Int Jr. of Mathematical Sciences & Applications
Vol. 5, No. 2, (July-December, 2015)

ISSN No: 2230-9888
www.journalshub.com

Convergence of Fuzzy Filters

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Abstract

We introduce the new concept convergent fuzzy filters in topological spaces.

Keywords: Convergent fuzzy filter, α fuzzy cofinite filter, α fuzzy neigbourhood filter.

I. Introduction

In a topological space filter is an important tool to study many properties. The closure of a set A can be characterized using convergent filters containing A. The continuity of a function from one topological space to another can be characterized using convergent filters.

In the year 1965 Lotfi A.Zadeh [2] introduced the concept of fuzzy sets and fuzzy logic.In the year 1968 C.L.Chang [1] introduced Fuzzy topological spaces. In the year 2014 We [3] introduced fuzzy sequences in a metric space. In the same year We [6] introduced fuzzy subsequences and limit of a fuzzy sequence in a metric space In the same year We [7] introduced dis convergent fuzzy sequences and divergent fuzzy sequence in R In the same year We [4] introduced fuzzy nets in a topological space and studied the properties of convergence.In the same year 2014 We [5] introduced the concept of fuzzification of filters in topological space and studied the properties.

II. Preliminaries

Let X be a non empty set. F ⊂ P(X) is called a filter(crisp filter) on X if 1. $\phi \notin F$ 2.F is closed under finite intersection. i.e., A, B ϵ F implies A∩B ϵ F. 3.B ϵ F and B ⊂ A implies A ϵ F. Let F be a crisp filter in a topological space. F is said to converge to an element a of X if F contains all the neighbourhoods of a.Let X be a non empty set.

A fuzzy set f on P(X) [$f : P(X) \to [0,1]$] is called a fuzzy filter if

1. $f(\phi) = 0$
2. $f(A \cap B) \geq \min \{f(A), f(B)\}$
3. $B \subset A$ implies $f(B) \leq f(A)$

In this paper we introduce convergent fuzzy filters and we study properties.

III. Convergence of fuzzy filter

Definition:3.1

Let X be a topological space. Let F be a fuzzy filter on X. Let a ϵ X. F is said to converge to a if for every open neighbourhood U of a, F(U) = 1. 'a' is called limit of F.

Definition:3.2

Let X be a topological space. Let F be a fuzzy filter on X. Let a ϵ X. Let α ϵ (0,1]. F is said to converge to 'a' at level α, if for every neighbourhood U of a, F(U) ≥ α. 'a' is called α-level limit of F.

Example: 3.3

Let X = {a,b,c}. T = {ϕ, {a}, X}. F: P(X) → [0,1] is defined as F(ϕ) = 0, F{a} = 1, F{b} = 0, F{c} = 0, F{a,b} = 1, F{a,c} = 1, F{b,c} = 0, F(X) = 1.

The neighbourhood of a are {a} and X.F{a} = 1 and F(X) = 1. Hence F converges to a.The neighbourhood of b is X only and F(X) = 1.Hence F → b.The neighbourhoods of c is X only and F(X) = 1.Hence F → c.

Result: 3.4

The above example shows that limit of a fuzzy filter need not be unique. But may be unique also.

Example: 3.5

X = {a,b} T = P(X). F : P(X) → [0,1] is defined as F(ϕ) = 0, F{a} = 1, F{b} = 0, F(X) = 1.Here F converges to a. F does not converge to b. So in this example limit of F is unique.

Example: 3.6

Let X = {a,b,c} T = {ϕ,{a},{b},{a,b},X}.F: P(X) → [0,1] is defined as F(ϕ) = 0, F{a} = .6 F{b} = .4 F{c} = .4 F{a,b} = .6 F{a,c} = .6 F{b,c} = .4 F(X) = 1. Take α = .6.Consider the neighbourhoods of a. {a}, {a,b}, X. F{a} ≥ α, F{a,b} ≥ α, F(X) ≥ α. Hence F converges to a at level α = .6 F does not converge to b at level α = .6 The neighbourhoods of b are {b}, {a,b}, X. Here F{b} = .4 is not greater than or equal to α. Hence F does not converge to b at level α.'The neighbourhoods of c is X only. F(X) = 1 ≥ α. Hence F converges to c at level α = .6

Result: 3.7

Let F be a fuzzy filter and let β ≤ α. Then if F converges to a at level α, then F converges to a at level β.

Theorem: 3.8

Let F be a fuzzy filter on X. Let α ϵ (0,1]. Let a ϵ X. Then the fuzzy filter F converges to a at level α iff the crisp filter α-cut of F converges to a.

Proof : Let F converge to a at level α.α_F = { A/ F(A) ≥ α}. Let U be a neighbourhood of a. Since F converges to a, F(U) ≥ α. Hence U ϵ α_F. The crisp filter α_F contains every neighbourhood of a. Hence α_F converges to a. Conversely, suppose α_F converges to a, We claim that the fuzzy filter F converges to a at level α. Let U be a neighbourhood of a. Since α_F converges to a, U ϵ αF. Hence F(U) ≥ α. Therefore the fuzzy filter F converges to a at level α.

