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Soft minimal and Soft Biminimal Spaces

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Abstract

The purpose of this paper is to introduce the concept of soft minimal and soft biminimal spaces and study some fundamental properties of $\tilde{m}$-soft closed set,$\tilde{m}$-soft open set, $\tilde{m}_1\tilde{m}_2$-soft closed sets and $\tilde{m}_1\tilde{m}_2$-soft open sets in soft minimal and soft biminimal spaces.
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1 Introduction

The concept of minimal structure(briefly m-structure) was introduced by V.Popa and T.Noiri[14]in 2000. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-closure and m_X-interior operators respectively. Further they introduced m-continuous functions and studied some of its basic properties. In 2010,C.Boonpok[2] introduced the concept of biminimal structure spaces. Also they introduced the notion of $m_X^1 m_X^2$-open sets and $m_X^1 m_X^2$-closed sets in biminimal structure spaces.

In the year 1999, Russian researcher Molodtsov [10],initaited the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics,computer science,economics,social sciences and medical sciences. Topological structures of soft set have been studied by some authors in recent years. M.Shabir and M.Naz[11],introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. They introduced the definition of soft open sets,soft closed sets,soft interior,soft closure and soft separation axioms. N.Cagman,S.Karatas and S.Enginoglu [4], defined the soft topology on a soft set and presented its related properties and foundations of the theory of soft topological spaces. Basavaraj M. Ittanagi [1], introduced and study the concept of soft bitopological spaces which are defined over an initial universe with a fixed set of parameters.

In this paper,we define soft minimal space,soft biminimal spaces,$\tilde{m}$-soft closed set, $\tilde{m}$-soft open set, $\tilde{m}_1\tilde{m}_2$-soft closed sets,$\tilde{m}_1\tilde{m}_2$-soft open sets.Also we study their properties and compared their properties with each other.

2 Preliminaries

Definition 2.1 [12] *Let X be a nonempty set and P(X) the power set of X. A subfamily m_X of P(X) is called a minimal structure(breifly m-structure) on X if $\emptyset \in m_X$ and $X \in m_X$*

By(X,m_X),We denote a nonempty set X with an m-structure m_X on X and it is called an m-space.Each member of m_X is said to be m_X-open and the complement of an m_X-open set is said to be m_X-closed.

Definition 2.2 [12] *Let X be a nonempty set and m_X an m-structure on X. For a subset A of X,the m_X-closure of A and the m_X-interior of A are defined as follows:*

(1) $mCl(A) = \cap \{F : A \subseteq F, X - F \in m_X\}$,

(2) $mInt(A) = \cup \{U : U \subseteq A, U \in m_X\}$.

Lemma 2.3 [9] *Let X be a nonempty set with a minimal structure m_X and A a subset of X. Then $x \in mcl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.*

Definition 2.4 [9] *An m-structure m_X on a nonempty set X is said to have property B if the union of any family of subsets belong to m_X belong to m_X.*

Lemma 2.5 [12] *Let X be a nonempty set and m_X an m-structure on X satisfying property B.For a subset A of X,the following properties hold:*

(1) $A \in m_X$ if and only if $mIntA = A$,

(2) If A is m_X-closed if and only if $mCl(A) = A$,

(3) $mInt(A) \in m_X$ and $mCl(A) \in m_X$-closed.

Definition 2.6 [3] *Let U be an initial universe and let E be a set of parameters.Let P(U) denote the power set of U,and $A \subseteq E$. A soft set F_A on the universe U is defined by the set of ordered pairs*

$$F_A = \{(x, f_A(x)) : x \in E\}$$

where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$.

Here, f_A is called approximate function of the soft set F_A.The value of $f_A(x)$ may be arbitary,some of them may be empty,some may have nonempty intresection.

Note that the set of all soft sets over U will be denoted by S(U).

