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# Some Common Fixed Point Results in Dislocated Quasi –Metric Spaces for Contraction Mappings

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*Abstract*

***The aim of this paper is to determine some common fixed point theorems for contraction mappings. This results presented in the paper generalize and extend the result of A. K. Dubey, R. Shukla and R.P.dubey [14].***

**Mathematical Subject Classification: 47H10**

**Key Word:** Dislocated quasi –metric, fixed point, common fixed point.

1. Introduction

Fixed point theorems are irrevocable in the theory of non linear analysis. In this area, the first important and significant result was proved by Banach in 1922[1] for contraction mappings in complete metric space, which is well- known as a Banach fixed point theorem. A comprehensive literature and generalization of fixed point theorems or Banach contraction theorem can be found in [2], [3], [4] and [5]. Kannan [2] proved a fixed point theorem for new type of contraction mappings called Kannan mappings in a complete metric space. In some papers, authors define new contractions and discuss the existence and uniqueness of fixed point of for such spaces.

The concept of dislocated metric space was introduced by P. Hitzler and seda [6] and [7] in which thev self distance of points need not to be zero necessarily. They also generalized famous Banach contraction principle in dislocated metric space. Recently, Zeyada et al. [8] develops the notion of complete $d_q -$ metric spaces and generalized fixed point theorem due to Hitzler and Seda[6] in dislocated quasi- metric spaces. After many papers have been published containing fixed point results in$d_q -$ metric spaces see([9], [10],[11], [13],[14] and [15]. In 2014, Dubey et al. [14] established some fixed point theorems in dislocated quasi-metric space as follows:

**Theorem 1.1:** Let$(X,\ d)$ be a complete dislocated quasi- metric space and suppose $T: X \longrightarrow X$ be a continuous mapping satisfying the following condition,

$$d(Tx,Ty) \leq \alpha d(x\,,y) + \beta[\, d(x,Tx) + d(y,Ty)\,] \text{ ………. (1)}$$

where $\alpha, \beta$ are non negative , which may depends on both $x$ and $y$ , such that

$$\text{Sup}\{\alpha + 2\beta : x, y \in X\} < 1.$$

Then $T$ has unique fixed point.

**Theorem 1.2:** Let$(X, d)$ be a complete dislocated quasi- metric space and suppose $T: X \longrightarrow X$ be a continuous function **satisfying** the following condition,

$$d(Tx, Ty) \leq \alpha d(x, y) + \beta[d(x, Ty) + d(y, Tx)] \text{ ………. (2)}$$

where $\alpha, \beta$ are non negative , which may depends on both $x$ and $y$ , such that

$$\text{Sup}\{\alpha + 2\beta : x, y \in X\} < 1.$$

In this paper, we prove common fixed point theorems for contraction mappings in dislocated quasi-metric space. Our results generalize and extend the respective above theorems 1.1 and 1.2 of [8].

## 2. Preliminaries

**Definition 2.1 [6] :** Let $X$ be a non – empty set let $d: X\, xX \rightarrow [0, \infty]$ be a function satisfies the following conditions:

(i) $d(x, y) = d(y, x) = 0$ implies $x = y$.

(ii) $d(x, y) \leq d(x, z) + d(z, y)$ for all$x, y, z \in X$.

Then $d$ is called a dislocated quasi - metric on $X$. If $d$ satisfies $d(x, x) = 0$, then it is called a quasi – metric on $X$. If $d$ satisfies $d(x, y) = d(y, x)$, then it is called dislocated metric.

**Definition 2.2[6]:** Let $(X, d)$ be a dislocated quasi -metric space, $x \in X$ and $\{\ _n\}_{n\geq 1}$ a sequence in X. Then,

(i) $\{x_n\}_{n\geq 1}$ dislocated quasi - converges to x if $\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0$

(ii) $\{x_n\}_{n\geq 1}$ is said to be a Cauchy sequence if for given $\varepsilon > 0$ $\exists$ $n_0 \in N$ such that

$$d(x_m, x_n) < \varepsilon \text{ or } (x_n, x_m) < \varepsilon \text{ for all } m, n \geq n_0$$

(iii) A dislocated quasi – metric space $(X, d)$ is called a complete if every Cauchy sequence in X is a dislocated convergent

**Definition 2.3[6]:** Let $(X, d_1)$ and $(X, d_2)$ be a dislocated metric space and $T: X \to Y$ be a function. Then $T$ is continuous to $x_o \in X$, if for each sequence $\{x_n\}_{n\geq 1}$ which is $d_1 - q$ convergent to $\square_0$ , the sequence$\{\square(\square_\square)\}$ is $\square_2 - \square$ convergent to $\square(\square_0)$ in Y.

