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On a unified integral formula involving the product of multivariable Aleph-functions with applications II

F.Y. AYANT[1]

1 Teacher in High School , France
E-mail : fredericayant@gmail.com

ABSTRACT
In this paper we first evaluate a new unified finite integral involving products of multivariable Aleph-function, general class of multivariable polynomials and the generalized hypergeometric function. Next, we make use of the results given by Orr and Caley in establishing three theorems. On account of most general nature of the functions and their arguments occuring in our main findings, several new results follow as their simple special cases. The present study thus provides interesting unifications and extansions of a number of integrals.

Keywords:Multivariable Aleph-function, class of multivariable polynomials, generalized hypergeometric function, finite integral, multivariable I-function, multivariable H-function..

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1. Introduction and preliminaries

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [1] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c}\mathrm{z}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r\end{array}\right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]$
$\left.[(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\right)$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)y_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^r\alpha_j^{(k)}s_k)}{\sum_{i=1}^R[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^r\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^r\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$a_j, j = 1, \cdots, p; b_j, j = 1, \cdots, q;$

$c_j^{(k)}, j = 1, \cdots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \cdots, p_{i^{(k)}};$

$d_j^{(k)}, j = 1, \cdots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \cdots, q_{i^{(k)}};$

with $k = 1 \cdots, r, i = 1, \cdots, R$, $i^{(k)} = 1, \cdots, R^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1 \cdots, r, i = 1, \cdots, R \text{ , } i^{(k)} = 1, \cdots, R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$

$\aleph(z_1, \cdots, z_r) = 0(|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}), min(|z_1|, \cdots, |z_r|) \rightarrow \infty$

where, with $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1) / \gamma_j^{(k)})], j = 1, \cdots, n_k$$

Serie representation of Aleph-function of several variables is given by

$$\aleph(y_1,\cdots,y_r) = \sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\psi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})$$

$$\times\ \theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r})y_1^{-\eta_{G_1,g_1}}\cdots y_r^{-\eta_{G_r,g_r}} \tag{1.6}$$

Where $\psi(.,\cdots,.)$, $\theta_i(.)$,$i=1,\cdots,r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1}=\frac{d_{g_1}^{(1)}+G_1}{\delta_{g_1}^{(1)}},\cdots,\ \eta_{G_r,g_r}=\frac{d_{g_r}^{(r)}+G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[d_j^i+p_i]\neq\delta_j^{(i)}[d_{g_i}^i+G_i]$ (1.7)

for $j\neq m_i, m_i=1,\cdots\eta_{G_i,g_i};p_i,n_i=0,1,2,\cdots,;y_i\neq 0,i=1,\cdots,r$ (1.8)

Consider the Aleph-function of s variables

$$\aleph(z_1,\cdots,z_s) = \aleph^{0,N:M_1,N_1,\cdots,M_s,N_s}_{P_i,Q_i,\iota_i;r:P_{i(1)},Q_{i(1)},\iota_{i(1)};r^{(1)};\cdots;P_{i(s)},Q_{i(s)};\iota_{i(s)};r^{(s)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{u}_j;\mu_j^{(1)},\cdots,\mu_j^{(r)})_{1,N}],[\iota_i(u_{ji};\mu_{ji}^{(1)},\cdots,\mu_{ji}^{(r)})_{N+1,P_i}]: \\ \cdot & \\ \cdot & \\ \cdot & \cdots\cdots\cdots,[\iota_i(v_{ji};\upsilon_{ji}^{(1)},\cdots,\upsilon_{ji}^{(r)})_{M+1,Q_i}]: \\ \mathrm{z}_s & \end{array}\right.$$

$$\left.\begin{array}{l}[(\mathrm{a}_j^{(1)});\alpha_j^{(1)})_{1,N_1}],[\iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)};\alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}];\cdots;[(\mathrm{a}_j^{(s)});\alpha_j^{(s)})_{1,N_s}],[\iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)};\alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_i^{(s)}}] \\ [(\mathrm{b}_j^{(1)});\beta_j^{(1)})_{1,M_1}],[\iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)};\beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}];\cdots;[(\mathrm{b}_j^{(s)});\beta_j^{(s)})_{1,M_s}],[\iota_{i^{(s)}}(b_{ji^{(s)}}^{(s)};\beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_i^{(s)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_r}\zeta(t_1,\cdots,t_s)\prod_{k=1}^{s}\phi_k(t_k)z_k^{t_k}\,\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.9}$$

with $\omega=\sqrt{-1}$

$$\zeta(t_1,\cdots,t_s)=\frac{\prod_{j=1}^{N}\Gamma(1-u_j+\sum_{k=1}^{s}\mu_j^{(k)}t_k)}{\sum_{i=1}^{r'}[\iota_i\prod_{j=N+1}^{P_i}\Gamma(u_{ji}-\sum_{k=1}^{s}\mu_{ji}^{(k)}t_k)\prod_{j=1}^{Q_i}\Gamma(1-\upsilon_{ji}+\sum_{k=1}^{s}\upsilon_{ji}^{(k)}t_k)]} \tag{1.10}$$

