfrak{B} = (b_{sk}; \beta'_{sk}, \beta''_{sk}, \dots, \beta_{sk}^s) \quad (1.18)$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \dots; (a_k^{(s)}, \alpha_k^{(s)})_{1,p^{(s)}}; B' = (b'_k, \beta'_k)_{1,q'}; \dots; (b_k^{(s)}, \beta_k^{(s)})_{1,q^{(s)}} \quad (1.19)$$

The multivariable I-function write :

$$I(z_1, \dots, z_s) = I_{U:p_s, q_s; W}^{V; 0, n_s; X} \left( \begin{array}{c|c} z_1 & A; \mathfrak{A}; A' \\ \cdot & \\ \cdot & \\ \cdot & \\ z_s & B; \mathfrak{B}; B' \end{array} \right) \quad (1.20)$$

The generalized polynomials defined by Srivastava [4], is given in the following manner :

$$S_{N'_1, \dots, N'_t}^{M'_1, \dots, M'_t} [y_1, \dots, y_t] = \sum_{K_1=0}^{[N'_1/M'_1]} \dots \sum_{K_t=0}^{[N'_t/M'_t]} \frac{(-N'_1)_{M'_1 K_1}}{K_1!} \dots \frac{(-N'_t)_{M'_t K_t}}{K_t!} A[N'_1, K_1; \dots; N'_t, K_t] y_1^{K_1} \dots y_t^{K_t} \quad (1.21)$$

Where  $M'_1, \dots, M'_t$  are arbitrary positive integers and the coefficients  $A[N'_1, K_1; \dots; N'_t, K_t]$  are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N'_1)_{M'_1 K_1}}{K_1!} \dots \frac{(-N'_t)_{M'_t K_t}}{K_t!} A[N'_1, K_1; \dots; N'_t, K_t] \quad (1.22)$$

Srivastava and Garg introduced and defined a general class of multivariable polynomials [5] as follows

$$S_E^{F_1, \dots, F_v} [z_1, \dots, z_v] = \sum_{L_1, \dots, L_v=0}^{F_1 L_1 + \dots + F_v L_v \leq E} (-E)_{F_1 L_1 + \dots + F_v L_v} B(E; L_1, \dots, L_v) \frac{z_1^{L_1} \dots z_v^{L_v}}{L_1! \dots L_v!} \quad (1.23)$$

The generalized hypergeometric serie is defined as follows.

$${}_pF_q(y) = \sum_{s'=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'}} y^{s'} \quad (1.24)$$

where  $[(a_p)]_{s'} = (a_1)_{s'} \dots (a_p)_{s'}; [(b_q)]_{s'} = (b_1)_{s'} \dots (b_q)_{s'}$ . The serie (1.24) converge if  $p \leq q$  and  $|y| < 1$

In the document , we note :

$$G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) = \phi(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) \theta_1(\eta_{G_1, g_1}) \dots \theta_r(\eta_{G_r, g_r}) \quad (1.25)$$

$$A_1 = \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \dots \frac{(-N_u)_{\mathfrak{M}_u K_u}}{K_u!} A[N_1, K_1; \dots; N_u, K_u] \quad (1.26)$$

$$B_1 = \frac{(-E)_{F_1 L_1 + \dots + F_v L_v} B(E; L_1, \dots, L_v)}{L_1! \dots L_v!} \quad (1.27)$$

