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Common quadruple fixed point theorem for hybrid pair of mappings under ϕ-ψ contraction

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Abstract: We establish a quadruple coincidence and common quadruple fixed point theorem for hybrid pair of mappings under ϕ-ψ contraction on a noncomplete metric space, which is not partially ordered. It is to be noted that to find quadruple coincidence point, we do not employ the condition of continuity of any mapping involved therein. We also give an example to validate our result. We improve and generalize several known results.

Mathematics Subject Classification: 47H10, 54H25.

Key words and phrases: Metric space, quadruple fixed point, quadruple coincidence point, ϕ-ψ contraction, w-compatibility, F-weakly commuting mappings.

1 Introduction and Preliminaries:

Let (X, d) be a metric space and CB(X) be the set of all nonempty closed bounded subsets of X. Let D(x, A) denote the distance from x to $A \subset X$ and H denote the Hausdorff metric induced by d, that is,

$D(x, A) = \inf_{a \in A} d(x, a)$

and $H(A, B) = \max\{\sup_{a \in A} D(a, B), \sup_{b \in B} D(b, A)\}$, for all $A, B \in CB(X)$.

The existence of fixed points for various multivalued contractive mappings has been studied by many authors under different conditions, which has found application in control theory, convex optimization, differential inclusions and economics.

In [7], Bhaskar and Lakshmikantham established some coupled fixed point theorems in the setting of single-valued mappings and apply these to study the existence and uniqueness of solution for periodic boundary value problems. Luong and Thuan [20] generalized the results of Bhaskar and Lakshmikantham [7]. Berinde [4] extended the results of Bhaskar and Lakshmikantham [7] and Luong and Thuan [20]. Lakshmikantham and Ciric [17] proved coupled coincidence and common coupled fixed point theorems for nonlinear contractive mappings in partially ordered complete metric spaces and extended the results of Bhaskar and Lakshmikantham [7]. Jain, Tas, Kumar and Gupta [15] extended and generalized the results of Berinde [4], Bhaskar and Lakshmikantham [7], Lakshmikantham and Ciric [17] and Luong and Thuan [20].

Berinde and Borcut [5] introduced the concept of tripled fixed point for single-valued mappings in partially ordered metric spaces and established the existence of tripled fixed point for single-valued mappings in partially ordered metric spaces.Karapinar [16] introduced the concept of quadruple fixed point for single valued mappings in partially ordered metric spaces and established some quadruple fixed point theorems.Many researchers have studied coupled, tripled and quadruple fixed point theorems for single valued mappings including [2, 3, 4, 6, 8, 9, 10, 14, 15, 16, 18, 20, 21, 22, 23, 24, 26].Recently Samet, Karapinar, Aydi and Rajic [25] claimed that most of the coupled fixed point theorems for single valued mappings on ordered metric spaces are consequences of well-known fixed point theorems.

Very few papers were devoted to coupled, tripled and quadruple fixed point problems for hybrid pair of mappings [1, 11, 12, 13, 19].Coupled fixed point theory for multivalued mappings was extended by Abbas, Ciric, Damjanovic and Khan [1] and obtained coupled coincidence points and common coupled fixed point theorems involving hybrid pair of mappings satisfying generalized contractive conditions in complete metric space.

The concepts of tripled fixed point theory in the settings of multivalued mappings was extended by Deshpande, Sharma and Handa [11] and obtained tripled coincidence points and common tripled fixed point theorems involving hybrid pair of mappings under generalized nonlinear contraction.

Further quadruple fixed point theory was extended by Deshpande and Handa [12] to multivalued mappings and obtained quadruple coincidence points and common quadruple fixed point theorems involving hybrid pair of mappings under ϕ-ψ contraction.

In [12], Deshpande and Handa introduced the following for multivalued mappings:

Definition 1.1. Let X be a nonempty set, $F : X^4 \to 2^X$ (a collection of all nonempty subsets of X) be a multivalued mapping and g be a self-mapping on X. An element $(x, y, z, w) \in X^4$ is called

(1) a quadruple fixed point of F if $x \in F(x, y, z, w)$, $y \in F(y, z, w, x)$, $z \in F(z, w, x, y)$, $w \in F(w, x, y, z)$.

(2) a quadruple coincidence point of hybrid pair {F, g} if $g(x) \in F(x, y, z, w)$, $g(y) \in F(y, z, w, x)$, $g(z) \in F(z, w, x, y)$, $g(w) \in F(w, x, y, z)$.