and $$\phi_k(t_k)=\frac{\prod_{j=1}^{M_k}\Gamma(b_j^{(k)}-\beta_j^{(k)}t_k)\prod_{j=1}^{N_k}\Gamma(1-a_j^{(k)}+\alpha_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{r^{(k)}}[\iota_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-b_{ji^{(k)}}^{(k)}+\beta_{ji^{(k)}}^{(k)}t_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(a_{ji^{(k)}}^{(k)}-\alpha_{ji^{(k)}}^{(k)}s_k)]} \tag{1.11}$$

Suppose , as usual , that the parameters

$u_j,j=1,\cdots,P;v_j,j=1,\cdots,Q;$

$a_j^{(k)},j=1,\cdots,N_k;a_{ji^{(k)}}^{(k)},j=n_k+1,\cdots,P_{i^{(k)}};$

$b_{ji^{(k)}}^{(k)},j=m_k+1,\cdots,Q_{i^{(k)}};b_j^{(k)},j=1,\cdots,M_k;$

with $k=1\cdots,s,i=1,\cdots,r',i^{(k)}=1,\cdots,r^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^{N} \mu_j^{(k)} + \iota_i \sum_{j=N+1}^{P_i} \mu_{ji}^{(k)} + \sum_{j=1}^{N_k} \alpha_j^{(k)} + \iota_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)} - \iota_i \sum_{j=1}^{Q_i} \upsilon_{ji}^{(k)} - \sum_{j=1}^{M_k} \beta_j^{(k)}$$

$$-\iota_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.12}$$

The reals numbers τ_i are positives for $i = 1, \cdots, r$, $\iota_{i^{(k)}}$ are positives for $i^{(k)} = 1 \cdots r^{(k)}$

The contour L_k is in the t_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(b_j^{(k)} - \beta_j^{(k)} t_k)$ with $j = 1$ to M_k are separated from those of $\Gamma(1 - u_j + \sum_{i=1}^{s} \mu_j^{(k)} t_k)$with $j = 1$ to N and $\Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} t_k)$ with $j = 1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

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$$+\sum_{j=1}^{M_k} \beta_j^{(k)} - \iota_{i^{(k)}} \sum_{j=M_k+1}^{q_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k = 1 \cdots, s, i = 1, \cdots, r \ , i^{(k)} = 1, \cdots, r^{(k)} \tag{1.13}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \cdots, z_s) = 0(\,|z_1|^{\alpha_1'}, \cdots, |z_s|^{\alpha_s'}\,)\,, max(\,|z_1|, \cdots, |z_s|\,) \to 0$$

$$\aleph(z_1, \cdots, z_s) \ = 0(\,|z_1|^{\beta_1'}, \cdots, |z_s|^{\beta_s'}\,)\,, min(\,|z_1|, \cdots, |z_s|\,) \to \infty$$

where, with $k = 1, \cdots, z : \alpha_k' = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, M_k$ and

$$\beta_k' = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, N_k$$

We will use these following notations in this paper

$$U = P_i, Q_i, \iota_i; r' ; \ V = M_1, N_1; \cdots; M_s, N_s \tag{1.15}$$

$$\mathrm{W} = P_{i^{(1)}}, Q_{i^{(1)}}, \iota_{i^{(1)}}; r^{(1)}, \cdots, P_{i^{(r)}}, Q_{i^{(r)}}, \iota_{i^{(s)}}; r^{(s)} \tag{1.16}$$

$$A_1 = \{(u_j; \mu_j^{(1)}, \cdots, \mu_j^{(s)})_{1,N}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \cdots, \mu_{ji}^{(s)})_{N+1,P_i}\} \tag{1.17}$$

$$B_1 = \{\iota_i(v_{ji}; \upsilon_{ji}^{(1)}, \cdots, \upsilon_{ji}^{(s)})_{M+1,Q_i}\} \tag{1.18}$$

$$C_1 = (a_j^{(1)}; \alpha_j^{(1)})_{1,N_1} \ , \iota_{i^{(1)}}(a_{ji^{(1)}}^{(1)}; \alpha_{ji^{(1)}}^{(1)})_{N_1+1,P_{i^{(1)}}}, \cdots, (a_j^{(s)}; \alpha_j^{(s)})_{1,N_s} \ , \iota_{i^{(s)}}(a_{ji^{(s)}}^{(s)}; \alpha_{ji^{(s)}}^{(s)})_{N_s+1,P_{i^{(s)}}} \tag{1.19}$$

$$D_1 =(b_j^{(1)};\beta_j^{(1)})_{1,M_1},\iota_{i^{(1)}}(b_{ji^{(1)}}^{(1)};\beta_{ji^{(1)}}^{(1)})_{M_1+1,Q_{i^{(1)}}},\cdots,(b_j^{(s)};\beta_j^{(s)})_{1,M_s},\iota_{i^{(s)}}(\beta_{ji^{(s)}}^{(s)};\beta_{ji^{(s)}}^{(s)})_{M_s+1,Q_{i^{(s)}}} \tag{1.20}$$