Example 2.7 *Suppose that there are six students in the universe U=$\{s_1, s_2, s_3, s_4, s_5, s_6\}$ under consideration, and that $E = \{x_1, x_2, x_3, x_4, x_5\}$ is a set of decision parameters.The x_i(i=1,2,3,4,5) stand for the parameters "average", "active", "brilliant", "weak" and "slowlearners" respectively.*

Consider the mapping f_A given by "students(.)", where (.) is to be filled in by one of the parameters $x_i \in E$. For instance,$f_A(e_1)$ means "students(average)", and its functional value is the set $\{h \in U$:h is an average student$\}$

Suppose that A=$\{x_1, x_3, x_5\} \subseteq E$ and $f_A(x_1)$ =$\{s_1, s_3\}$,$f_A(x_3)$= $\{s_2, s_4, s_6\}$ and $f_A(x_5)$=U. Then we can view the soft set F_A as consisting of the following collection of approximations,

$$F_A = \{(x_1, \{s_1, s_3\}), (x_3, \{s_2, s_4, s_6\}), (x_5, U)\}$$

Definition 2.8 [3] *Let $F_A \in S(U)$. Then,*

1. *If $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called an empty set, denoted by $F_\emptyset$.*
2. *If $f_A(x) = U$ for all $x \in A$, then F_A is called A-universal soft set, denoted by $F_{\tilde{A}}$.*
3. *If $A = E$, then the A-universal soft set is called universal soft set, denoted by $F_{\tilde{E}}$*

Definition 2.9 [3] *Let $F_A, F_B \in S(U)$. Then,*

1. *F_A is a soft subset of F_B, denoted by $F_A \tilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.*
2. *F_A and F_B are soft equal, denoted by $F_A = F_B$ if and only if $f_A(x) = f_B(x)$ for all $x \in E$.*

Definition 2.10 [3] *Let $F_A, F_B \in S(U)$. Then, soft union $F_A \tilde{\cup} F_B$ and soft intersection $F_A \tilde{\cap} F_B$ of F_A and F_B are defined by the approximate functions, respectively,*

$$f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x),\ f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$$

and the soft complement $F_A^{\tilde{c}}$ of F_A is defined by the approximate function

$$f_{A^{\tilde{c}}}(x) = f_A^c(x)$$

where $f_A^c(x)$ is complement of the set $f_A(x)$. that is, $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

It is easy to see that $\left(F_A^{\tilde{c}}\right)^{\tilde{c}} = F_A$ and $F_\emptyset^{\tilde{c}} = F_{\tilde{E}}$

Definition 2.11 [4] *Let $F_A \in S(U)$. Power soft set of F_A is defined by*

$$\tilde{P}(F_A) = \left\{F_{Ai} \tilde{\subseteq} F_A : i \in I\right\}$$

and its cardinality is defined by

$$|\tilde{P}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$$

where $|f_A(x)|$ is cardinality of $f_A(x)$.

Example 2.12 *.Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$. Then*

$$\begin{aligned}
F_{A_1} &= \{(x_1, \{u_1\})\},\\
F_{A_2} &= \{(x_1, \{u_2\})\},\\
F_{A_3} &= \{(x_1, \{u_1, u_2\})\},\\
F_{A_4} &= \{(x_2, \{u_2\})\},\\
F_{A_5} &= \{(x_2, \{u_3\})\},\\
F_{A_6} &= \{(x_2, \{u_2, u_3\})\},
\end{aligned}$$

$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_3\})\}$,
$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$,
$F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$,
$F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\}$,
$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_3\})\}$,
$F_{A_{15}} = F_A$,
$F_{A_{16}} = F_\emptyset$.

are all soft subsets of F_A *. so* $|\tilde{P}(F_A)| = 2^4 = 16$.

3 Soft minimal Spaces

In this section,we introduce the concept of soft minimal space and study some properties of $\tilde{m}$-soft closed set and $\tilde{m}$-soft open set in soft minimal space.