## 2. Main integral

$$\begin{aligned}
& \int_0^a x^{\rho-1} (a-x)^{\sigma-1} {}_pF_q\left((a_p); (b_q); bx^\eta(a-x)^\lambda\right) S_{N_1, \dots, N_u}^{\mathfrak{M}_1, \dots, \mathfrak{M}_u} \begin{pmatrix} y_1 x^{e_1} (a-x)^{f_1} \\ \vdots \\ y_u x^{e_u} (a-x)^{f_u} \end{pmatrix} \\
& S_E^{F_1, \dots, F_v} \begin{pmatrix} x_1 x^{g_1} (a-x)^{h_1} \\ \vdots \\ x_v x^{g_v} (a-x)^{h_v} \end{pmatrix} \aleph \begin{pmatrix} z'_1 x^{c_1} (a-x)^{d_1} \\ \vdots \\ z'_r x^{c_r} (a-x)^{d_r} \end{pmatrix} I \begin{pmatrix} z_s x^{\gamma_1} (a-x)^{\delta_1} \\ \vdots \\ z_s x^{\gamma_s} (a-x)^{\delta_s} \end{pmatrix} dx \\
& = \sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \dots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \dots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \dots, L_v=0}^{F_1 L_1 + \dots + F_v L_v \leq E} \sum_{s'=0}^{\infty} \frac{[(a_p)]_{s'} A_1 B_1 (-)^{G_1 + \dots + G_r}}{[(b_q)]_{s'} s'! \delta_{g_1} G_1! \dots \delta_{g_r} G_r!} \\
& G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) y_1^{K_1} \dots y_u^{K_u} x_1^{L_1} \dots x_v^{L_v} z'_1{}^{\eta_{G_1, g_1}} \dots z'_r{}^{\eta_{G_r, g_r}} b^{s'} \\
& a^{\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^u (e_i+f_i)K_i+\sum_{i=1}^r (c_i+d_i)\eta_{G_i, g_i}+\sum_{i=1}^v (g_i+h_i)L_i} I_{U:p_s+2, q_s+1; W}^{V; 0, n_s+2; X} \left( \begin{array}{c|c} z_1 a^{\gamma_1+\delta_1} & \text{A} ; \\ \cdot & \cdot \\ \cdot & \cdot \\ z_s a^{\gamma_s+\delta_s} & \text{B} ; \end{array} \right. \\
& (1-\rho-\eta s' - \sum_{i=1}^r c_i \eta_{G_i, g_i} - \sum_{i=1}^u e_i K_i - \sum_{i=1}^v g_i L_i; \gamma_1, \dots, \gamma_s), \\
& (-\rho-\sigma-(\eta+\lambda)s' - \sum_{i=1}^r (c_i+d_i)\eta_{G_i, g_i} - \sum_{i=1}^u K_i(e_i+f_i) - \sum_{i=1}^v L_i(g_i+h_i); \gamma_1+\delta_1, \dots, \gamma_s+\delta_s), \\
& (-\sigma-\lambda s' - \sum_{i=1}^r d_i \eta_{G_i, g_i} - \sum_{i=1}^u f_i K_i - \sum_{i=1}^v h_i L_i; \delta_1, \dots, \delta_s), \mathfrak{A} : A' \Bigg) \\
& \qquad \qquad \qquad \mathfrak{B} : \text{B} \Bigg) \tag{2.1}
\end{aligned}$$

where  $G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r})$ ,  $A_1, B_1$  are defined by (1.25), (1.26) respectively.

Provided that

a)  $\min(e_i, f_i, c_j, d_j, g_k, h_k, \gamma_l, \delta_l, \rho, \sigma) \geq 0$ , (not all zero simultaneously) with  $i = 1, \dots, u; j = 1, \dots, r$

$k = 1, \dots, v$  and  $l = 1, \dots, s$

$$\text{b) } Re[\rho + \sum_{i=1}^r c_i \min_{1 \leq j \leq M_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^s \gamma_i \min_{1 \leq j \leq m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$$

$$c) \operatorname{Re}[1 + \sigma + \sum_{i=1}^r d_i \min_{1 \leq j \leq M_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^s \delta_i \min_{1 \leq j \leq m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > 0$$

$$d) |\arg z'_k| < \frac{1}{2} A_i^{(k)} \pi, \text{ where } A_i^{(k)} \text{ is defined by (1.5); } i = 1, \dots, r$$

$$e) |\arg z_k| < \frac{1}{2} \Omega_i \pi, \text{ where } \Omega_i \text{ is defined by (1.11); } i = 1, \dots, s$$

$$\mathbf{Proof} : \text{Let } M\{\} = \frac{1}{(2\pi\omega)^s} \int_{L_1} \cdots \int_{L_s} \xi(t_1, \dots, t_s) \prod_{k=1}^s \phi_k(t_k) \{\} \quad (2.2)$$

To prove (2.1), first we express the Aleph-function of r variables, two general class of polynomials of several variables, the generalized hypergeometric function in form of serie with the help of (1.6), (1.22), (1.23) and (1.24) respectively. Interchanging the order of summations and integration wich is possible under the stated conditions, we obtain.

$$\sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \dots, L_v=0}^{F_1 L_1 + \cdots + F_v L_v \leq E} \sum_{s'=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'} s'!} A_1 B_1 \frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!}$$

$$G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) y_1^{K_1} \cdots y_u^{K_u} x_1^{L_1} \cdots x_v^{L_v} z_1'^{\eta_{G_1, g_1}} \cdots z_r'^{\eta_{G_r, g_r}} b^{s'}$$

$$\int_0^a x^{\rho + \eta s' + \sum_{i=1}^u K_i e_i + \sum_{i=1}^r c_i \eta_{G_i, g_i} + \sum_{i=1}^v g_i L_i - 1} (a - x)^{\sigma + \lambda s' + \sum_{i=1}^u K_i f_i + \sum_{i=1}^r d_i \eta_{G_i, g_i} + \sum_{i=1}^v h_i L_i - 1} \\ I \left( \begin{matrix} z_s x^{\gamma_1} (a - x)^{\delta_1} \\ \vdots \\ z_s x^{\gamma_s} (a - x)^{\delta_s} \end{matrix} \right) dx \quad (2.3)$$

