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CONTROL VOLUME METHOD FOR MODELLING SLIGHTLY COMPRESSIBLE FLOW IN RESERVOIRS

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ABSTRACT

Numerical simulation is widely used for predicting reservoir behavior and forecasting its performance. In this paper numerical modeling is performed to simulate single phase flow in oil reservoir in horizontal well, with the main objective of determining pressure variation in the reservoir. The conservation of mass and momentum equation in form of Darcy's law are combined to form PDE's that govern the flow. One dimensional slightly compressible flow in Cartesian co-ordinates and radial co-ordinates with two injections and one extraction attached to the reservoir at distinct points is discussed. The governing equation is solved numerically using FVM with an assumption that the permeability and porosity are constant throughout the reservoir. Tri-diagonal Matrix Algorithm is developed from which iterative numerical solutions of pressure is obtain. The results were comparable with the analytical solution.

KEY Words: Finite Volume Methods (FVM), slightly compressible flow and Single phase flows, Tri-Diagonal Matrix Algorithm (TDMA).

Mathematics Subject Classification: 62J12, 62G99

Computing Classification System: I.4

1. INTRODUCTION

Reservoir is underground porous media usually containing rock, oil, gas and water. Reservoir simulation is the means by which one uses a numerical model of the geological and petro physical characteristics of hydrocarbons reservoir to analyze and predict fluid behavior in the reservoir over time.

(Dickstein, 1997) presented a FV model for the simulation of single phase flow of slightly compressible fluid in a reservoir drained by horizontal well. The grid was locally refined around the well to efficiently handle different time scales in a robust way.

(Cai, et al., 1997) studied the control-volume mixed finite element methods for obtaining accurate velocity approximations on irregular block-centered quadrilateral grids. The control volume formulation for Darcy's law was viewed as a discretization into element sized tanks with imposed pressures at the ends, giving a local discrete Darcy law analogous to the block-by-block conservation in the usual mixed discretization of the mass conservation equation. Numerical results in two dimensions showed second order convergence in the velocity, even with discontinuous anisotropic permeability on an irregular grid.

(Monkeberg, 2012) developed a routine to compute FVM's on triangular grids for solving hyperbolic PDEs. He implemented the FVM and used it to define an immiscible and incompressible two-phase flow in a rectangular domain. The tests of the implementation for incompressible single phase and incompressible two phase flows were correct.
The focus in this work was to simulate single phase flow through horizontal oil reservoirs and study the variation of pressure with time and space. FVM is applied to the discretization of the differential equations that describe the flow. 1-D slightly compressible flow is solved in Cartesian and radial co-ordinates, using FV cell-centered approach. For simplicity permeability and porosity were set constant throughout the reservoir.

2. MATHEMATICAL FORMULATION AND ANALYSIS

In general the flow of a fluid through a porous medium is modeled by Darcy's law, which is an expression of conservation of momentum. The law is used to describe oil, water and gas flows through petroleum reservoirs. It states that the volumetric flow rate is proportional to the gradient of the potential, which describes proportional relationship between the instantaneous discharge rate through a porous medium, the viscosity of the fluid and the pressure drop over a given time. The law is given by;

$$u = -\frac{k}{\mu}\nabla p \tag{1}$$

where k, μ, p and u are permeability of a porous medium, fluid viscosity, Darcy's velocity and pressure, respectively.

2.1 Continuity equation

The mass conservation equation in 3D is given by;

$$\frac{\partial(\varphi\rho)}{\partial t} = -\nabla\cdot(\rho u) + f \tag{2}$$

where ∇ is del operator, φ is the porosity and ρ is density.
Using equation (1) in (2) we obtain (3) which is the governing equation for single phase flow in reservoir

$$\frac{\partial(\varphi\rho)}{\partial t} = \nabla.\frac{\rho k}{\mu}(\nabla p - \rho g\nabla z) + f \tag{3}$$

Equation (3) can be transformed into equation (4) which is a parabolic equation in p governing slightly compressible flow of oil in the reservoir.

$$\varphi\rho C_t\frac{\partial p}{\partial t} = \nabla.\left(\frac{\rho k}{\mu}(\nabla p - \rho g\nabla z)\right) + Q \tag{4}$$