Definition 3.1 *Let* F_A *be a nonempty soft set and* $\tilde{P}(F_A)$ *is the soft power set of* F_A*. A subfamily* $\tilde{m}$ *of* $\tilde{P}(F_A)$ *is called the soft minimal set on* F_A *if* $F_\emptyset \in \tilde{m}$ *and* $F_A \in \tilde{m}$*.*

$(F_A, \tilde{m})$ *is called a soft minimal space. Each member of* $\tilde{m}$ *is said to be* $\tilde{m}$ *-soft open set and the complement of an* $\tilde{m}$*-soft open set is said to be* $\tilde{m}$*-soft closed.*

Example 3.2 *Let* $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\} \subseteq E$ *and the* $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$, *Then*

$F_{A_1} = \{(e_1, \{u_1\})\}$,
$F_{A_2} = \{(e_1, \{u_2\})\}$,
$F_{A_3} = \{(e_1, \{u_1, u_2\})\}$,
$F_{A_4} = \{(e_2, \{u_2\})\}$,
$F_{A_5} = \{(e_2, \{u_3\})\}$,
$F_{A_6} = \{(e_2, \{u_2, u_3\})\}$,
$F_{A_7} = \{(e_1, \{u_1\}), (e_2, \{u_2\})\}$,
$F_{A_8} = \{(e_1, \{u_1\}), (e_2, \{u_3\})\}$,
$F_{A_9} = \{(e_1, \{u_1\}), (e_2, \{u_2, u_3\})\}$,
$F_{A_{10}} = \{(e_1, \{u_2\}), (e_2, \{u_2\})\}$,
$F_{A_{11}} = \{(e_1, \{u_2\}), (e_2, \{u_3\})\}$,
$F_{A_{12}} = \{(e_1, \{u_2\}), (e_2, \{u_2, u_3\})\}$,
$F_{A_{13}} = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2\})\}$,
$F_{A_{14}} = \{(e_1, \{u_1, u_2\}), (e_2, \{u_3\})\}$,
$F_{A_{15}} = F_A$,
$F_{A_{16}} = F_\emptyset$.

soft minimal set $(\tilde{m}) = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_9}, F_{A_{11}}, F_{A_{13}}\}$

Definition 3.3 *Let F_A be a nonempty soft set and $\tilde{m}$-be a soft minimal set on F_A.For a soft subset F_B of F_A, the $\tilde{m}$-soft closure of F_B and $\tilde{m}$-soft interior of F_B are defined as follows:*

(1) $\tilde{m}Cl(F_B) = \cap \{F_\alpha : F_B \subseteq F_\alpha, F_A - F_\alpha \in \tilde{m}\}$,

(2) $\tilde{m}Int(F_B)= \cup \{F_\beta : F_\beta \subseteq F_B, F_\beta \in \tilde{m}\}$.

Lemma 3.4 *Let F_A be a nonempty soft set and $\tilde{m}$ be a soft minimal set on F_A. For soft subset F_B and F_C of F_A,the following properties hold:*

(1) $\tilde{m}cl(F_A - F_B) = F_A - \tilde{m}Int(F_B)$ *and* $\tilde{m}Int(F_A - F_B) = F_A - \tilde{m}cl(F_B)$,

(2) If $(F_A - F_B) \in \tilde{m}$, *then* $\tilde{m}cl(F_B) = (F_B)$ *and if* $F_B \in \tilde{m}$,
then $\tilde{m}Int(F_B) = F_B$,

(3) $\tilde{m}cl(F_\emptyset) = F_\emptyset$, $\tilde{m}cl(F_A) = F_A$, $\tilde{m}Int(F_\emptyset) = F_\emptyset$ *and* $\tilde{m}Int(F_A) = F_A$,

(4) If $F_B \subseteq F_C$, *then* $\tilde{m}cl(F_B) \subseteq \tilde{m}cl(F_C)$ *and* $\tilde{m}Int(F_B) \subseteq \tilde{m}Int(F_C)$,

(5) $F_B \subseteq \tilde{m}cl(F_B)$ *and* $\tilde{m}Int(F_B) \subseteq F_B$,

(6) $\tilde{m}cl(\tilde{m}cl(F_B)) = \tilde{m}cl(F_B)$ *and* $\tilde{m}Int(\tilde{m}Int(F_B)) = \tilde{m}Int(F_B)$.