The multivariable Aleph-function write :

$$\aleph(z_1,\cdots,z_s) = \aleph_{U:W}^{0,N:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A}_1 : C_1 \\ \cdot & \cdots \\ \cdot & \mathrm{B}_1 : D_1 \\ \mathrm{z}_s & \end{array}\right) \tag{1.21}$$

The generalized polynomials of multivariables defined by Srivastava [4], is given in the following manner :

$$S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}[y_1,\cdots,y_u] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}\frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_u)_{\mathfrak{M}_uK_u}}{K_u!}$$

$$A[N_1,K_1;\cdots;N_u,K_u]y_1^{K_1}\cdots y_u^{K_u} \tag{1.22}$$

Where $\mathfrak{M}_1,\cdots,\mathfrak{M}_u$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_u,K_u]$ are arbitrary constants, real or complex.

Srivastava and Garg introduced and defined a general class of multivariable polynomials [5] as follows

$$S_E^{F_1,\cdots,F_v}[z_1,\cdots,z_v] = \sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}(-E)_{F_1L_1+\cdots+F_vL_v}B(E;L_1,\cdots,L_v)\frac{z_1^{L_1}\cdots z_v^{L_v}}{L_1!\cdots L_v!} \tag{1.23}$$

The generalized hypergeometric serie is defined as follows.

$${}_pF_q(y)=\sum_{s'=0}^{\infty}\frac{[(a_p)]_{s'}}{[(b_q)]_{s'}}y^{s'} \tag{1.24}$$

gwhere $[(a_p)]_{s'} = (a_1)_{s'}\cdots(a_p)_{s'}$; $[(b_q)]_{s'} = (b_1)_{s'}\cdots(b_q)_{s'}$. The serie (1.24) converge if $p\leqslant q$ and $|y|<1$

In the document , we note :

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}) = \phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r}) \tag{1.25}$$

$$A' = \frac{(-N_1)_{\mathfrak{M}_1K_1}}{K_1!}\cdots\frac{(-N_u)_{\mathfrak{M}_uK_u}}{K_u!}A[N_1,K_1;\cdots;N_u,K_u] \tag{1.26}$$

$$B' = \frac{(-E)_{F_1L_1+\cdots+F_vL_v}B(E;L_1,\cdots,L_v)}{L_1!\cdots L_v!} \text{ and } U_{21} = P_i+2,Q_i+1,\iota_i;r' \tag{1.27}$$

2. Main integral

$$\int_0^a x^{\rho-1}(a-x)^{\sigma-1}{}_pF_q\Big((a_p);(b_q);bx^{\eta}(a-x)^{\lambda}\Big)S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}_{N_1,\cdots,N_u}\left(\begin{array}{c} \mathrm{y}_1x^{e_1}(a-x)^{f_1} \\ \cdots \\ \cdots \\ \mathrm{y}_ux^{e_u}(a-x)^{f_u} \end{array}\right)$$

$$S^{F_1,\cdots,F_v}_{E}\left(\begin{array}{c} \mathrm{x}_1x^{g_1}(a-x)^{h_1} \\ \cdots \\ \cdots \\ \mathrm{x}_vx^{g_v}(a-x)^{h_v} \end{array}\right)\aleph\left(\begin{array}{c} \mathrm{z'}_1x^{c_1}(a-x)^{d_1} \\ \cdots \\ \cdots \\ \mathrm{z'}_rx^{c_r}(a-x)^{d_r} \end{array}\right)\aleph\left(\begin{array}{c} \mathrm{z}_sx^{\gamma_1}(a-x)^{\delta_1} \\ \cdots \\ \cdots \\ \mathrm{z}_sx^{\gamma_s}(a-x)^{\delta_s} \end{array}\right)\mathrm{d}x$$

$$=\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\sum_{s'=0}^{\infty}\frac{[(a_p)]_{s'}}{[(b_q)]_{s'}s'!}A'B'\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}$$

$$G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})\,y_1^{K_1}\cdots y_u^{K_u}\,x_1^{L_1}\cdots x_v^{L_v}\,z_1'^{\,\eta_{G_1,g_1}}\cdots z_r'^{\,\eta_{G_r,g_r}}b^{s'}$$