2.2 Theorem of existence and uniqueness of solution

To prove existence and uniqueness of solution of equation (4), we rewrite equations (1) and (2) as a system for the unknowns u and p

$$u + K\nabla p = 0, \tag{5}$$

$$div(u) = f. \tag{6}$$

In reservoirs, K can be discontinuous although the normal component of u is continuous.
Equations (5) and (6) in weak formulation are respectively:

$$u.v + K\nabla p.v = 0 \qquad \forall v \in V, \tag{7}$$

$$div(u)q = fq \qquad \forall q \in Q, \tag{8}$$

where $(u,p) \in U \times P$.
This methods have been considered in (Thomas, 1994) (Brezzi, 1991), with particular choice

$$\left.\begin{aligned} &H(div,\Omega) = \left\{v \in \left(L^2(\Omega)\right)^d,\ div(v) \in L^2(\Omega)\right\} \\ &U = H(div,\Omega), P = H_0^1(\Omega), V = \left(L^2(\Omega)\right)^d, Q = L^2(\Omega) \end{aligned}\right\} \tag{9}$$

where $H_0^1(\Omega) = \{p \in L^2(\Omega),\ Dp \in L^2(\Omega).\ p = 0 \text{ on } \partial\Omega\}$. However, it is generally difficult to find stable pair discrete spaces (U_h, P_h), $U_h \subset H(div,\Omega)$, $P_h \subset H_0^1(\Omega)$ for unstructured domain.
In this section we state a theorem which is an easy generalization of the results by (Nicolaides, 1982), (Brezzi, 1991) , (Babuska, 1992) and (Thomas, 1994).
Consider an abstract problem of finding $(u.p) \in U \times P$, given

$$\left.\begin{aligned} &a(u,v) + b(v,p) = < g, v > \quad \in V, \\ &c(u,q) = < f, q > \qquad \forall \in Q, \end{aligned}\right\} \tag{10}$$

where $(U, \|.\|u)$, $(P, \|.\|p)$, $(V, \|.\|v)$ and $(Q, \|.\|q)$ are four Hilbert spaces, $a(.,.)$, $b(.,.)$ and $c(.,.)$ are bilinear forms defined respectively on $U \times V$, $P \times V$ and $U \times Q$ spaces. The right hand sides are defined for $g \in V', f \in Q'$, where V' and Q' are the dual spaces of V and Q correspondingly.
Consider the discrete problem to find $(u_h, p_h) \in U_h \times P_h$:

$$\left.\begin{aligned} &a_h(u_h,v_h) + b_h(v_h,p_h) = < g, v_h > 1_h \quad \forall v_h \in V_h \\ &c_h(u_h,q_h) = < f, q_h > 1_h \quad \forall q_h \in Q_h, \end{aligned}\right\} \tag{11}$$

where $(U_h, \|.\|u_h)$, $(P_h, \|.\|p_h)$, $(V_h, \|.\|v_h)$ and $(Q_h, \|.\|q_h)$ are four Hilbert spaces, $a_h(.,.)$, $b_h(.,.)$ and $c_h(.,.)$ are bilinear forms defined respectively on $U_h \times V_h, P_h \times V_h$ and $U_h \times Q_h$. Suppose that $U_h \not\subset U$ and $V_h \not\subset V$, i.e., discretization of equation (13) is non-conforming, and let U_{0h} and V_{1h} be the spaces defined by

$$\left.\begin{aligned} &U_{0h} = \{u_h \in U_h, \quad \forall q_h \in Q_h, \quad c_h(u_h,q_h) = 0\} \\ &V_{1h} = \{v_h \in V_h, \quad \forall U_h \in U_{0h}, \quad a_h(u_h,v_h) = 0\ \} \end{aligned}\right\}. \tag{12}$$