(3) a common quadruple fixed point of hybrid pair {F, g} if $x = g(x) \in F(x, y, z, w)$, $y = g(y) \in F(y, z, w, x)$, $z = g(z) \in F(z, w, x, y)$, $w = g(w) \in F(w, x, y, z)$.

We denote the set of quadruple coincidence points of mappings F and g by C{F, g}. Note that if $(x, y, z, w) \in$ C{F, g}, then (y, z, w, x), (z, w, x, y) and (w, x, y, z) are also in C{F, g}.

Definition 1.2. Let $F : X^4 \to 2^X$ be a multivalued mapping and g be a self-mapping on X. The hybrid pair {F, g} is called w-compatible if $g(F(x, y, z, w)) \subseteq F(gx, gy, gz, gw)$ whenever $(x, y, z, w) \in C\{F, g\}$.

Definition 1.3. Let $F : X^4 \to 2^X$ be a multivalued mapping and g be a self-mapping on X. The mapping g is called F-weakly commuting at some point $(x, y, z, w) \in X^4$ if $g^2x \in F(gx, gy, gz, gw)$, $g^2y \in F(gy, gz, gw, gx)$, $g^2z \in (gz, gw, gx, gy)$, $g^2w \in F(gw, gx, gy, gz)$.

Lemma 1.1. Let (X, d) be a metric space. Then, for each $a \in X$ and $B \in CB(X)$, there is $b_o \in B$ such that $D(a, B) = d(a, b_o)$, where $D(a, B) = \inf_{b \in B} d(a, b)$.

In this paper, we establish a quadruple coincidence and common quadruple fixed point for hybrid pair of mappings under ϕ-ψ contraction on a noncomplete metric space, which is not partially ordered. It is to be noted that to find quadruple coincidence point, we do not use the condition of continuity of any mapping involved therein. We improve, extend and generalize the results of Berinde [4], Bhaskar and Lakshmikantham [7], Lakshmikantham and Ciric [17], Jain, Tas, Kumar and Gupta [15], Luong and Thuan [20] and many others. An example is furnished which demonstrate the validity of the hypotheses and degree of generality of our main result.

2. Main results

Let Φ denote the set of all functions $\phi : [0, +\infty) \to [0, +\infty)$ satisfying

(i_ϕ) ϕ is continuous and (strictly) increasing,

(ii_ϕ) $\phi(t) < t$ for all $t > 0$,

(iii_ϕ) $\phi(t+s) \le \phi(t) + (s)$ for all $t, s>0$.

Note that, by (i_ϕ) and (ii_ϕ) we have that $\phi(t) = 0$ if and only if $t = 0$.

Let Ψ denote the set of all functions $\psi : [0, +\infty) \to [0, +\infty)$ which satisfies

(i_ψ) $\lim_{t\to r} \psi(t) > 0$ for all $r > 0$ and $\lim_{t\to 0+} \psi(t)=0$.

Now, we prove the following theorem:

Theorem 2.1. Let (X, d) be a metric space. Assume $F : X^4 \to CB(X)$ and $g : X \to X$ be two mappings satisfying

$$\phi\left(\frac{1}{4}\begin{bmatrix} H(F(x,y,z,w), F(p,q,r,s)) \\ +H(F(y,z,w,x), F(q,r,s,p)) \\ +H(F(z,w,x,y), F(r,s,p,q)) \\ +H(F(w,x,y,z), F(s,p,q,r)) \end{bmatrix}\right) \tag{2.1}$$

$$\le \phi\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right)$$

for all $x, y, z, w, p, q, r, s \in X$, where $\phi \in \Phi$ and $\psi \in \Psi$. Furthermore assume that $F(X\times X \times X\times X) \subseteq g(X)$ and g(X) is a complete subset of X. Then F and g have a quadruple coincidence point. Moreover, F and g have a common quadruple fixed point, if one of the following conditions holds:

(a) F and g are w-compatible. $\lim_{n\to\infty} g^n x = p$, $\lim_{n\to\infty} g^n y = q$, $\lim_{n\to\infty} g^n z = r$, $\lim_{n\to\infty} g^n w = sw$ for some $(x, y, z, w) \in C\{F, g\}$ and for some $p, q, r, s \in X$ and g is continuous at p, q, r, s.

(b) g is F-weakly commuting for some $(x, y, z, w) \in C\{F, g\}$ and gx, gy, gz, gw are fixed points of g, that is, $g^2x = gx$, $g^2y = gy$, $g^2z = gz$, $g^2w = gw$

(c) g is continuous at x, y, z, w. $\lim_{n\to\infty} g^n p = x$, $\lim_{n\to\infty} g^n q = y$, $\lim_{n\to\infty} g^n r = z$, $\lim_{n\to\infty} g^n r = w$ for some $(x, y, z, w) \in C\{F, g\}$ and for some $p, q, r, s \in X$.