Definition: 3.9 Fuzzy co finite filter

Let X be an infinite set. A function F : P(X) → [0,1] is called a fuzzy co-finite filter, if F(A) = 1 if A^C is finite and F(A) = 0 otherwise.

Theorem: 3.10

Fuzzy co finite filter is a fuzzy filter.

Proof : F : P(X) → [0,1] is defined as F(A) = 1 if A^C is finite and 0 otherwiseNow Consider ϕ ϵ P(X). ϕ^C = X which is not finite. Hence F(ϕ) = 0. Take A, B ϵ P(X). If F(A) = 0 and F(B) = 0 then whatever be the value of F(A∩B), we have F(A∩B) ≥ min {F(A), F(B)}. Now suppose F(A) = 1 and F(B) = 1, then A^C is finite and B^C is finite. Hence $A^C \cup B^C$ is finite which implies $(A\cap B)^C$ is finite. Hence F(A∩B) = 1. Hence F(A∩B) ≥ min {F(A), F(B)}. Now suppose F(A) = 0 and F(B) = 1, then min {F(A), F(B)} = 0. Hence F(A∩B) ≥ min {F(A), F(B)}. Let A⊂B. Then $A^C \supset B^C$. If F(A) = 0 then whatever be the value of F(B), we have F(B) ≥ F(A). If F(A) = 1 then A^c is finite. This implies that B^C is finite. Hence F(B) = 1. Therefore F(A) ≤ F(B). Hence A ⊂ B implies F(A) ≤ F(B). Hence F is a fuzzy filter.

Definition: 3.11 α- fuzzy co finite filter :

Let X be an infinite set. A fuzzy filter F on X is called a α-fuzzy co finite filter if F(A) = 1 if A^C is finite and F(A) < α otherwise.

Result: 3.12

It is clear that Fuzzy cofinite filter is an α fuzzy co finite filter for every α ϵ (0,1].

Result: 3.13

If F is a α fuzzy co finite filter and α ≤ β then F is a β fuzzy co finite filter.

Example: 3.14

Let X = N. Define F : P(N) → [0,1] as F(A) = 1 of A^C is finite, F(ϕ) = 0 and F(A) = 0.1 otherwise. It is clear that for α = 0.1. F is a α fuzzy co finite filter but it is not a fuzzy co finite filter. Hence we get the result .

Result: 3.15

A α- fuzzy co finite filter need not be fuzzy co finite filter.

Theorem: 3.16

Let X be an infinite set. Let F be a α fuzzy co finite filter. Then α cut of F is a crisp co finite filter on X.

Proof : X is an infinite set. F : P(X) → [0,1] is a α fuzzy co finite filter.

Let αF = { A / F(A) ≥ α}. αF is the α cut of F.

1. F is a filter and hence F(ϕ) = 0. Therefore F(ϕ) is not greater than or equal to α. Hence ϕ ∉ A.
2. Let A, B ϵ αF. This implies that F(A) = 1 and F(B) = 1. F(A∩B) = min {F(A), F(B)} = min {1,1} = 1 ≥ α. Hence A∩B ϵ αF.
3. A ⊂B and A ϵ αF implies F(A) ≥ α. Since F is a filter F(B) ≥ F(A) and hence F(B) ≥ α which implies F(A) = 1. Hence A^C is finite. Hence If A ≠ ϕ then A ϵ A iff A^C is finite. Hence A is a co finite filter. Hence α cut of a α fuzzy co finite filter is a crisp co finite filter.

Theorem: 3.17

Let X be an infinite set. Let T be the co finite topology in X. Then fuzzy co finite filter on X converges to every point of X at level α.

Proof : (X,T) is a topological space X is infinite. T is co finite topology. Take α ϵ (0,1]. Let F be the fuzzy co finite filter.Then F(A)=1 if A^C is finite and 0 otherwise.Let U be an open set containing x0. U is open implies U^C is finite. Hence F(U) = 1. Hence F(U) ≥ α. Therefore for every open set U contains x_0, we have F(U) ≥ α. Hence F converges to x_0 at level α. This is true for every x_0 ϵ X. Hence F converges to every point of X at level α.

Theorem: 3.18

Let X be an infinite set. Let T be the co finite topology. Let F be α fuzzy co finite filter. Then F converges to every pair of X at level α.

Proof : X is infinite set. T is co finite topology. α ϵ (0,1]. F(ϕ) = 0. For A≠ ϕ, F(A) = 1 if A^C is finite. If A is non empty and AC is not finite then F(A) < α. Take any x_0 ϵ X. Let U be an open set containing x_0. U is open implies U^C is finite. Since F is α fuzzy co finite filter and UC is finite, F(U) = 1. F(U) ≥ α. Hence F converges to x_0 at level α. This is true for every x_0 ϵ X. Hence F converges to every point of X at level α.