$$a^{\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^{u}(e_i+f_i)K_i+\sum_{i=1}^{r}(c_i+d_i)\eta_{G_i,g_i}+\sum_{i=1}^{v}(g_i+h_i)L_i}\ \aleph^{0,N+2:V}_{U_{21}:W}\left(\begin{array}{c} \mathrm{z}_1a^{\gamma_1+\delta_1} \\ \cdot \\ \cdot \\ \mathrm{z}_sa^{\gamma_s+\delta_s} \end{array}\right|$$

$$(1\text{-}\rho-\eta s'-\textstyle\sum_{i=1}^{r}c_i\eta_{G_i,g_i}-\sum_{i=1}^{u}e_iK_i-\sum_{i=1}^{v}g_iL_i;\gamma_1,\cdots,\gamma_s),$$

$$\cdots$$

$$(\text{-}\rho-\sigma-(\eta+\lambda)s'-\textstyle\sum_{i=1}^{r}(c_i+d_i)\eta_{G_i,g_i}-\sum_{i=1}^{u}K_i(e_i+f_i)-\sum_{i=1}^{v}L_i(g_i+h_i);\gamma_1+\delta_1,\cdots,\gamma_s+\delta_s),$$

$$\left.\begin{array}{c}(\text{-}\sigma-\lambda s'-\sum_{i=1}^{r}d_i\eta_{G_i,g_i}-\sum_{i=1}^{u}f_iK_i-\sum_{i=1}^{v}h_iL_i;\delta_1,\cdots,\delta_s),A_1:C_1 \\ \cdots \\ \mathrm{B}_1:D_1\end{array}\right) \tag{2.1}$$

where $G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}),A_1,B_1$ and U_{21} are defined by (1.25), (1.26) and (1.27) respectively.

Provided that

a) $min(e_i,f_i,c_j,d_j,g_k,h_k,\gamma_l,\delta_l,\rho,\sigma)\geqslant 0,$ (not all zero simultaneously) with $i=1,\cdots,u;j=1,\cdots,r$ $k=1,\cdots,v$ and $l=1,\cdots,s$

b) $Re[\rho+\sum_{i=1}^{r}c_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^{s}\gamma_i\min_{1\leqslant j\leqslant M_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

c) $Re[1+\sigma+\sum_{i=1}^{r}d_i\min_{1\leqslant j\leqslant m_i}\frac{d_j^{(i)}}{\delta_j^{(i)}}+\sum_{i=1}^{s}\delta_i\min_{1\leqslant j\leqslant M_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}]>0$

d)$|argz'_k| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)}$ is defined by (1.5) ; $i = 1, \cdots, r$

e)$|argz_k| < \frac{1}{2} B_i^{(k)} \pi$, where $B_i^{(k)}$ is defined by (1.13) ; $i = 1, \cdots, s$

Proof : Let $M\{\}= \frac{1}{(2\pi\omega)^s} \int_{L_1} \cdots \int_{L_r} \zeta(t_1, \cdots, t_s) \prod_{k=1}^{s} \phi_k(t_k)\{\}$ (2.2)

To prove (2.1), first we express the Aleph-function of r variables, two general class of polynomials of several variables, the generalized hypergeometric function in form of serie with the help of (1.6), (1.22), (1.23) and (1.24) respectively. Interchanging the order of summations and integration wich is possible under the stated conditions, we obtain.

$$\sum_{G_1, \cdots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \cdots, L_v=0}^{F_1 L_1 + \cdots F_v L_v \leqslant E} \sum_{s'=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'} s'!} A' B' \frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!}$$

$$G(\eta_{G_1, g_1}, \cdots \eta_{G_r, g_r}) \, y_1^{K_1} \cdots y_u^{K_u} \, x_1^{L_1} \cdots x_v^{L_v} \, z_1'^{\eta_{G_1, g_1}} \cdots z_r'^{\eta_{G_r, g_r}} b^{s'}$$

$$\int_0^a x^{\rho + \eta s' + \sum_{i=1}^u K_i e_i + \sum_{i=1}^r c_i \eta_{G_i, g_i} + \sum_{i=1}^v g_i L_i - 1} (a-x)^{\sigma + \lambda s' + \sum_{i=1}^u K_i f_i + \sum_{i=1}^r d_i \eta_{G_i, g_i} + \sum_{i=1}^v h_i L_i - 1}$$

$$\aleph \begin{pmatrix} \mathrm{z}_s x^{\gamma_1} (a-x)^{\delta_1} \\ \cdot \cdot \cdot \\ \cdot \cdot \cdot \\ \mathrm{z}_s x^{\gamma_s} (a-x)^{\delta_s} \end{pmatrix} \mathrm{d}x \tag{2.3}$$