Definition 3.5 *A soft subset F_B of a soft minimal space $(F_A, \tilde{m})$ is called $\tilde{m}$-soft closed if $\tilde{m}cl(F_B) = F_B$. The complement of $\tilde{m}$-soft closed set is called $\tilde{m}$-soft open.*

Example 3.6 *. Refer Example 3.2, Let $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})$.Define soft minimal set $(\tilde{m})$ on F_A as follows:*
$\tilde{m} = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}, F_{A_7}\}$ *. Then F_{A_2} is $\tilde{m}$-soft closed.*

Let $(F_A, \tilde{m})$ be soft minimal space and F_B be a soft subset of F_A.Then F_B is $\tilde{m}$-soft closed if and only if $\tilde{m}cl(F_B)=F_B$.

Proposition 3.7 *Let $\tilde{m}$ be a soft minimal set on F_A satisfying property B. Then F_B is a $\tilde{m}$-soft closed soft subset of a soft minimal sapce$(F_A, \tilde{m})$ if and only if F_B is $\tilde{m}$-soft closed.*

Proposition 3.8 *Let $(F_A, \tilde{m})$ be a soft minimal space. If F_B and F_C are $\tilde{m}$-soft closed soft subset of $(F_A, \tilde{m})$,then $F_B \cap F_C$ is $\tilde{m}$-soft closed.*

Proof. Let F_B and F_C be $\tilde{m}$-soft closed. Then $\tilde{m}cl(F_B) = F_B$ and $\tilde{m}cl(F_C) = F_C$. since $F_B \cap F_C \subseteq F_B$ and $F_B \cap F_C \subseteq F_C$. $\tilde{m}cl(F_B \cap F_C) \subseteq \tilde{m}cl(F_B)$ and $\tilde{m}cl(F_B \cap F_C) \subseteq \tilde{m}cl(F_C)$. Therefore,$\tilde{m}cl(F_B \cap F_C) \subseteq \tilde{m}cl(F_B) \cap \tilde{m}cl(F_C) = F_B \cap F_C$. But $F_B \cap F_C \subseteq \tilde{m}cl(F_B \cap F_C)$. Consequently,$\tilde{m}cl(F_B \cap F_C) = F_B \cap F_C$. Hence,$F_B \cap F_C$ is $\tilde{m}$-soft closed. □

Remark 3.9 *The union of two $\tilde{m}$-soft closed set is not a $\tilde{m}$-soft closed set in general as can be seen from the following example.*

Example 3.10 *Refer Example 3.2, Let* $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$
Define soft minimal set $\tilde{m}$ *on* F_A *as follows:* $\tilde{m} = \{F_\emptyset, F_A, F_{A_1}, F_{A_5}, F_{A_7}\}$.
Then F_{A_2} *and* F_{A_4} *are* $\tilde{m}$*-soft closed. But* $F_{A_2} \cup F_{A_4} = F_{A_{10}}$ *is not* $\tilde{m}$*-soft closed.*

Proposition 3.11 *Let* $(F_A, \tilde{m})$ *be a soft minimal space. Then* F_B *is a* $\tilde{m}$*-soft open soft subset of* $(F_A, \tilde{m})$ *if and only if* $F_B = \tilde{m}Int(F_B)$.