Assuming that there exists three constants A, B and C independent of h such that

$$\left.\begin{aligned} a_h(u_h,v_h) &\le A\|u_h\|U_h\|v_h\|V_h, \\ b_h(v_h,p_h) &\le B\|v_h\|V_h\|p_h\|P_h, \\ c_h(u_h,q_h) &\le C\|u_h\|U_h\|q_h\|Q_h, \end{aligned}\right\} \tag{13}$$

and consider the Babuska-Brezzi conditions:

$$\left.\begin{aligned} &\inf_{u_h \in U_{0h}} \sup_{v_h \in V_h} \frac{a_h(u_h,v_h)}{\|u_h\|U_h\|v_h\|V_h} \ge \alpha, \\ &\inf_{p_h \in P_h} \sup_{v_h \in V_h} \frac{b_h(v_h,p_h)}{\|v_h\|V_h\|p_h\|P_h} \ge \beta, \\ &\inf_{q_h \in Q_h} \sup_{u_h \in U_h} \frac{c_h(u_h,q_h)}{\|u_h\|U_h\|q_h\|Q_h} \ge \gamma, \end{aligned}\right\} \tag{14}$$

such that $\dim(U_h) + \dim(P_h) = \dim(V_h) + \dim(Q_h)$ is satisfied.
Then problem (14) has a unique solution (u_h, p_h). Moreover, if α, β and γ are independent of h, there exists positive constant C independent of h such that

$$\|u - u_h\|U_h + \|p - p_h\|P_h \le C\left(\inf_{v_h \in V_h}\|u - v_h\|v_h + \inf_{p_h \in P_h}\|p - p_h\|P_h + M_{1h} + M_{2h} + M_{3h} + M_{4h}\right), \tag{15}$$

where

$$M_{1h} = \sup_{v_h \in V_h} \frac{a_h(u,v_h) + b_h(v_h,p) - < q, v_h >}{\|v_h\|v_h},$$

$$M_{2h} = \begin{matrix} sup \\ v_h \in V_h \end{matrix} \frac{< g, v_h > - < g, v_h > h}{\|v_h\| v_h},$$
$$M_{3h} = \begin{matrix} sup \\ q_h \in q_h \end{matrix} \frac{c_h(u, q_h) - < f, q_h >}{\|q_h\| Q_h},$$
$$M_{4h} = \begin{matrix} sup \\ q_h \in Q_h \end{matrix} \frac{< f. q_h > - < f, q_h > h}{\|q_h\| Q_h}.$$

and we can further use the results above for the bilinear forms:

$$\left.\begin{matrix} a(u,v) = \int u.v, & a_h(u_h, v_h) = \int u_h.v_h, \\ b(v,h) = \int K\nabla p.v, & b_h(v_h, p_h) = \int K\nabla p_h.v_h, \\ c(u,q) = \int div(u)q, & c_h(u_h, q_h) = \sum_{V\in v} \int u_h.nq_h, \end{matrix}\right\} \qquad (16)$$

where $U_h \not\subset U = H(div, \Omega), V_h \subset V = \left(L^2(\Omega)\right)^2, P_h \subset P = H_0^1(\Omega), Q_h \subset Q = L^2(\Omega)$.

3. NUMERICAL DISCRETIZATION

Suppose the domain Ω is divided into set of control volumes Ω_i, $i = 1, \ldots\ldots, n$, and assume Ω_i is smooth enough and integrating by parts, equation (4) over Ω_i we obtain equation (17) and (20) in. rectangular no source term and cylindrical coordinates with source terms, respectively.

3.1 Case 1

Integrating equation (4) over the control volume in the reservoir domain,Ω using FVM with the assumptions that source term is zero, reservoir is horizontal and one dimensional gives rise to

$$\iint_{\Omega_i} \varphi\rho C_t \frac{\partial p}{\partial t} d\Omega = \iint_{\Omega_i} \frac{k\rho}{\mu}\left(\frac{\partial^2 p}{\partial x^2}\right) d\Omega \qquad (17)$$

Simplifying (17) we obtain integral equation over the boundary of the control volume and we get

$$\frac{\varphi\mu C_t}{k}\int_{t_0}^{t} \frac{\partial p}{\partial t}\, dt = \int_{t_0}^{t}\int_{w}^{e} \frac{\partial^2 p}{\partial x^2} dx dt \qquad (18)$$