**Definition 2.4[6]:** Let $(X, d)$ be a dislocated quasi -metric space. A map $\square: \square \to \square$ is a contraction, if there exist $0 \leq \square < 1$ such that

$$\square(\square\square, \square\square) \leq \square\square(\square, \square) \text{ for all} \square, \square, \in \square.$$

**Lemma 2.5 [6]:** Limit in a $\square_\square$- metric space is unique.

**Lemma 2.6 [6]:** let $(X, d)$ be a dislocated quasi -metric space and let $\square: \square \to \square$ be a continuous function, then $\left\{\left(\square^\square(\square_0)\right)\right\}$ is a Cauchy sequence for each $\square_0 \in \square$.

## 3. Main results

**Theorem 3.1:** Let$(X, d)$ be a complete dislocated quasi- metric space and suppose $\square_1, \square_2: \square \longrightarrow \square$ be any two continuous function **satisfying** the following condition,

$$\square(\square_1\square, \square_2\square) \leq \square\square(\square, \square) + \square[\square(\square, \square_1\square) + \square(\square, \square_2\square)] \;\ldots\ldots\ldots\; (1)$$

where □, □ are non negative , which may depends on both □ and □ , such that

$$\mathrm{Sup}\{\square + 2\square : \square, \square \in \square\} < 1.$$

Then $\square_1$ and $\square_2$ have a unique common fixed point in □.

**Proof:** For each $\square_0 \in \square$ and n ≥ 1, set $\square_1 = \square_1\square_0$ and $\square_{2\square+1} = \square_1\square_{2\square} = T_1^{2n+1}x_0$.

Similarly $\square_{2\square+2} = \square_2\square_{2\square+1} = T_2^{2n+2}\square_0$ . Then we have

$$\square\left(\square_{2\square}, \square_{2\square+1}\right) = \square\left(\square_1\square_{2\square-1}, \square_2\square_{2\square}\right)$$

$$\leq \square\square(\square_{2\square-1}, \square_{2\square}) + \square[\square(\square_{2\square-1}, \square_1\square_{2\square-1}) + \square(\square_{2\square}, \square_2\square_{2\square})]$$

$$= \square\square(\square_{2\square-1}, \square_{2\square}) + \square[\square(\square_{2\square-1}, \square_{2\square}) + \square(\square_{2\square}, \square_{2\square+1})]$$

$$\leq (\square + \square)\square((\square_{2\square}, \square_{2\square-1}) + \square[\square(\square_{2\square}, \square_{2\square+1})]$$

$$\square\left(\square_{2\square}, \square_{2\square+1}\right) \leq \frac{\square+\square}{1-\square}\square((\square_{2\square}, \square_{2\square-1})$$

$$\square\left(\square_{2\square+1}, \square_{2\square}\right) \leq \square\,\square((\square_{2\square}, \square_{2\square-1})$$

Where $\square = \frac{\square+\square}{1-\square}$

Similarly, we can show that $\square\left(\square_{2\square-1}, \square_{2\square}\right) \leq \square\,\square(\square_{2\square-2}, \square_{2\square-1})$

In this way, we get $\square\left(\square_{2\square+1}, \square_{2\square}\right) \leq \square^{\square}\square((\square_0, \square_1)$

Since $0 \leq \square < 1$, so for $\square \to \infty$, $\square^{\square} \to 0$ we have $\square\left(\square_{2\square+1}, \square_{2\square}\right) \to \infty$. Hence $\{\square_{2\square}\}$ is a Cauchy sequence in the complete dislocated quasi-metric space in □. So, there is a point $\square_0 \in \square$ such that $\square_{2\square} \to \square_0$ as $\square \to \infty$. also the sub sequences $\{\square_{2\square}\}$ and $\{\square_{2\square+1}\}$ converge to $\square_0$.