Now expressing the Aleph-function of s-variables in terms of Mellin-Barnes contour integrals and changing the order of integrations ,we get

$$\sum_{G_1, \cdots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{M}_u]} \sum_{L_1, \cdots, L_v=0}^{F_1 L_1 + \cdots F_v L_v \leqslant E} \sum_{s'=0}^{\infty} \frac{[(a_p)]_{s'}}{[(b_q)]_{s'} s'!} A' B' \frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!}$$

$$G(\eta_{G_1, g_1}, \cdots \eta_{G_r, g_r}) \, y_1^{K_1} \cdots y_u^{K_u} \, x_1^{L_1} \cdots x_v^{L_v} \, z_1'^{\eta_{G_1, g_1}} \cdots z_r'^{\eta_{G_r, g_r}} b^{s'}$$

$$M \left(\int_0^a x^{\rho + \eta s' + \sum_{i=1}^u K_i e_i + \sum_{i=1}^r c_i \eta_{G_i, g_i} + \sum_{i=1}^v g_i L_i + \sum_{i=1}^s \gamma_i t_i - 1} \right.$$

$$\left. (a-x)^{\sigma + \lambda s' + \sum_{i=1}^u K_i f_i + \sum_{i=1}^r d_i \eta_{G_i, g_i} + \sum_{i=1}^v h_i L_i + \sum_{i=1}^s \delta_i t_i - 1} \mathrm{d}x \right) \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{2.4}$$

Now, evaluating the above integral with the help of Eulerian integral

$$\int_0^a x^{\lambda-1} (a-x)^{\mu-1} \mathrm{d}x = a^{\lambda+\mu} \frac{\Gamma(\lambda)\Gamma(\mu)}{\Gamma(\lambda+\mu)} \tag{2.5}$$

Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

3. Theorems

In this section , using Cayley-Orr type of identities involving bilateral hypergeometric serie due to Shukla [3], an attempt has been made to establish certain results which include the results due to Mrs Srivastava[7]. In the sequel, certain expansions involving generalized hypergeometric function have been deduced from some of these results.

Theorem 1

If $(1-E)\Gamma\left[\begin{array}{c}\text{1+E-D,D-E}\\ \cdots \\ \text{1+2C-D},\frac{3}{2}+A+B-C-D,D-2A,D-2B\end{array}\right]{}_1H_1\left[\begin{array}{c}\text{D-}\frac{1}{2}-A-B+C\\ \cdots \\ \text{D}\end{array};x\right]$

$${}_2H_2\left[\begin{array}{c}\text{1+2A-D,1+2B-D}\\ \cdots \\ \text{2-D,1+2C-D}\end{array};x\right]+\text{Idem }(D;E)=\sum_{n=-\infty}^{\infty}a_nx^n \text{ then}$$

$$\Gamma\left[\begin{array}{c}\text{D,E,2-D,2-E}\\ \cdots \\ \text{2C},\frac{1}{2}\text{+A+B-C,1-2A,1-2B}\end{array}\right]\int_0^a x^{\rho-1}(a-x)^{\sigma-1}{}_pF_q\Big((a_p);(b_q);bx^{\eta}(a-x)^{\lambda}\Big)\,{}_2F_1\left[A,B;C;x\right]$$

$${}_2F_1\left[\frac{1}{2}+C-A,C-B+\frac{1}{2};\frac{1}{2};x\right]S_E^{F_1,\cdots,F_v}\left(\begin{array}{c}\mathrm{x}_1x^{g_1}(a-x)^{h_1}\\ \cdots \\ \cdots \\ \mathrm{x}_vx^{g_v}(a-x)^{h_v}\end{array}\right)\aleph\left(\begin{array}{c}\mathrm{z'}_1x^{c_1}(a-x)^{d_1}\\ \cdots \\ \cdots \\ \mathrm{z'}_rx^{c_r}(a-x)^{d_r}\end{array}\right)$$

$$\aleph\left(\begin{array}{c}\mathrm{z}_sx^{\gamma_1}(a-x)^{\delta_1}\\ \cdots \\ \cdots \\ \mathrm{z}_sx^{\gamma_s}(a-x)^{\delta_s}\end{array}\right)\mathrm{d}x$$

$$=\Gamma(\rho)\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_u=0}^{[N_u/\mathfrak{M}_u]}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\sum_{s'=0}^{\infty}\sum_{n=0}^{\infty}\frac{[(a_p)]_{s'}}{[(b_q)]_{s'}s'!}A'B'$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})\,y_1^{K_1}\cdots y_u^{K_u}\,x_1^{L_1}\cdots x_v^{L_v}\,z_1'^{\,\eta_{G_1,g_1}}\cdots z_r'^{\,\eta_{G_r,g_r}}b^{s'}\,a_n\frac{(C+\frac{1}{2})_n}{(C+1)_n}$$

$$a^{n+\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^{u}(e_i+f_i)K_i+\sum_{i=1}^{r}(c_i+d_i)\eta_{G_i,g_i}+\sum_{i=1}^{v}(g_i+h_i)L_i}\ \aleph_{U_{21}:W}^{0,N+2:V}\left(\begin{array}{c} \mathrm{z}_1 a^{\gamma_1+\delta_1} \\ \cdot \\ \cdot \\ \mathrm{z}_s a^{\gamma_s+\delta_s} \end{array}\right|$$