Proof. Let F_B be a $\tilde{m}$-soft open soft subset of $(F_A, \tilde{m})$. Then F_A - F_B is $\tilde{m}$-soft closed. Therefore, $\tilde{m}cl(F_A - F_B) = F_A$ - F_B.
By Lemma 3.4 (1), F_A - $\tilde{m}Int(F_B) = F_A$ - F_B. Consequently,$F_B = \tilde{m}Int(F_B)$.
Conversely,let $F_B = \tilde{m}Int(F_B)$. Therefore F_A - $F_B = F_A$ - $\tilde{m}Int(F_B)$.
By Lemma 3.4 (1), F_A - $F_B = \tilde{m}cl(F_A - F_B)$. Hence, F_A - F_B is $\tilde{m}$-soft closed.
Consequently,F_B is $\tilde{m}$-soft open. □

Proposition 3.12 *Let* $(F_A, \tilde{m})$ *be a soft minimal space.If* F_B *and* F_C *are* $\tilde{m}$*- soft open soft subsets of* $(F_A, \tilde{m})$*,then* $F_B \cup F_C$ *is* $\tilde{m}$*-soft open.*

Proof. Let F_B and F_C be $\tilde{m}$-soft open. Then $\tilde{m}Int(F_B) = F_B$ and $\tilde{m}Int(F_C) = F_C$.
since $F_B \subseteq F_B \cup F_C$ and $F_C \subseteq F_B \cup F_C$, $\tilde{m}Int(F_B) \subseteq \tilde{m}Int(F_B \cup F_C)$ and
$\tilde{m}Int(F_C) \subseteq \tilde{m}Int(F_B \cup F_C)$. Therefore $F_B \cup F_C = \tilde{m}Int(F_B) \cup \tilde{m}Int(F_C) \subseteq \tilde{m}Int(F_B \cup F_C)$
But $\tilde{m}Int(F_B \cup F_C) \subseteq F_B \cup F_C$. Consequently,$\tilde{m}Int(F_B \cup F_C) = F_B \cup F_C$.
Hence $F_B \cup F_C$ is $\tilde{m}$-soft open.

□

Remark 3.13 *The intersection of two* $\tilde{m}$*-soft open set is not a* $\tilde{m}$*-soft open set in general as can be seen from the following example.*

Example 3.14 *.Refer Example 3.2 Let* $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$
Define soft minimal set $\tilde{m}$ *on* F_A *as follows:* $\tilde{m} = \{F_A, F_\emptyset, F_{A_2}, F_{A7}, F_{A_{10}}, F_{A_{14}}\}$
Then F_{A_7} *and* $F_{A_{10}}$ *are soft open. But* $F_{A_7} \cap F_{A_{10}} = F_{A_4}$ *is not* $\tilde{m}$*-soft open.*

Definition 3.15 *Let* $(F_A, \tilde{m})$ *be a soft minimal space and* F_Y *be a soft subset of* F_A.
Define soft minimal set $\tilde{m}_{F_Y}$ *on* F_Y *as follows:* $\tilde{m}_{F_Y} = \{F_B \cap F_Y | F_B \in \tilde{m}\}$. *Then* $(F_Y, \tilde{m}_{F_Y})$ *is called a soft minimal subspace of* $(F_A, \tilde{m})$.

Let $(F_Y, \tilde{m})$ *be a soft minimal subspace of* $(F_A, \tilde{m})$ *and let* F_B *be a soft subset of* F_Y. *The* $\tilde{m}_{F_Y}$*-soft closure and* $\tilde{m}_{F_Y}$*-soft interior of* F_B *are denote by* $\tilde{m}_{F_Y}cl(F_B)$ *and* $\tilde{m}_{F_Y}IntF_B$. *Then* $\tilde{m}_{F_Y}cl(F_B) = F_Y \cap \tilde{m}cl(F_B)$.

Proposition 3.16 *Let* $(F_Y, \tilde{m}_{F_Y})$ *be a soft minimal subspace of* $(F_A, \tilde{m})$ *and* $F\alpha$ *be a soft subset of* F_Y*.If* $F\alpha$ *is* $\tilde{m}$*-soft closed,then* $F\alpha$ *is* $\tilde{m}_{F_Y}$ *-soft closed.*

Proof. Let $F\alpha$ be $\tilde{m}$-soft closed.Then $\tilde{m}cl(F\alpha) = F\alpha$. Therefore,$\tilde{m}cl(F\alpha) = F\alpha$.
Hence $F_Y \cap \tilde{m}cl(F\alpha) = F\alpha$. Conesquently $\tilde{m}_{F_Y}cl(F\alpha) = F\alpha$.
Hence $F\alpha$ is $\tilde{m}_{F_Y}$-soft closed. □

4 Soft Biminimal Spaces

In this section, We introduce the concept of soft biminimal spaces and study some properties of $\tilde{m}_1\tilde{m}_2$-soft closed sets and $\tilde{m}_1\tilde{m}_2$-soft open sets in soft biminimal spaces.