Since $\square_1$ is continuous we have

$$\square_1(\square_0) = \lim_{\square\to\infty} \square_1\square_{2\square} = \lim_{\square\to\infty} \square_{2\square+1} = \square_0$$

Thus $\square_1(\square_0) = \square_0$. Similarly, taking the continuity of $\square_2$. We can show that $\square_2(\square_0) = \square_0$.

Hence $\square_0$ is the common fixed point of $\square_1$ and $\square_2$.

**Uniqueness:** Let $\square_1$ and $\square_2$ have two common fixed points of $\square_0$ and $\square_0$ for $\square_0 \neq \square_0$. Then by given condition we have

$$\square(\square_0, \square_0) = \square(\square_1\square_0, \square_2\square_0)$$

$$\leq \square\square(\square_0, \square_0) + \square[\square(\square_0, \square_1\square_0) + \square(\square_0, \square_2\square_0)] \ldots\ldots\; (2)$$

Since $\square_0$ and $\square_0$ are common fixed points of $\square_1$ and $\square_2$, condition (1) implies that $\square(\square_0, \square_0) = 0$

and $(\square_0, \square_0)$ . Thus equation (2) becomes

$$\square(\square_0, \square_0) \leq \square\square(\square_0, \square_0) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (3)$$

Similarly we have

$$□(□_0 , □_0) \leq □□(□_0 , □_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (4)$$

Subtracting (4) from (3) we get

$$|□(□_0 , □_0) - □(□_0 , □_0)| \leq |□||□(□_0 , □_0) - □(□_0 , □_0)|$$

Since $□ < 1$, so the above inequality is possible if

$$|□(□_0 , □_0) - □(□_0 , □_0)| = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (5)$$

By combining equations (3), (4) and (5), one can get $□(□_0 , □_0) = 0$ and $□(□_0 , □_0) = 0$. Using (i) we have $□_0 = □_0$. Hence $□_1$ and $□_2$ have a unique common fixed point in □.

**Theorem 3.2:** Let$(X, d)$ be a complete dislocated quasi- metric space and suppose $□_1, □_2 : □ \longrightarrow □$ be any two continuous function **satisfying** the following condition,

$$□(□_1□,\ □_2□) \leq □□(□ , □) + □[\ □(□, □_2□) + □(□, □_1 □)\ ] \ \ldots\ldots\ (2)$$

where □, □ are non negative , which may depends on both □ and □ , such that

$$\text{Sup}\{□ + 2□ : □, □ \in □\} < 1.$$

Then $□_1$ and $□_2$ have an unique common fixed point in □.

**Proof:** For each $□_0 \in □$ and n ≥ 1, set $□_1 = □_1□_0$ and $□_{2□+1} = □_1□_{2□} = T_1^{2n+1}x_0$.

Similarly $□_{2□+2} = □_2□_{2□+1} = T_2^{2n+2}□_0$ . Then we have

$$□\left(□_{2□}, □_{2□+1}\right) = □\left(□_1□_{2□-1}, □_2□_{2□}\right)$$

$$\leq □□(□_{2□-1}, □_{2□}) + □[□(□_{2□-1}, □_2□_{2□}) + □(□_{2□}, □_1□_{2□-1})]$$

$$= □□(□_{2□-1}, □_{2□}) + □[□(□_{2□-1}, □_{2□+1}) + □(□_{2□}, □_{2□})]$$

$$\leq □□(□_{2□}, □_{2□-1}) + □[□(□_{2□}, □_{2□-1}) + □(□_{2□+1}, □_{2□})]$$

$$\leq (□ + □)□((□_{2□}, □_{2□-1}) + □[□(□_{2□+1}, □_{2□})]$$

$$□\left(□_{2□+1}, □_{2□}\right) \leq \frac{□+□}{1-□}□((□_{2□}, □_{2□-1})$$

$$□\left(□_{2□+1}, □_{2□}\right) \leq □\ □((□_{2□}, □_{2□-1})$$

Where $□ = \frac{□+□}{1-□}$

Rest of the proof of this theorem is same as theorem 3.1.

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