Definition: 3.19 Fuzzy neighbourhood filter

Let (X,T) be a topological space. Let A be a filter on X. Let a ϵ X. A is called fuzzy neighbourhood filter at a if A(A) = 1 if A is an open set containing a.

Definition: 3.20 α fuzzy neighbourhood filter

Let (X,T) be a topological space. Let ***A*** be a filter on X. Let a ϵ X. **A** is called α fuzzy neighbourhood filter at a if **A**(A) ≥ α if A is an open set containing a.

Example: 3.21

Let X = {a,b,c}. T = { ϕ, {a,b}, {b,c}, {b}, X} . **A** : P(X) → [0,1] is defined as **A**{ϕ} = 0, **A**{a} = 0, **A**{b} = 1, **A**{a,b} = 1, **A**{b,c} = 1 **A**{a,b,c} = 1 **A**(c) = 0, **A**(X) = 1. **A**{a,c} = 1. It is clear that A is a fuzzy neighbourhood filter at a. A is a fuzzy neighbourhood filter at b. A is a fuzzy neighbourhood filter at c.

Example: 3.22

Let X = {a,b,c }, T = { ϕ, {a,b}, {b,c}, {b}, X} . **A** : P(X) → (0,1] is defined as **A**(ϕ) = 0, **A**(a) = 0, **A**{a,b} = 0.4, **A**{b,c} = 0.4 **A**{b} = 0.4, **A**(c) = 0, **A**{a,c} = 0, **A**(X) = 1. Take α = 0.3. Consider a ϵ X. The open sets containing a are {a,b}, {b,c}, X. **A**{a,b} ≥ α, **A**{b,c} ≥ α. Hence **A** is α fuzzy neighbourhood filter at a. **A** is a α fuzzy neighbourhood filter at b and c also.

Example: 3.23

Let X = {a,b,c}, T = {ϕ, {a,b}, {b,c}, {b}, X}. Define fuzzy filter A as ϕ → 0, {a} → 0, {a,b} → 0.5, {b,c} → 0.6 {b} → 0.6 {c} →0 and X → 1, {a,c} → 0. Take α = 0.6. Consider a ϵ X. **A**{a,b} = 0.5 is not greater than or equal to 0.6 .

∴ **A** is not a α fuzzy neighbourhood filter at a. **A** is in α fuzzy neighbourhood filter at b and c.

Theorem: 3.24

Let (X,T) be a topological space. Let a ϵ X. Let **A** be a fuzzy neighbourhood filter at a. Then **A** converges to a.

Proof : (X,T) is a topological space. a ϵ X. **A** is a fuzzy neighbourhood filter at a. Let U be an open set containing a. Since **A** is a fuzzy neighbourhood filter at a, **A**(U) = 1. This is true for every open set U containing a. Hence **A** converges to a.

Theorem: 3.25

Let (X,T) be a topological space and let a ϵ X. Let **A** be a fuzzy filter on X. If **A** converges to a then **A** is a fuzzy neighbourhood filter at a.

Proof : (X,T) is a topological space. a ϵ X. Fuzzy filter **A** converges to a. Let U be an open set containing a. Since fuzzy filter **A** converges a, **A**(U) = 1. This is true for all open sets containing a. Hence **A** is a fuzzy neighbourhood filter at a.

Theorem: 3.26

Let (X,T) be a topological space. Let a ϵ X. Let **A** be a fuzzy filter on X. **A** converges to a iff **A** is a fuzzy neighbourhood filter at a.

Proof : Follows from previous theorems.

Theorem: 3.27

Let (X,T) be a topological space. Let a ϵ X. Let A be a fuzzy filter on X. **A** converges to a at level α ϵ (0,1] iff **A** is a α fuzzy neighbourhood filter at a.

Proof : (X,T) is a topological space. a ϵ X. **A** is a fuzzy filter on X.

A converges to a at level α iff For every open set is containing a, **A**(U) $\geq \alpha$
iff **A** is a α fuzzy neighbourhood filter at a.

References

1. C.L. Chang, "Fuzzy topological spaces", J.Math.Anal.Appl.24(1968),182-190.

2. L. A. Zadeh, " Fuzzy sets", Information and control, 8(1965)338-353.

3.M.Muthukumari, A.Nagarajan, M.Murugalingam , "Fuzzy Sequences in Metric Spaces" Int.Journal of Math.Analysis, Vol.8, 2014, no 15,699-706

4.M.Muthukumari, A.Nagarajan, M.Murugalingam , "Fuzzy Nets" Int.Journal of Math.Analysis, Vol.8, 2014, no 35,1715-1721

5. M.Muthukumari, A.Nagarajan, M.Murugalingam , "Fuzzification of Filters " Mathematical Sciences International Research Journal : Volume 3 Issue 2 (2014) , 669-671

6. M.Muthukumari, A.Nagarajan, M.Murugalingam , "Fuzzy Subsequences and Limit points " Int. Jr. of Mathematics Sciences & Applications (Accepted)

7. M.Muthukumari, A.Nagarajan, M.Murugalingam , "Disconvergent and Divergent Fuzzy sequences "Int. Mathematical Forum" (Accepted)