Summing numerical approximation of equation (18) over the entire domain, we get the components of the summation in the whole domain. Then applying TDMA for inversion we obtain the numerical solution. The discretized equation becomes

$$\frac{1}{\gamma}\left(p(t) - p(t_0)\right) = \frac{\Delta t}{\Delta x}(p_{i+1} - p_i) - \frac{\Delta t}{\Delta x}(p_i - p_{i-1}) \qquad (19)$$

where$\gamma = \frac{k}{\varphi\mu C_t}$

3.2 Case 2

We consider one dimensional flow, with a source term in cylindrical co-ordinates in equation (4), and then integrate over the control volume in the domain to obtain,

$$\frac{1}{\gamma}\int_{\Omega_i} r\frac{\partial p}{\partial t} d\Omega = \iint_{\Omega_i} \left(r\frac{\partial^2 p}{\partial r^2} + \frac{\partial p}{\partial r}\right) d\Omega + \iint_{\Omega_i} Q\, d\Omega \qquad (20)$$

Equation (20) is integrated over the boundary of the control volume and the approximation summed over the whole domain to obtain.

$$\frac{1}{2\gamma}({r^2}_{i+1} - {r^2}_{i-1})\{p(t) - p(t_0)\} = r_{i+1}\left(\frac{\partial p}{\partial r}\right) - r_{i-1}\left(\frac{\partial p}{\partial r}\right) + Q\Delta r\Delta t \quad (21)$$

4. RESULTS AND DISCUSSION

Consider a one dimensional flow model problem defined by equation for slightly compressible flow

$$\left.\begin{array}{ll} \varphi\rho C_t \dfrac{\partial p}{\partial t} = \dfrac{k\rho}{\mu}\left(\dfrac{\partial^2 p}{\partial x^2}\right) & \boldsymbol{x} \in \Omega \\ p(1,t) = p_1, & t > 0 \\ p(100,t) = p_2, & t > 0 \\ p(x,1) = 1000 & \end{array}\right\} \quad (22)$$

where $k = 1, c_t = 10^{-10}, \mu = 0.001,\ \varphi = 0.2$

The numerical and analytical pressure as a function of space variable x, from the node j=2, is illustrated in Figure 2 and Figure 3, pressure is shown as function of radius with the pressure values obtained by inversion using the TDMA. Figure 4 illustrates the injection and extraction attached to the reservoir at distinct points.

