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On Locally b** Closed Sets

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Abstract : We introduce the notion of locally b**-closed , b**-B-set, locally b** closed continuous , b**-B-continuous functions and obtain decomposition of continuity and complete continuity.

Key words : b**-open sets , t-set,B-set, locally closed , decomposition of continuity

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1.I ntroduction

Tong [17,18] , Ganster- Reily [5] , Hatice [7], Hatir-Noiri [8] , Przemski [20] , Noiri – Sayed [13] and Erguang – Pengfei [4] , gave some decompositions of continuity. Andrijevic [2] introduced a class of generalized open sets in a topological space, the so called b-open sets. The class of b-open sets is contained in the class of semi open and preopen sets. Tong [18] introduced the concept of t-set and B-set in topological space.In this paper we introduce the notion of locally b**-closed sets , b-B-set , b**-closed continuous and b-B-continuous function , and obtain another decomposition of continuity. All throughout this paper (X, τ) and (Y , σ) stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let A $\subseteq$ X , the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively.

A is regular open if A= Int(Cl(A)) and A is regular closed if A= Cl(Int(A)).The complement of b**-open set is said to be b**-closed. The intersection of all b**-closed sets of X containing A is called b**-closure of A and is denoted by b**Cl(A) of A. The union of all b**-open (resp. α-open , semi open ,preopen)sets of X contain in A is called b**-interior (resp. αInt(A) , sInt(A) ,pInt(A)). of A and is denoted by b**Int(A) of A. The family of all b**-open (resp. α-open , semi open ,preopen ,regular open, b**-closed, preclosed) subsets of a space X is denoted by B**O(X) (resp.αO(X),S O(X) and P O(X),B**Cl(X), PCl(X)).

Definition 1.1. A subset A of a space X is said to be :

1. α-open [12] if $A \subseteq Int(Cl(Int(A)))$;
2. Semi-open [9] if $A \subseteq Cl(Int(A))$;
3. preopen [15] if $A \subseteq Int(Cl(A))$;
4. b-open [1] if $A \subseteq Cl(Int(A)) \bigcup Int(Cl(A))$;
5. Semi-preopen [2] if $A \subseteq Cl(Int(Cl(A)))$;
6. b**-open if $A \subseteq Int(Cl(Int(A))) \bigcup Cl(Int(Cl(A)))$.

The following result will be useful in the sequel.

Lemma 1.1. [1] If A is a subset of a space (X, τ), then

1. $sInt(A) = A \bigcap Cl(Int(A))$;
2. $pInt(A) = A \bigcap Int(Cl(A))$;
3. $\alpha Int(A) = A \bigcap Int(Cl(Int(A)))$;
4. $bInt(A) = sInt(A) \bigcup pInt(A)$;
5. $b**Int(A) = \alpha Int(A) \bigcup \beta Int(A)$.

2.Locally b**-closed sets

Definition 2.1. A subset A of a space X is called:

1.t-set [18] if $Int(A) = Int(Cl(A))$.

2.B-set [18] if A=$U \bigcap V$ where $U \in \tau$ and V is a t-set.

3.locally closed [3] if $U \bigcap V$, where $U \in \tau$ and V is closed.

4. locally b closed if $U \bigcap V$, where $U \in \tau$ and V is b closed

5. locally b**- closed if $U \bigcap V$, where $U \in \tau$ and V is b**-closed.

We recall that a topological space (X, τ) is said to be extremally disconnected (briefly E.D.) if the closure of every open set of X is open in X. We note that a subset A of X is locally closed if and only if $A = U \bigcap Cl(A)$ for some open set U. The following example shows that the two notions of b**-open and locally closed are independent.

Example 2.1. Let X={a,b,c,d} and $\tau = \{\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}, X, \phi\}$ with B**O(X, τ) = $\{\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},\{a,b,d\}, X, \phi\}$ and the family of all locally closed sets is LC(X, τ) $=\{\{a\},\{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{a,c\},\{c,d\},\{a,b,c\}, X, \phi\}$.It is clear that {a,b,d} is b** open but not locally closed and {c,d} is locally closed but not b**-open.

Theorem 2.1. For a subset A of an extremally disconnected space (X, τ), the following are equivalent:

1. A is open,
2. A is b**-open and locally closed.

Proof. $(1) \Rightarrow (2)$. This is obvious from the definitions.

$(2)\Rightarrow(1)$ Let A be b**-closed and locally closed so $A\subseteq Cl(Int(Cl(A)))\cup Int(Cl(Int(A)))$, $A=U\cap Cl(A)$. Then

A $\subseteq U\cap[Int(Cl(Int(A)))\cup Cl(Int(Cl(A)))]$

$$\subseteq [(Int(U\cap Cl(Int(A)))\cup(U\cap Cl(Int(Cl(A))))]$$
$$\subseteq [Int(U\cap Int(Cl(A)))\cup(U\cap Cl(Cl(Int(A))))]$$
$$\subseteq [Int(Int(U\cap(Cl(A)))\cup(U\cap Cl(Int(A))))]$$
$$\subseteq Int(U\cap Cl(A))\cup Int(U\cap Cl(A))$$

$= IntA\cup IntA$ = Int A

Therefore A is open.