Now expressing the I-function of s-variables in terms of Mellin-Barnes contour integrals and changing the order of integrations ,we get

$$\sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \dots, L_v=0}^{F_1 L_1 + \cdots + F_v L_v \leq E} \sum_{s'=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'} s'!} A_1 B_1 \frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!}$$

$$G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) y_1^{K_1} \cdots y_u^{K_u} x_1^{L_1} \cdots x_v^{L_v} z_1'^{\eta_{G_1, g_1}} \cdots z_r'^{\eta_{G_r, g_r}} b^{s'}$$

$$M \left( \int_0^a x^{\rho + \eta s' + \sum_{i=1}^u K_i e_i + \sum_{i=1}^r c_i \eta_{G_i, g_i} + \sum_{i=1}^v g_i L_i + \sum_{i=1}^s \gamma_i t_i - 1} dx \right) dt_1 \cdots dt_s$$

$$(a - x)^{\sigma + \lambda s' + \sum_{i=1}^u K_i f_i + \sum_{i=1}^r d_i \eta_{G_i, g_i} + \sum_{i=1}^v h_i L_i - 1} \quad (2.4)$$

Now, evaluating the above integral with the help of Eulerian integral

$$\int_0^a x^{\lambda-1} (a - x)^{\mu-1} dx = a^{\lambda+\mu} \frac{\Gamma(\lambda)\Gamma(\mu)}{\Gamma(\lambda + \mu)} \quad (2.5)$$

Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

### 3. Theorems

Now we shall obtain three theorems with the help of our main integral formula given by (2.1) and the following three results recorded in the work by Slater [3,p.75, Th.3 ; p.78, Th 4 ; p.79, Th.5].

Result 1. If  $(1-x)^{\alpha'+\beta'-\gamma'-\frac{1}{2}} {}_2F_1[2\alpha'-1, 2\beta'; 2\gamma'-1; x] = \sum_{n=0}^{\infty} \gamma'_n x^n$

then  ${}_2F_1[\alpha', \beta'; \gamma'; x] {}_2F_1[\gamma' - \alpha' + \frac{1}{2}, \gamma' - \frac{1}{2}; \gamma'; x] = \sum_{n=0}^{\infty} \frac{(\gamma' - 1/2)_n}{(\gamma')_n} \gamma'_n x^n$  (3.1)

Result 2. If  $(1-x)^{\alpha'+\beta'-\gamma'-\frac{1}{2}} {}_3F_2[2\alpha', 2\beta', \gamma'; 2\gamma', \alpha' + \beta' + \frac{1}{2}; x] = \sum_{n=0}^{\infty} \alpha'_n x^n$  then

${}_2F_1[\beta', \gamma' - \beta'; \gamma' + \frac{1}{2}; x] {}_2F_1[\alpha' + \frac{1}{2}, \gamma' - \alpha' + \frac{1}{2}; \gamma' + \frac{1}{2}; x] = \sum_{n=0}^{\infty} \frac{(\alpha' + \beta' + 1/2)_n}{(\gamma' + 1/2)_n} \alpha'_n x^n$  (3.2)

Result 3. If  ${}_2F_1[\alpha', \beta'; \gamma'; x] {}_2F_1[\alpha', \beta'; \delta'; x] = \sum_{n=0}^{\infty} \gamma'_n x^n$  then

${}_4F_3[\alpha', \beta', \frac{\gamma'}{2} + \frac{\delta'}{2}, \frac{\gamma'}{2} + \frac{\beta'}{2} - \frac{1}{2}; \alpha' + \beta', \gamma', \delta'; 4x(1-x)] = \sum_{n=0}^{\infty} \frac{(\gamma' + \delta' - 1)_n}{(\alpha' + \gamma')_n} \gamma'_n x^n$  (3.3)

Theorem 1. If  $(1-x)^{\alpha'+\beta'-\gamma'-\frac{1}{2}} {}_2F_1[2\alpha'-1, 2\beta'; 2\gamma'-1; x] = \sum_{n=0}^{\infty} \gamma'_n x^n$ , then

$$\int_0^a x^{\rho-1} (a-x)^{\sigma-1} {}_pF_q((a_p); (b_q); bx^\eta (a-x)^\lambda) {}_2F_1[\alpha', \beta'; \gamma'; x] {}_2F_1[\gamma' - \alpha' + \frac{1}{2}, \gamma' - \frac{1}{2}; \gamma'; x]$$

$$S_{N_1, \dots, N_u}^{\mathfrak{M}_1, \dots, \mathfrak{M}_u} \left( \begin{matrix} y_1 x^{e_1} (a-x)^{f_1} \\ \vdots \\ y_u x^{e_u} (a-x)^{f_u} \end{matrix} \right) S_E^{F_1, \dots, F_v} \left( \begin{matrix} x_1 x^{g_1} (a-x)^{h_1} \\ \vdots \\ x_v x^{g_v} (a-x)^{h_v} \end{matrix} \right) \mathfrak{K} \left( \begin{matrix} z'_1 x^{c_1} (a-x)^{d_1} \\ \vdots \\ z'_r x^{c_r} (a-x)^{d_r} \end{matrix} \right)$$