$$(1\text{-}\rho-n-\eta s'-\textstyle\sum_{i=1}^{r} c_i\eta_{G_i,g_i}-\sum_{i=1}^{u} e_iK_i-\sum_{i=1}^{v} g_iL_i;\gamma_1,\cdots,\gamma_s),$$
$$\cdots$$
$$(\text{-}\rho-\sigma-n-(\eta+\lambda)s'-\textstyle\sum_{i=1}^{r}(c_i+d_i)\eta_{G_i,g_i}-\sum_{i=1}^{u} K_i(e_i+f_i)-\sum_{i=1}^{v} L_i(g_i+h_i);\gamma_1+\delta_1,\cdots,\gamma_s+\delta_s),$$

$$\left.\begin{array}{c}(\text{-}\sigma-\lambda s'-\sum_{i=1}^{r} d_i\eta_{G_i,g_i}-\sum_{i=1}^{u} f_iK_i-\sum_{i=1}^{v} h_iL_i;\delta_1,\cdots,\delta_s),A_1:C_1 \\ \cdots \\ \mathrm{B}_1:D_1\end{array}\right) \tag{3.1}$$

under the same notations and conditions of validity that (2.1) and $Re(\rho)>1$

Theorem 2

If $(1-E)\ \Gamma\left[\begin{array}{c}1+\text{E-D,D-E} \\ \cdots \\ 1+\text{2C-D},\frac{3}{2}+A+B-C-D,D-2A,D-2B\end{array}\right]{}_1H_1\left[\begin{array}{c}\text{D-}\frac{1}{2}-A-B+C \\ \cdots \\ \text{D}\end{array};x\right]$

$${}_2H_2\left[\begin{array}{c}\text{2A-D,1+2B-D} \\ \cdots \\ \text{2-D,1+2C-D}\end{array};x\right]+\text{ Idem }(D;E)=\sum_{n=-\infty}^{\infty}a_nx^n \text{ then}$$

$$\Gamma\left[\begin{array}{c}\text{D,E,2-D,2-E} \\ \cdots \\ \text{2C-1},\tfrac{1}{2}\text{+A+B-C,2-2A,1-2B}\end{array}\right]\int_0^a x^{\rho-1}(a-x)^{\sigma-1}{}_pF_q\Big((a_p);(b_q);bx^{\eta}(a-x)^{\lambda}\Big){}_2F_1\left[A,B;C;x\right]$$

$${}_2F_1\left[\frac{1}{2}+C-A,C-B-\frac{1}{2};C;z\right]\ S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}\left(\begin{array}{c}\mathrm{y}_1x^{e_1}(a-x)^{f_1} \\ \cdots \\ \cdots \\ \mathrm{y}_ux^{e_u}(a-x)^{f_u}\end{array}\right)$$

$$S_E^{F_1,\cdots,F_v}\left(\begin{array}{c}\mathrm{x}_1x^{g_1}(a-x)^{h_1} \\ \cdots \\ \cdots \\ \mathrm{x}_vx^{g_v}(a-x)^{h_v}\end{array}\right)\aleph\left(\begin{array}{c}\mathrm{z'}_1x^{c_1}(a-x)^{d_1} \\ \cdots \\ \cdots \\ \mathrm{z'}_rx^{c_r}(a-x)^{d_r}\end{array}\right)\aleph\left(\begin{array}{c}\mathrm{z}_sx^{\gamma_1}(a-x)^{\delta_1} \\ \cdots \\ \cdots \\ \mathrm{z}_sx^{\gamma_s}(a-x)^{\delta_s}\end{array}\right)\mathrm{d}x$$

$$=\Gamma(\sigma)\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\sum_{s'=0}^{\infty}\sum_{n=0}^{\infty}\frac{[(a_p)]_{s'}}{[(b_q)]_{s'}s'!}A'B'$$

$$\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})\,y_1^{K_1}\cdots y_u^{K_u}\,x_1^{L_1}\cdots x_v^{L_v}\,{z'_1}^{\eta_{G_1,g_1}}\cdots{z'_r}^{\eta_{G_r,g_r}}b^{s'}\,a_n\frac{\left(C-\frac{1}{2}\right)_n}{(C)_n}$$

$$a^{n+\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^{u}(e_i+f_i)K_i+\sum_{i=1}^{r}(c_i+d_i)\eta_{G_i,g_i}+\sum_{i=1}^{v}(g_i+h_i)L_i}\;\aleph_{U_{21}:W}^{0,N+2:V}\left(\begin{array}{c|c}\mathrm{z}_1a^{\gamma_1+\delta_1} & \\ \cdot & \\ \cdot & \\ \mathrm{z}_sa^{\gamma_s+\delta_s} & \end{array}\right.$$