Definition 4.1 *Let F_A be a nonempty soft set on the universe U, $\tilde{m}_1$ and $\tilde{m}_2$ be two different soft minimals on F_A. Then $(F_A, \tilde{m}_1, \tilde{m}_2)$ is called a soft biminimal space.*

Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space and F_B be a soft subset of F_A. The $\tilde{m}$-soft closure and $\tilde{m}$-soft interior of F_B with respect to $\tilde{m}_i$ are denote by $\tilde{m}cl_i(F_B)$ and $\tilde{m}Int_i(F_B)$,respectively,for i=1,2.

Definition 4.2 *A soft subset F_B of a soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$ is called $\tilde{m}_1\tilde{m}_2$-soft closed if $\tilde{m}cl_1(\tilde{m}cl_2(F_B)) = F_B$. The complement of $\tilde{m}_1\tilde{m}_2$-soft closed set is called $\tilde{m}_1\tilde{m}_2$-soft open.*

Example 4.3 *.Refer Example 3.2, Let $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$, Define soft minimals $\tilde{m}_1$ and $\tilde{m}_2$ on F_A as follows: $\tilde{m}_1 = \{F_A, F_\emptyset, F_{A_1}\}$ and $\tilde{m}_2 = \{F_A, F_\emptyset, F_{A_1}\}$. Then F_{A_2} is $\tilde{m}_1\tilde{m}_2$-soft closed.*

Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal spaces and F_B be a soft subset of F_A. Then F_B is $\tilde{m}_1\tilde{m}_2$-soft closed if and only if $\tilde{m}cl_1(F_B) = F_B$ and $\tilde{m}cl_2(F_B) = F_B$.

Proposition 4.4 *Let $\tilde{m}_1$ and $\tilde{m}_2$ be a soft minimal on F_A satisfying property B. Then F_B is a $\tilde{m}_1\tilde{m}_2$ -soft closed soft subset of a soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$ if and only if F_B is both $\tilde{m}_1$-soft closed and $\tilde{m}_2$-soft closed.*

Proposition 4.5 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space.If F_B and F_C are $\tilde{m}_1\tilde{m}_2$-soft closed soft subsets of $(F_A, \tilde{m}_1, \tilde{m}_2)$ then $F_B \cap F_C$ is $\tilde{m}_1\tilde{m}_2$-soft closed.*

Proof. Let F_B and F_C be $\tilde{m}_1\tilde{m}_2$-soft closed.Then $\tilde{m}cl_1(\tilde{m}cl_2(F_B)) = F_B$ and $\tilde{m}cl_1(\tilde{m}cl_2(F_C)) = F_C$. since $F_B \cap F_C \subseteq F_B$ and $F_B \cap F_C \subseteq F_C$,
$\tilde{m}cl_1(\tilde{m}cl_2(F_B \cap F_C)) \subseteq \tilde{m}cl_1(\tilde{m}cl_2(F_B))$ and $\tilde{m}cl_1(\tilde{m}cl_2(F_B \cap F_C)) \subseteq \tilde{m}cl_1(\tilde{m}cl_2(F_C))$.
Therefore, $\tilde{m}cl_1(\tilde{m}cl_2(F_B \cap F_C)) \subseteq \tilde{m}cl_1(\tilde{m}cl_2(F_B)) \cap \tilde{m}cl_1(\tilde{m}cl_2(F_C)) = F_B \cap F_C$.
But $F_B \cap F_C \subseteq \tilde{m}cl_1(\tilde{m}cl_2(F_B \cap F_C))$. Consequently,$\tilde{m}cl_1(\tilde{m}cl_2(F_B \cap F_C)) = F_B \cap F_C$.
Hence, $F_B \cap F_C$ is $\tilde{m}_1\tilde{m}_2$-soft closed. □