$$I \left( \begin{matrix} z_s x^{\gamma_1} (a-x)^{\delta_1} \\ \vdots \\ z_s x^{\gamma_s} (a-x)^{\delta_s} \end{matrix} \right) dx$$



$$= \sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \dots, L_v=0}^{F_1 L_1 + \dots + F_v L_v \leq E} \sum_{s'=0}^{\infty} \sum_{n=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'} s'!} \frac{(-)^{G_1 + \dots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!}$$

$$A_1 B_1 \frac{(\gamma' - 1/2)_n}{(\gamma')_n} \gamma'_n G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) y_1^{K_1} \cdots y_u^{K_u} x_1^{L_1} \cdots x_v^{L_v} z_1'^{\eta_{G_1, g_1}} \cdots z_r'^{\eta_{G_r, g_r}} b^{s'}$$

$$a^{n+\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^u(e_i+f_i)K_i+\sum_{i=1}^r(c_i+d_i)\eta_{G_i,g_i}+\sum_{i=1}^v(g_i+h_i)L_i}I_{U:p_s+2,q_s+1;W}^{V;0,n_s+2;X}\left(\begin{array}{c|c} z_1a^{\gamma_1+\delta_1} & \text{A;} \\ \cdot & \cdot \\ \cdot & \cdot \\ z_sa^{\gamma_s+\delta_s} & \text{B;} \end{array}\right)$$

$$(1-\rho-n-\eta s'-\sum_{i=1}^r c_i\eta_{G_i,g_i}-\sum_{i=1}^u e_iK_i-\sum_{i=1}^v g_iL_i;\gamma_1,\cdots,\gamma_s),$$

$$(-\rho-\sigma-n-(\eta+\lambda)s'-\sum_{i=1}^r(c_i+d_i)\eta_{G_i,g_i}-\sum_{i=1}^u K_i(e_i+f_i)-\sum_{i=1}^v L_i(g_i+h_i);\gamma_1+\delta_1,\cdots,\gamma_s+\delta_s),$$

$$\left. \begin{array}{c} (-\sigma-\lambda s'-\sum_{i=1}^r d_i\eta_{G_i,g_i}-\sum_{i=1}^u f_iK_i-\sum_{i=1}^v h_iL_i;\delta_1,\cdots,\delta_s), \mathfrak{A}: A' \\ \cdot \\ \cdot \\ \cdot \\ \mathfrak{B}: B' \end{array} \right) \quad (3.4)$$

under the same notations and conditions of validity that (2.1)

**Theorem 2.** If  $(1-x)^{\alpha'+\beta'-\gamma'-\frac{1}{2}} {}_3F_2[2\alpha', 2\beta', \gamma'; 2\gamma', \alpha'+\beta'+\frac{1}{2}; x] = \sum_{n=0}^{\infty} \alpha'_n x^n$  then

$$\int_0^a x^{\rho-1} (a-x)^{\sigma-1} {}_pF_q((a_p); (b_q); bx^\eta (a-x)^\lambda) {}_2F_1[\alpha', \beta'; \gamma'; x] {}_2F_1[\gamma' - \alpha' + \frac{1}{2}, \gamma' - \frac{1}{2}; \gamma'; x]$$

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}\left(\begin{array}{c} y_1x^{e_1}(a-x)^{f_1} \\ \cdot \\ \cdot \\ \cdot \\ y_ux^{e_u}(a-x)^{f_u} \end{array}\right)S_E^{F_1,\cdots,F_v}\left(\begin{array}{c} x_1x^{g_1}(a-x)^{h_1} \\ \cdot \\ \cdot \\ \cdot \\ x_vx^{g_v}(a-x)^{h_v} \end{array}\right)\mathfrak{K}\left(\begin{array}{c} z'_1x^{c_1}(a-x)^{d_1} \\ \cdot \\ \cdot \\ \cdot \\ z'_rx^{c_r}(a-x)^{d_r} \end{array}\right)$$

$$I\left(\begin{array}{c} z_sx^{\gamma_1}(a-x)^{\delta_1} \\ \cdot \\ \cdot \\ \cdot \\ z_sx^{\gamma_s}(a-x)^{\delta_s} \end{array}\right)dx$$

$$= \sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \dots, L_v=0}^{F_1 L_1 + \dots + F_v L_v \leq E} \sum_{s'=0}^{\infty} \sum_{n=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'} s'!} \frac{(\alpha' + \beta' + 1/2)_n}{(\gamma' + 1/2)_n} \alpha'_n$$

$$A_1 B_1 \frac{(-)^{G_1 + \dots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) y_1^{K_1} \cdots y_u^{K_u} x_1^{L_1} \cdots x_v^{L_v} z_1'^{\eta_{G_1, g_1}} \cdots z_r'^{\eta_{G_r, g_r}} b^{s'}$$