$$(1\text{-}\rho-n-\eta s'-\textstyle\sum_{i=1}^{r}c_i\eta_{G_i,g_i}-\sum_{i=1}^{u}e_iK_i-\sum_{i=1}^{v}g_iL_i;\gamma_1,\cdots,\gamma_s),$$
$$\cdots$$
$$(\text{-}\rho-\sigma-n-(\eta+\lambda)s'-\textstyle\sum_{i=1}^{r}(c_i+d_i)\eta_{G_i,g_i}-\sum_{i=1}^{u}K_i(e_i+f_i)-\sum_{i=1}^{v}L_i(g_i+h_i);\gamma_1+\delta_1,\cdots,\gamma_s+\delta_s),$$

$$\left.\begin{array}{c}(\text{-}\sigma-\lambda s'-\sum_{i=1}^{r}d_i\eta_{G_i,g_i}-\sum_{i=1}^{u}f_iK_i-\sum_{i=1}^{v}h_iL_i;\delta_1,\cdots,\delta_s),A_1:C_1\\ \cdots\\ \mathrm{B}_1:D_1\end{array}\right)\qquad(3.2$$

under the same notations and conditions of validity that (2.1) and $Re(\rho)>1$

Theorem 3

If

$$(1-E)\Gamma\left[\begin{array}{c}\text{1+E-D,D-E}\\ \cdots\\ \text{1+2C-D,2+A+B-C-D,D-2A,D-2B}\end{array}\right]\;{}_1H_1\left[\begin{array}{c}\text{C-A-B-1+D}\\ \cdots\\ \text{D}\end{array};x\right]$$

$${}_2H_2\left[\begin{array}{c}\text{1+2A-D,1+2B-D}\\ \cdots\\ \text{2-D,1+2C-D}\end{array};x\right]+\text{Idem }(D;E)=\sum_{n=-\infty}^{\infty}a_nx^n\text{ then}$$

$$\Gamma\left[\begin{array}{c}\text{D,E,2-D,2-E}\\ \cdots\\ \text{2C-1,1+A+B-C,1-2A,1-2B}\end{array}\right]\int_0^a x^{\rho-1}(a-x)^{\sigma-1}{}_pF_q\Big((a_p);(b_q);bx^\eta(a-x)^\lambda\Big){}_2F_1\left[A,B;C+\frac{1}{2};x\right]$$

$$ {}_2F_1\left[C-A,C-B;C+\frac{1}{2};x\right]\ S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}_{N_1,\cdots,N_u}\left(\begin{array}{c} \mathrm{y}_1x^{e_1}(a-x)^{f_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{y}_ux^{e_u}(a-x)^{f_u}\end{array}\right) $$

$$ S^{F_1,\cdots,F_v}_{E}\left(\begin{array}{c} \mathrm{x}_1x^{g_1}(a-x)^{h_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{x}_vx^{g_v}(a-x)^{h_v}\end{array}\right)\aleph\left(\begin{array}{c} \mathrm{z'}_1x^{c_1}(a-x)^{d_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{z'}_rx^{c_r}(a-x)^{d_r}\end{array}\right)\aleph\left(\begin{array}{c} \mathrm{z}_sx^{\gamma_1}(a-x)^{\delta_1}\\ \cdot\ \cdot\ \cdot\\ \cdot\ \cdot\ \cdot\\ \mathrm{z}_sx^{\gamma_s}(a-x)^{\delta_s}\end{array}\right)\mathrm{d}x $$

$$ =\Gamma(\sigma)\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{L_1,\cdots,L_v=0}^{F_1L_1+\cdots F_vL_v\leqslant E}\sum_{s'=0}^{\infty}\sum_{n=0}^{\infty}\frac{[(a_p)]_{s'}}{[(b_q)]_{s'}s'!}A'B' $$

$$ \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}G(\eta_{G_1,g_1},\cdots\eta_{G_r,g_r})\,y_1^{K_1}\cdots y_u^{K_u}\,x_1^{L_1}\cdots x_v^{L_v}\,z_1'^{\,\eta_{G_1,g_1}}\cdots z_r'^{\,\eta_{G_r,g_r}}b^{s'}\,a_n\frac{(C)_n}{\left(C+\frac{1}{2}\right)_n} $$

$$ a^{n+\rho+\sigma+(\eta+\lambda)s'+\sum_{i=1}^{u}(e_i+f_i)K_i+\sum_{i=1}^{r}(c_i+d_i)\eta_{G_i,g_i}+\sum_{i=1}^{v}(g_i+h_i)L_i}\ \aleph^{0,N+2:V}_{U_{21}:W}\left(\begin{array}{c} \mathrm{z}_1a^{\gamma_1+\delta_1}\\ \cdot\\ \cdot\\ \mathrm{z}_sa^{\gamma_s+\delta_s}\end{array}\right| $$