Remark 4.6 *The union of two $\tilde{m}_1\tilde{m}_2$-soft closed set is not a $\tilde{m}_1\tilde{m}_2$-soft closed set in general as can be seen from the following example.*

Example 4.7 *. Refer Example 3.2, Let $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$. Define soft minimals $\tilde{m}_1$ and $\tilde{m}_2$ on F_A as follows : $\tilde{m}_1 = \{F_\emptyset, F_A, F_{A_2}, F_{A_4}, F_{A_{11}}\}$ and $\tilde{m}_2 = \{F_A, F_\emptyset, F_{A_1}, F_{A_5}, F_{A_7}\}$.Then F_{A_5} and F_{A_7} are $\tilde{m}_1\tilde{m}_2$- soft closed. But $F_{A_5} \cup F_{A_7} = F_{A_9}$ is not a $\tilde{m}_1\tilde{m}_2$-soft closed.*

Proposition 4.8 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space. Then F_B is a $\tilde{m}_1\tilde{m}_2$-soft open soft subsets of $(F_A, \tilde{m}_1, \tilde{m}_2)$ if and only if $F_B = \tilde{m}Int_1(\tilde{m}Int_2(F_B))$.*

Proof. Let F_B be a $\tilde{m}_1\tilde{m}_2$-soft open soft subset of $(F_A, \tilde{m}_1, \tilde{m}_2)$. Then $F_A - F_B$ is $\tilde{m}_1\tilde{m}_2$-soft closed.Therefore,$\tilde{m}cl_1(\tilde{m}cl_2(F_A - F_B)) = F_A - F_B$.By Lemma 3.4(1), $F_A - \tilde{m}Int_1(\tilde{m}Int_2(F_B))$ $= F_A - F_B$. Consequently, $F_B = \tilde{m}Int_1(\tilde{m}Int_2(F_B))$.
Conversely,let $F_B = \tilde{m}Int_1(\tilde{m}Int_2(F_B))$. Therefore, $F_A - F_B = F_A - \tilde{m}Int_1(\tilde{m}Int_2(F_B))$. By Lemma 3.4 (1), $F_A - F_B = \tilde{m}cl_1(\tilde{m}cl_2(F_A - F_B))$.Hence $F_A - F_B$ is $\tilde{m}_1\tilde{m}_2$-soft closed. Consequently,F_B is $\tilde{m}_1\tilde{m}_2$-soft open. □

Proposition 4.9 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space. If F_B and F_C are $\tilde{m}_1\tilde{m}_2$-soft open soft subsets of $(F_A, \tilde{m}_1, \tilde{m}_2)$,then $F_B \cup F_C$ is $\tilde{m}_1\tilde{m}_2$ -soft open.*

Proof. Let F_B and F_C be $\tilde{m}_1\tilde{m}_2$-soft open. Then $\tilde{m}Int_1(\tilde{m}Int_2(F_B)) = F_B$ and $\tilde{m}Int_1(\tilde{m}Int_2(F_C))$ $= F_C$. Since $F_B \subseteq F_B \cup F_C$ and $F_C \subseteq F_B \cup F_C$, $\tilde{m}Int_1(\tilde{m}Int_2(F_B)) \subseteq \tilde{m}Int_1(\tilde{m}Int_2(F_B \cup F_C))$ and $\tilde{m}Int_1(\tilde{m}Int_2(F_C)) \subseteq \tilde{m}Int_1(\tilde{m}Int_2(F_B \cup F_C))$.
Therefore, $F_B \cup F_C = \tilde{m}Int_1(\tilde{m}Int_2(F_B)) \cup \tilde{m}Int_1(\tilde{m}Int_2(F_C)) \subseteq \tilde{m}Int_1(\tilde{m}Int_2(F_B \cup F_C))$.
But, $\tilde{m}Int_1(\tilde{m}Int_2(F_B \cup F_C)) \subseteq F_B \cup F_C$. Consequently,$\tilde{m}Int_1(\tilde{m}Int_2(F_B \cup F_C)) = F_B \cup F_C$.
Hence,$F_B \cup F_C$ is $\tilde{m}_1\tilde{m}_2$-soft open. □