$$a^{n+\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^u(e_i+f_i)K_i+\sum_{i=1}^r(c_i+d_i)\eta_{G_i,g_i}+\sum_{i=1}^v(g_i+h_i)L_i}I_{U:p_s+2,q_s+1;W}^{V;0,n_s+2;X}\left(\begin{array}{c|c} z_1a^{\gamma_1+\delta_1} & \mathbf{A}; \\ \cdot & \cdot \\ \cdot & \cdot \\ z_sa^{\gamma_s+\delta_s} & \mathbf{B}; \end{array}\right)$$

$$(1-\rho-n-\eta s'-\sum_{i=1}^r c_i\eta_{G_i,g_i}-\sum_{i=1}^u e_iK_i-\sum_{i=1}^v g_iL_i;\gamma_1,\cdots,\gamma_s),$$

$$(-\rho-\sigma-n-(\eta+\lambda)s'-\sum_{i=1}^r(c_i+d_i)\eta_{G_i,g_i}-\sum_{i=1}^u K_i(e_i+f_i)-\sum_{i=1}^v L_i(g_i+h_i);\gamma_1+\delta_1,\cdots,\gamma_s+\delta_s),$$

$$(-\sigma-\lambda s'-\sum_{i=1}^r d_i\eta_{G_i,g_i}-\sum_{i=1}^u f_iK_i-\sum_{i=1}^v h_iL_i;\delta_1,\cdots,\delta_s),\mathfrak{A}:A'\Bigg)_{\mathfrak{B}:\mathbf{B}'} \quad (3.5)$$

under the same notations and conditions of validity that (2.1)

Theorem 3. If  ${}_2F_1[\alpha',\beta';\gamma';x]{}_2F_1[\alpha',\beta';\delta';x]=\sum_{n=0}^{\infty}\gamma'_n x^n$  then

$$\int_0^a x^{\rho-1}(a-x)^{\sigma-1}{}_pF_q\Big((a_p);(b_q);bx^\eta(a-x)^\lambda\Big)S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}\left(\begin{array}{c} y_1x^{e_1}(a-x)^{f_1} \\ \cdot \\ \cdot \\ y_ux^{e_u}(a-x)^{f_u} \end{array}\right)$$

$${}_4F_3\left[\alpha',\beta',\frac{\gamma'}{2}+\frac{\delta'}{2},\frac{\gamma'}{2}+\frac{\beta'}{2}-\frac{1}{2};\alpha'+\beta',\gamma',\delta';4x(1-x)\right]$$

$$S_E^{F_1,\cdots,F_v}\left(\begin{array}{c} x_1x^{g_1}(a-x)^{h_1} \\ \cdot \\ \cdot \\ x_vx^{g_v}(a-x)^{h_v} \end{array}\right)\aleph\left(\begin{array}{c} z'_1x^{c_1}(a-x)^{d_1} \\ \cdot \\ \cdot \\ z'_rx^{c_r}(a-x)^{d_r} \end{array}\right)I\left(\begin{array}{c} z_sx^{\gamma_1}(a-x)^{\delta_1} \\ \cdot \\ \cdot \\ z_sx^{\gamma_s}(a-x)^{\delta_s} \end{array}\right)dx$$

$$=\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}F_1L_1+\cdots+F_vL_v\leq E\sum_{L_1,\cdots,L_v=0}^{\infty}\sum_{s'=0}^{\infty}\sum_{n=0}^{\infty}\frac{[(a_p)]_{s'}}{[(b_q)]_{s'}s'!}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}$$

$$\frac{(\gamma'+\delta'-1)_n}{(\alpha'+\gamma')_n}\gamma'_n x^n G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})y_1^{K_1}\cdots y_u^{K_u}x_1^{L_1}\cdots x_v^{L_v}z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}b^{s'}A_1B_1$$

$$a^{n+\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^u(e_i+f_i)K_i+\sum_{i=1}^r(c_i+d_i)\eta_{G_i,g_i}+\sum_{i=1}^v(g_i+h_i)L_i}I_{U:p_s+2,q_s+1;W}^{V;0,n_s+2;X}\left(\begin{array}{c|c} z_1a^{\gamma_1+\delta_1} & \mathbf{A}; \\ \cdot & \cdot \\ \cdot & \cdot \\ z_sa^{\gamma_s+\delta_s} & \mathbf{B}; \end{array}\right)$$

$$\begin{aligned}
& (1-\rho-n-\eta s' - \sum_{i=1}^r c_i \eta_{G_i, g_i} - \sum_{i=1}^u e_i K_i - \sum_{i=1}^v g_i L_i; \gamma_1, \dots, \gamma_s), \\
& (-\rho-\sigma-n-(\eta+\lambda)s' - \sum_{i=1}^r (c_i + d_i) \eta_{G_i, g_i} - \sum_{i=1}^u K_i (e_i + f_i) - \sum_{i=1}^v L_i (g_i + h_i); \gamma_1 + \delta_1, \dots, \gamma_s + \delta_s), \\
& (-\sigma - \lambda s' - \sum_{i=1}^r d_i \eta_{G_i, g_i} - \sum_{i=1}^u f_i K_i - \sum_{i=1}^v h_i L_i; \delta_1, \dots, \delta_s), \mathfrak{A} : A' \\
& \qquad \qquad \qquad \mathfrak{B} : B'
\end{aligned} \tag{3.4}$$

under the same notations and conditions of validity that (2.1)