$$ \begin{array}{c}(1\text{-}\rho-n-\eta s'-\sum_{i=1}^{r}c_i\eta_{G_i,g_i}-\sum_{i=1}^{u}e_iK_i-\sum_{i=1}^{v}g_iL_i;\gamma_1,\cdots,\gamma_s),\\ \cdot\ \cdot\ \cdot\\ (\text{-}\rho-\sigma-n-(\eta+\lambda)s'-\sum_{i=1}^{r}(c_i+d_i)\eta_{G_i,g_i}-\sum_{i=1}^{u}K_i(e_i+f_i)-\sum_{i=1}^{v}L_i(g_i+h_i);\gamma_1+\delta_1,\cdots,\gamma_s+\delta_s),\end{array} $$

$$ \left.\begin{array}{c}(\text{-}\sigma-\lambda s'-\sum_{i=1}^{r}d_i\eta_{G_i,g_i}-\sum_{i=1}^{u}f_iK_i-\sum_{i=1}^{v}h_iL_i;\delta_1,\cdots,\delta_s),A_1:C_1\\ \cdot\ \cdot\ \cdot\\ \mathrm{B}_1:D_1\end{array}\right) \tag{3.3} $$

under the same notations and conditions of validity that (2.1) and $Re(\rho)>1$

Proof of theorem 1

We have due to Shukla [3] . If If $(1-E)\Gamma\left[\begin{array}{c}1+\mathrm{E}\text{-}\mathrm{D},\mathrm{D}\text{-}\mathrm{E}\\ \cdot\ \cdot\ \cdot\\ 1+2\mathrm{C}\text{-}\mathrm{D},\frac{3}{2}+A+B-C-D,D-2A,D-2B\end{array}\right]$

$$ {}_1H_1\left[\begin{array}{cc} \text{D-}\frac{1}{2}-A-B+C & \\ \cdots & ;x \\ \text{D} & \end{array}\right]{}_2H_2\left[\begin{array}{cc} \text{1+2A-D,1+2B-D} & \\ \cdots & ;x \\ \text{2-D,1+2C-D} & \end{array}\right] + \text{Idem } (D;E) = \sum_{n=-\infty}^{\infty} a_n x^n \tag{3.4} $$

then

$$ \Gamma\left[\begin{array}{c} \text{D,E,2-D,2-E} \\ \cdots \\ \text{2C,}\frac{1}{2}\text{+A+B-C,1-2A,1-2B} \end{array}\right] {}_2F_1\left[A,B;C;x\right]{}_2F_1\left[\frac{1}{2}+C-A,C-B+\frac{1}{2};\frac{1}{2};x\right] $$

$$ = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}+C\right)_n}{(1+C)_n} a_n x^n \tag{3.5} $$

Multiplying both sides of (3.5) by :

$$ x^{\rho-1}(a-x)^{\sigma-1}{}_pF_q\Big((a_p);(b_q);bx^{\eta}(a-x)^{\lambda}\Big) S_{N_1,\cdots,N_u}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_u}\left(\begin{array}{c} \text{y}_1 x^{e_1}(a-x)^{f_1} \\ \cdots \\ \cdots \\ \text{y}_u x^{e_u}(a-x)^{f_u} \end{array}\right) $$

$$ S_E^{F_1,\cdots,F_v}\left(\begin{array}{c} \text{x}_1 x^{g_1}(a-x)^{h_1} \\ \cdots \\ \cdots \\ \text{x}_v x^{g_v}(a-x)^{h_v} \end{array}\right) \aleph\left(\begin{array}{c} \text{z'}_1 x^{c_1}(a-x)^{d_1} \\ \cdots \\ \cdots \\ \text{z'}_r x^{c_r}(a-x)^{d_r} \end{array}\right) \aleph\left(\begin{array}{c} \text{z}_s x^{\gamma_1}(a-x)^{\delta_1} \\ \cdots \\ \cdots \\ \text{z}_s x^{\gamma_s}(a-x)^{\delta_s} \end{array}\right) $$

and integrating the equation with respect to x between the limits 0 to a. Evaluating the right side thus obtained by interchanging the order of integration ans summations (which is justified due to a absolute convergence of the integral involved in the process) and then integrating the inner integral with the help of the result (2.1). We get the desired equation (3.1).
The proof of theorem 2. and theorem 3. can be established on the similar methods .

Remarks : We have the similar formulas with the multivariable I-function defined by Sharma et al [1], the multivariable H-function defined by Srivastava et al [6] and the Aleph-function of two variables defined by Sharma [2].

4. Conclusion

The aleph-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various other special functions o several variables such as multivariable I-function ,multivariable Fox's H-function, Fox's H-function , Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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