Definition 2.2. A subset A of a topological space X is called D(c,b**)-set if Int(A)=b**Int(A).

From the following examples one can deduce that b*8-open and D(c,b**)- set are independent.

Example 2.2. Let X={a,b,c,d}, $\tau=\{\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},X,\phi\}$.Then B**O(X, τ)= $\{\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},\{a,b,d\},X,\phi\}$, it is clear that A={c} is D(c,b**) but not b**-open .Also B={a,b,d} is b**-open but not D(c,b**).

Theorem 2.2. For a subset A of a space (X, τ), the following are equivalent:

1. A is open,
2. A is b**-open and D(c,b**)- set.

Proof . $(1)\Rightarrow(2)$ If A is open then A is b**-open and A=Int(A)=bInt(A) so A is D(c,b**)- set.

$(2)\Rightarrow(1)$ The condition $A\in B**O(X)$ and $A\in D(c,b**)$ imply A=b**Int(A) and Int(A) = b**Int(A) and consequently A is open.

Proposition 2.1. Let H be a subset of (X, τ), H is locally b**-closed if and only if there exist an open set U⊆X such that $H=U\cap b**Cl(H)$.

Proof. Since H being locally b**-closed, then $H = U \bigcap F$, where U is open and F is b**-closed. So H $\subseteq$ U and H $\subseteq$ F then H $\subseteq$ b**Cl(H) $\subseteq$ b**Cl(F) = F. Therefore $H \subseteq U \bigcap b**Cl(H) \subseteq U \bigcap b**Cl(F) = U \bigcap F$ =H.Hence $H = U \bigcap b**Cl(H)$. Conversely since b**Cl(H) is b**-closed and $H = U \bigcap bCl(H)$, then H is locally b**-closed.

Proposition 2.2.

1. The union of any family of b**-open sets is b**-open.
2. The intersection of an open set and a b**-open set is a b**-open set.

Proposition 2.3. Let A be a subset of a topological space X, if A is locally b**-closed, then

1. b**Cl(A) – A is b**-closed set.
2. $[[A \bigcup (X - b**Cl(A))]$ is b**-open.
3. $A \subseteq b**Int(A \bigcup (X - b**Cl(A)))$.

Proof. 1. If A is an locally b**-closed, there exist an U is open such that $A = U \bigcap b**Cl(A)$. Now

$$
\begin{aligned}
\text{B**Cl(A) – A} &= b**Cl(A) - [U \bigcap b**Cl(A)] \\
&= b**Cl(A) \bigcap [X - (U \bigcap b**Cl(A))] \\
&= b**Cl(A) \bigcap [(X - U) \bigcup (X - b**Cl(A))] \\
&= [b**Cl(A) \bigcap (X - U)] \bigcup [b**Cl(A) \bigcap (X - b**Cl(A))] \\
&= b**Cl(A) \bigcap (X - U)
\end{aligned}
$$

.

which is closed by proposition 2.2.

2. Since b**Cl(A) – A is b**-closed , then [X-b**Cl(A)-A] is b** open and [X-(b**Cl(A) –A)] = $X - [(b**Cl(A) \bigcap (X - A)]$ = $[A \bigcup (X - b**Cl(A))]$.

3. It is clear that $A \subseteq [A \bigcup (X - b**Cl(A))]$ = $b**Int[A \bigcup (X - b**Cl(A))]$.

As a consequence of the proposition 2.2 we have the following

Corollary 2.2.The intersection of a locally b**-closed set and locally closed set is locally b**-closed.

Let A,B $\subseteq$ X. Then A and B are said to be separated if $A \bigcap Cl(B) = \phi$ and $B \bigcap Cl(A) = \phi.$

Theorem 2.3. Suppose (X, τ), is closed under finite unions of b**-closed sets. Let A and B be the locally b**-closed. If A and B are separated, then $A \bigcup B$ is locally b**-closed.

Proof. Since A and b are locally b**-closed, $A = G \bigcap b^{**}Cl(A)$ and $B = H \bigcap b^{**}Cl(B),$ where G and H are open in X. Put $U = G \bigcap (X \setminus Cl(B)$ and $V = H \bigcap (X \setminus Cl(A).$ Then $U \bigcap b^{**}Cl(A) = ((G \bigcap (X \setminus Cl(B))) \bigcap b^{**}Cl(A) =$ $A \bigcap (X \setminus Cl(B) = A,$ Since $A \subseteq (X \setminus Cl(B).$ Similarly $V \bigcap b^{**}Cl(B) = B.$ And $U \bigcap b^{**}Cl(B) \subseteq U \bigcap Cl(B) = \phi$ and $V \bigcap b^{**}Cl(A) \subseteq V \bigcap Cl(A) = \phi$. Since U and V are open.

$$
\begin{aligned}
(U \bigcup V) \bigcap b^{**}Cl(A \bigcup B) &= (U \bigcup V) \bigcap (b^{**}Cl(A) \bigcup b^{**}Cl(B)) \\
&= (U \bigcap b^{**}Cl(A)) \bigcup (U \bigcap b^{**}Cl(B)) \bigcup (V \bigcap b^{**}Cl(A)) \bigcup (V \bigcap b^{**}Cl(B)) \\
&= A \bigcup B
\end{aligned}
$$

Hence A∪B is locally b**-closed.

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