### Proof of theorem 1

We first consider the result.1, Multiplying both sides of (3.1) by :

$$\begin{aligned}
& x^{\rho-1} (a-x)^{\sigma-1} {}_pF_q \left( (a_p); (b_q); bx^\eta (a-x)^\lambda \right) S_{N_1, \dots, N_u}^{\mathfrak{M}_1, \dots, \mathfrak{M}_u} \left( \begin{matrix} y_1 x^{e_1} (a-x)^{f_1} \\ \cdot \cdot \cdot \\ y_u x^{e_u} (a-x)^{f_u} \end{matrix} \right) \\
& S_E^{F_1, \dots, F_v} \left( \begin{matrix} x_1 x^{g_1} (a-x)^{h_1} \\ \cdot \cdot \cdot \\ x_v x^{g_v} (a-x)^{h_v} \end{matrix} \right) \aleph \left( \begin{matrix} z'_1 x^{c_1} (a-x)^{d_1} \\ \cdot \cdot \cdot \\ z'_r x^{c_r} (a-x)^{d_r} \end{matrix} \right) I \left( \begin{matrix} z_s x^{\gamma_1} (a-x)^{\delta_1} \\ \cdot \cdot \cdot \\ z_s x^{\gamma_s} (a-x)^{\delta_s} \end{matrix} \right)
\end{aligned} \tag{3.5}$$

and integrating the equation with respect to x between the limits 0 to a. Evaluating the right side thus obtained by interchanging the order of integration and summations ( which is justified due to a absolute convergence of the integral involved in the process ) and then integrating the inner integral with the help of the result (2.1). We get the desired equation (3.1).

The proof of theorem 2. and theorem 3. can be established on the similar methods using results 2. and 3.

## 4. Multivariable H-function

If  $U = V = A = B = 0$ , the multivariable I-function defined by Prasad degenerates in multivariable H-function defined by Srivastava et al [6]. We have the following results.

Corollary 1. If  $(1-x)^{\alpha'+\beta'-\gamma'-\frac{1}{2}} {}_2F_1[2\alpha'-1, 2\beta'; 2\gamma'-1; x] = \sum_{n=0}^{\infty} \gamma'_n x^n$ , then

$$\int_0^a x^{\rho-1} (a-x)^{\sigma-1} {}_pF_q \left( (a_p); (b_q); bx^\eta (a-x)^\lambda \right) {}_2F_1[\alpha', \beta'; \gamma'; x] {}_2F_1[\gamma' - \alpha' + \frac{1}{2}, \gamma' - \frac{1}{2}; \gamma'; x]$$

$$S_{N_1, \dots, N_u}^{\mathfrak{M}_1, \dots, \mathfrak{M}_u} \begin{pmatrix} y_1 x^{e_1} (a-x)^{f_1} \\ \vdots \\ y_u x^{e_u} (a-x)^{f_u} \end{pmatrix} S_E^{F_1, \dots, F_v} \begin{pmatrix} x_1 x^{g_1} (a-x)^{h_1} \\ \vdots \\ x_v x^{g_v} (a-x)^{h_v} \end{pmatrix} \aleph \begin{pmatrix} z'_1 x^{c_1} (a-x)^{d_1} \\ \vdots \\ z'_r x^{c_r} (a-x)^{d_r} \end{pmatrix}$$

$$H \begin{pmatrix} z_s x^{\gamma_1} (a-x)^{\delta_1} \\ \vdots \\ z_s x^{\gamma_s} (a-x)^{\delta_s} \end{pmatrix} dx$$

$$= \sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \dots, L_v=0}^{F_1 L_1 + \dots + F_v L_v \leq E} \sum_{s'=0}^{\infty} \sum_{n=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'} s'!} \frac{(-)^{G_1 + \dots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!}$$

$$A_1 B_1 \frac{(\gamma' - 1/2)_n}{(\gamma')_n} \gamma'_n G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) y_1^{K_1} \cdots y_u^{K_u} x_1^{L_1} \cdots x_v^{L_v} z'_1{}^{\eta_{G_1, g_1}} \cdots z'_r{}^{\eta_{G_r, g_r}} b^{s'}$$