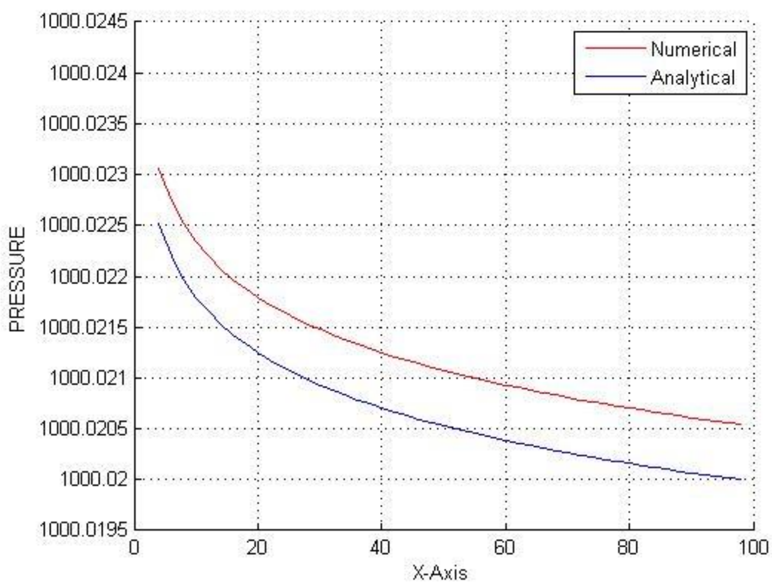

Figure 1: Numerical and analytical pressure as a function of space variable x

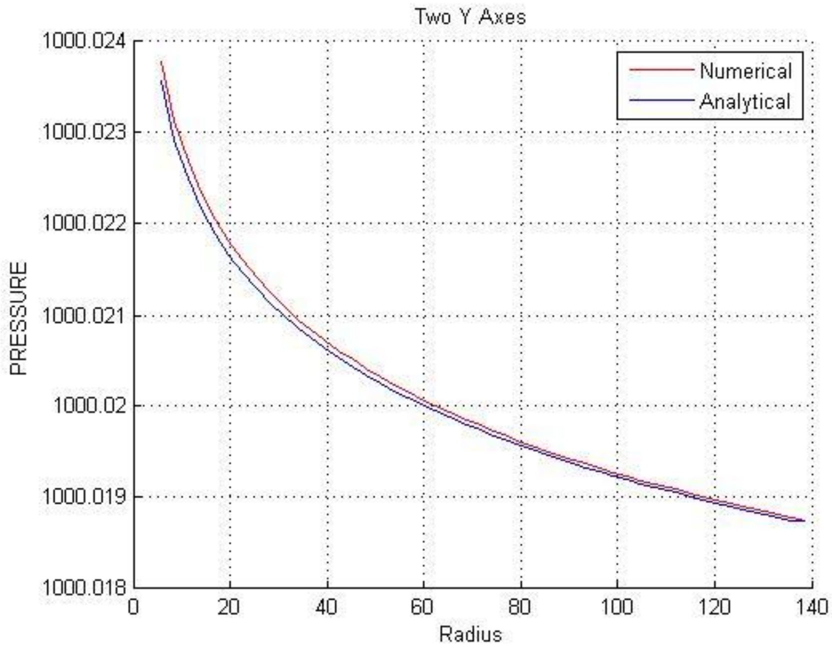

Figure 2: Numerical and analytical pressure distribution as a function of radius and time.

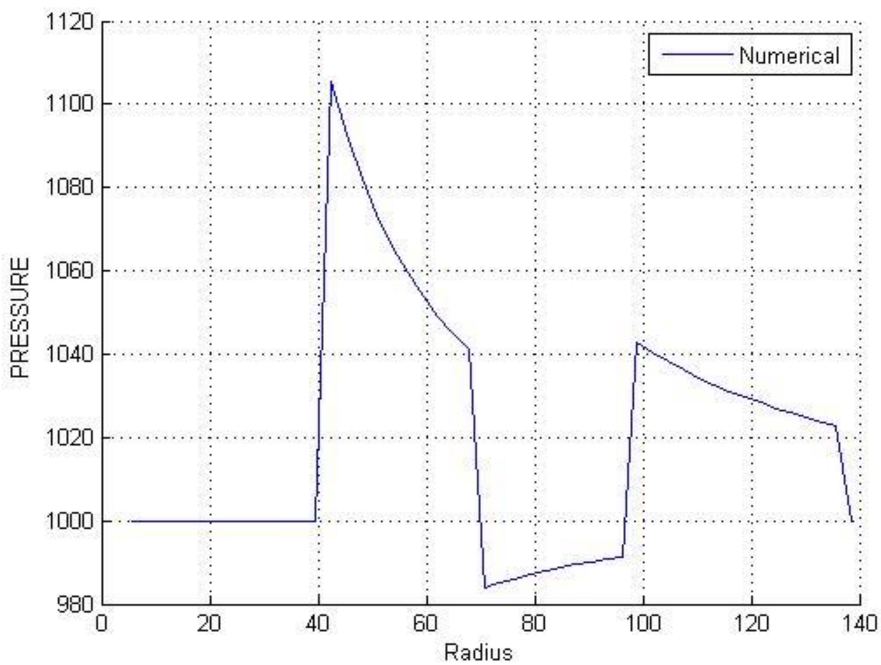

Figure 3: Injection of pressure at two points and extraction at one point in the reservoir with respect to radius.

5. CONCLUSIONS

The main objectives of this work was to model single phase flow in oil reservoirs and study the behavior of pressure in the reservoir of different orientations at constant permeability and porosity. CVM with rectangular gridding in the horizontal plane was developed for describing the fluid flow in oil reservoirs. The results obtained shows that Iterative scheme is the most accurate in approximation of numerical solutions in the CVM.

The scope of this work was to model the flow of oil through horizontal reservoir and simulate the variation of pressure under reservoir conditions. Further work can be carried out to determine pressure variations when permeability is a function of the space variable, and to investigate two phase flow of immiscible fluid in a vertical reservoir which captures realistic reservoir phenomenon.

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