Remark 4.10 *The intersection of two $\tilde{m}_1\tilde{m}_2$-soft open is not a $\tilde{m}_1\tilde{m}_2$-soft open set in general as can be seen from the following example.*

Example 4.11 *Refer Example 3.2, Let $F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$ Define soft minimals $\tilde{m}_1$ and $\tilde{m}_2$ on F_A as follows:*
$\tilde{m}_1 = \{F_\emptyset, F_A, F_{A_2}, F_{A_7}, F_{A_8}, F_{A_{11}}\}$
$\tilde{m}_2 = \{F_\emptyset, F_A, F_{A_2}, F_{A_9}, F_{A_{11}}\}$. *Then F_{A_8} and $F_{A_{11}}$ are $\tilde{m}_1\tilde{m}_2$ -soft open.*
But $F_{A_8} \cap F_{A_{11}} = F_{A_5}$ is not $\tilde{m}_1\tilde{m}_2$-soft open.

Definition 4.12 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space and F_Y be a soft subset of F_A. Define soft minimals $\tilde{m}_{1_{F_Y}}$ and $\tilde{m}_{2_{F_Y}}$ on F_Y as follows:*
$\tilde{m}_{1_{F_Y}} = \{F_B \cap F_Y | F_B \in \tilde{m}_1\}$ *and* $\tilde{m}_{2_{F_Y}} = \{F_C \cap F_Y | F_C \in \tilde{m}_2\}$.
A triple $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ is called a soft biminimal subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$.

Let $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ be a soft biminimal subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$ and let F_B be a soft subset of F_Y The $\tilde{m}_{F_Y}$ -soft closure and $\tilde{m}_{F_Y}$ -soft interior of F_B with respect to $\tilde{m}_{i_{F_Y}}$ are denote by $\tilde{m}_{F_Y}cl_i(F_B)$ and $\tilde{m}_{F_Y}Int_i(F_B)$, respectively, for i = 1,2, Then $\tilde{m}_{F_Y}cl_1(F_B) = F_Y \cap \tilde{m}cl_1(F_B)$ and $\tilde{m}_{F_Y}cl_2(F_B) = F_Y \cap \tilde{m}cl_2(F_B)$.

Proposition 4.13 *Let $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ be a soft biminimal subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$ and $F\alpha$ be a soft subset of F_Y. If $F\alpha$ is $\tilde{m}_1\tilde{m}_2$ -soft closed,then $F\alpha$ is $\tilde{m}_{1_{F_Y}}\tilde{m}_{2_{F_Y}}$ -soft closed.*

Proof. let $F\alpha$ be $\tilde{m}_1\tilde{m}_2$ -soft closed. Then $\tilde{m}cl_1(\tilde{m}cl_2(F\alpha)) = F\alpha$. Therefore, $\tilde{m}cl_1(F\alpha)$ $= F\alpha$ and $\tilde{m}cl_2(F\alpha) = F\alpha$. Hence, $F_Y \cap \tilde{m}cl_1(F\alpha) = F\alpha$ and $F_Y \cap \tilde{m}cl_2(F\alpha) = F\alpha$. Consequently, $\tilde{m}_{F_Y}cl_1(\tilde{m}_{F_Y}cl_2(F\alpha)) = F\alpha$.
Hence, $F\alpha$ is $\tilde{m}_{1_{F_Y}}\tilde{m}_{2_{F_Y}}$ -soft closed. □

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