$$a^{n+\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^u(e_i+f_i)K_i+\sum_{i=1}^r(c_i+d_i)\eta_{G_i,g_i}+\sum_{i=1}^v(g_i+h_i)L_i}H_{p_s+2,q_s+1;W}^{0,n_s+2;X}\left(\begin{array}{c} z_1a^{\gamma_1+\delta_1} \\ \cdot \\ \cdot \\ z_sa^{\gamma_s+\delta_s} \end{array}\right|$$

$$(1-\rho-n-\eta s'-\sum_{i=1}^r c_i\eta_{G_i,g_i}-\sum_{i=1}^u e_iK_i-\sum_{i=1}^v g_iL_i;\gamma_1,\cdots,\gamma_s),$$

$$(-\rho-\sigma-n-(\eta+\lambda)s'-\sum_{i=1}^r(c_i+d_i)\eta_{G_i,g_i}-\sum_{i=1}^uK_i(e_i+f_i)-\sum_{i=1}^vL_i(g_i+h_i);\gamma_1+\delta_1,\cdots,\gamma_s+\delta_s),$$

$$\left. \begin{array}{c} (-\sigma-\lambda s'-\sum_{i=1}^r d_i\eta_{G_i,g_i}-\sum_{i=1}^u f_iK_i-\sum_{i=1}^v h_iL_i;\delta_1,\cdots,\delta_s),\mathfrak{A}:A' \\ \cdot \\ \cdot \\ \cdot \\ \mathfrak{B}:B' \end{array} \right) \quad (4.1)$$

under the same notations and conditions of validity that (2.1) with  $U = V = A = B = 0$

Corollary 2. **If**  $(1-x)^{\alpha'+\beta'-\gamma'-\frac{1}{2}}{}_3F_2[2\alpha',2\beta',\gamma';2\gamma',\alpha'+\beta'+\frac{1}{2};x]=\sum_{n=0}^{\infty}\alpha'_n x^n$  **then**

$$\int_0^a x^{\rho-1}(a-x)^{\sigma-1}{}_pF_q\Big((a_p);(b_q);bx^\eta(a-x)^\lambda\Big){}_2F_1\left[\alpha',\beta';\gamma';x\right]{}_2F_1\Big[\gamma'-\alpha'+\frac{1}{2},\gamma'-\frac{1}{2};\gamma';x\Big]$$

$$S_{N_1, \dots, N_u}^{\mathfrak{M}_1, \dots, \mathfrak{M}_u} \begin{pmatrix} y_1 x^{e_1} (a-x)^{f_1} \\ \vdots \\ y_u x^{e_u} (a-x)^{f_u} \end{pmatrix} S_E^{F_1, \dots, F_v} \begin{pmatrix} x_1 x^{g_1} (a-x)^{h_1} \\ \vdots \\ x_v x^{g_v} (a-x)^{h_v} \end{pmatrix} \aleph \begin{pmatrix} z'_1 x^{c_1} (a-x)^{d_1} \\ \vdots \\ z'_r x^{c_r} (a-x)^{d_r} \end{pmatrix}$$

$$\begin{aligned}
& H \begin{pmatrix} z_s x^{\gamma_1} (a-x)^{\delta_1} \\ \vdots \\ z_s x^{\gamma_s} (a-x)^{\delta_s} \end{pmatrix} dx \\
&= \sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \dots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \dots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \dots, L_v=0}^{F_1 L_1 + \dots + F_v L_v \leq E} \sum_{s'=0}^{\infty} \sum_{n=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'} s'!} \frac{(\alpha' + \beta' + 1/2)_n}{(\gamma' + 1/2)_n} \alpha'_n \\
& \frac{(-)^{G_1 + \dots + G_r}}{\delta_{g_1} G_1! \dots \delta_{g_r} G_r!} G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) y_1^{K_1} \dots y_u^{K_u} x_1^{L_1} \dots x_v^{L_v} z_1'^{\eta_{G_1, g_1}} \dots z_r'^{\eta_{G_r, g_r}} b^{s'} \\
& A_1 B_1 a^{n+\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^u (e_i+f_i)K_i+\sum_{i=1}^r (c_i+d_i)\eta_{G_i, g_i}+\sum_{i=1}^v (g_i+h_i)L_i} H_{p_s+2, q_s+1; W}^0 \left( \begin{matrix} z_1 a^{\gamma_1+\delta_1} \\ \vdots \\ z_s a^{\gamma_s+\delta_s} \end{matrix} \right) \\
& (1-\rho-n-\eta s' - \sum_{i=1}^r c_i \eta_{G_i, g_i} - \sum_{i=1}^u e_i K_i - \sum_{i=1}^v g_i L_i; \gamma_1, \dots, \gamma_s), \\
& (-\rho-\sigma-n-(\eta+\lambda)s' - \sum_{i=1}^r (c_i+d_i)\eta_{G_i, g_i} - \sum_{i=1}^u K_i(e_i+f_i) - \sum_{i=1}^v L_i(g_i+h_i); \gamma_1+\delta_1, \dots, \gamma_s+\delta_s), \\
& (-\sigma-\lambda s' - \sum_{i=1}^r d_i \eta_{G_i, g_i} - \sum_{i=1}^u f_i K_i - \sum_{i=1}^v h_i L_i; \delta_1, \dots, \delta_s), \mathfrak{A} : A' \\
& \mathfrak{B} : B'
\end{aligned} \tag{4.2}$$

under the same notations and conditions of validity that (2.1) with  $U = V = A = B = 0$

Corollary 3. If  ${}_2F_1[\alpha', \beta'; \gamma'; x] {}_2F_1[\alpha', \beta'; \delta'; x] = \sum_{n=0}^{\infty} \gamma'_n x^n$  then

$$\int_0^a x^{\rho-1} (a-x)^{\sigma-1} {}_pF_q((a_p); (b_q); bx^{\eta}(a-x)^{\lambda}) S_{N_1, \dots, N_u}^{\mathfrak{M}_1, \dots, \mathfrak{M}_u} \begin{pmatrix} y_1 x^{e_1} (a-x)^{f_1} \\ \vdots \\ y_u x^{e_u} (a-x)^{f_u} \end{pmatrix}$$

$${}_4F_3 \left[ \alpha', \beta', \frac{\gamma'}{2} + \frac{\delta'}{2}, \frac{\gamma'}{2} + \frac{\beta'}{2} - \frac{1}{2}; \alpha' + \beta', \gamma', \delta'; 4x(1-x) \right]$$

$$S_E^{F_1, \dots, F_v} \begin{pmatrix} x_1 x^{g_1} (a-x)^{h_1} \\ \vdots \\ x_v x^{g_v} (a-x)^{h_v} \end{pmatrix} \mathfrak{N} \begin{pmatrix} z'_1 x^{c_1} (a-x)^{d_1} \\ \vdots \\ z'_r x^{c_r} (a-x)^{d_r} \end{pmatrix} H \begin{pmatrix} z_s x^{\gamma_1} (a-x)^{\delta_1} \\ \vdots \\ z_s x^{\gamma_s} (a-x)^{\delta_s} \end{pmatrix} dx$$

$$= \sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \dots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \dots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \dots, L_v=0}^{F_1 L_1 + \dots + F_v L_v \leq E} \sum_{s'=0}^{\infty} \sum_{n=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'} s'!} \frac{(-)^{G_1 + \dots + G_r}}{\delta_{g_1} G_1! \dots \delta_{g_r} G_r!}$$

$$A_1 B_1 \frac{(\gamma' + \delta' - 1)_n}{(\alpha' + \gamma')_n} \gamma'_n x^n G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) y_1^{K_1} \dots y_u^{K_u} x_1^{L_1} \dots x_v^{L_v} z_1'^{\eta_{G_1, g_1}} \dots z_r'^{\eta_{G_r, g_r}} b^{s'}$$

$$a^{n+\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^u(e_i+f_i)K_i+\sum_{i=1}^r(c_i+d_i)\eta_{G_i,g_i}+\sum_{i=1}^v(g_i+h_i)L_i} H_{p_s+2,q_s+1;W}^{0,n_s+2;X} \left( \begin{matrix} z_1 a^{\gamma_1+\delta_1} \\ \cdot \\ \cdot \\ z_s a^{\gamma_s+\delta_s} \end{matrix} \right|$$

$$(1-\rho-n-\eta s'-\sum_{i=1}^r c_i \eta_{G_i,g_i}-\sum_{i=1}^u e_i K_i-\sum_{i=1}^v g_i L_i;\gamma_1,\cdots,\gamma_s),$$

$$(-\rho-\sigma-n-(\eta+\lambda)s'-\sum_{i=1}^r (c_i+d_i)\eta_{G_i,g_i}-\sum_{i=1}^u K_i(e_i+f_i)-\sum_{i=1}^v L_i(g_i+h_i);\gamma_1+\delta_1,\cdots,\gamma_s+\delta_s),$$

$$\left. \begin{matrix} (-\sigma-\lambda s'-\sum_{i=1}^r d_i \eta_{G_i,g_i}-\sum_{i=1}^u f_i K_i-\sum_{i=1}^v h_i L_i;\delta_1,\cdots,\delta_s), \mathfrak{A}: A' \\ \cdot \\ \cdot \\ \mathfrak{B} : B' \end{matrix} \right) \quad (4.3)$$

under the same notations and conditions of validity that (2.1) with  $U = V = A = B = 0$

## 5. Conclusion

The aleph-function of several variables and multivariable I-function presented in this paper, is quite basic in nature. Therefore, on specializing the parameters of this function, we may obtain various other special functions of several variables such as multivariable I-function defined by sharma et al [2], multivariable Fox's H-function, Fox's H-function, Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modified Bessel function, Whittaker function, exponential function, binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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