(d) $g(C\{F, g\})$ is a singleton subset of $C\{F, g\}$.

Proof: Let $x_0, y_0, z_0, w_0 \in X$ be arbitrary. Then $F(x_0, y_0, z_0, w_0)$, $F(y_0, z_0, w_0, x_0)$, $F(z_0, w_0, x_0, y_0)$, $F(w_0, x_0, y_0, z_0)$ are well defined. Choose $gx_1 \in F(x_0, y_0, z_0, w_0)$, $gy_1 \in F(y_0, z_0, w_0, x_0)$, $gz_1 \in F(z_0, w_0, x_0, y_0)$ and $gw_1 \in F(w_0, x_0, y_0, z_0)$ because $F(X\times X\times X\times X) \subseteq (X)$. Since $F : X^4 \to CB(X)$, therefore by Lemma 1.1, there exist $u_1 \in F(x_1, y_1, z_1, w_1)$, $u_2 \in F(y_1, z_1, w_1, x_1)$, $u_3 \in F(z_1, w_1, x_1, y_1)$, $u_4 \in F(w_1, x_1, y_1, z_1)$ such that

$d(gx_1, u_1) \leq H(F(x_0, y_0, z_0, w_0), F(x_1, y_1, z_1, w_1))$,

$d(gy_1, u_2) \leq H(F(y_0, z_0, w_0, x_0), F(y_1, z_1, w_1, x_1))$,

$d(gz_1, u_3) \leq H(F(z_0, w_0, x_0, y_0), F(z_1, w_1, x_1, y_1))$,

$d(gw_1, u_4) \leq H(F(w_0, x_0, y_0, z_0), F(w_1, x_1, y_1, z_1))$.

Since $F(X\times X\times X\times X) \subseteq g(X)$, there exist $x_2, y_2, z_2, w_2 \in X$ such that $u_1 = gx_2$, $u_2 = gy_2$, $u_3 = gz_2$, $u_4 = gw_2$. Thus

$d(gx_1, gx_2) \leq H(F(x_0, y_0, z_0, w_0), F(x_1, y_1, z_1, w_1))$,

$d(gy_1, gy_2) \leq H(F(y_0, z_0, w_0, x_0), F(y_1, z_1, w_1, x_1))$,

$d(gz_1, gz_2) \leq H(F(z_0, w_0, x_0, y_0), F(z_1, w_1, x_1, y_1))$,

$d(gw_1, gw_2) \leq H(F(w_0, x_0, y_0, z_0), F(w_1, x_1, y_1, z_1))$.

Continuing this process, we obtain sequences $\{x_n\}$, $\{y_n\}$, $\{z_n\}$, $\{w_n\}$ in X such that for all $n \in \mathbb{N}$, we have $gx_{n+1} \in F(x_n, y_n, z_n, w_n)$, $gy_{n+1} \in F(y_n, z_n, w_n, x_n)$, $gz_{n+1} \in F(z_n, w_n, x_n, y_n)$, $gw_{n+1} \in F(w_n, x_n, y_n, z_n)$ such that

$$\phi\left(\frac{1}{4}\begin{bmatrix} d(gxn+1,\ gxn) \\ +d(gyn+1,\ gyn) \\ +d(gzn+1,\ gzn) \\ +d(gwn+1,\ gwn) \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} H\big(F(xn,\ yn,\ zn,\ wn),\ F(xn-1,\ yn-1,\ zn-1,\ wn-1)\big) \\ +H\big(F(yn,\ zn,\ wn,\ xn),\ F(yn-1,\ zn-1,\ wn-1,\ xn-1)\big) \\ +H\big(F(zn,\ wn,\ xn,\ yn),\ F(zn-1,\ wn-1,\ xn-1,\ yn-1)\big) \\ +H(F(wn,\ xn,\ yn,\ zn), F(wn-1,\ xn-1,\ yn-1,\ zn-1)) \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(gxn,\ gxn-1) \\ +d(gyn,\ gyn-1) \\ +d(gzn,\ gzn-1) \\ +d(gwn,\ gwn-1) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(gxn,\ gxn-1) \\ +d(gyn,\ gyn-1) \\ +d(gzn,\ gzn-1) \\ +d(gwn,\ gwn-1) \end{bmatrix}\right)$$

Thus

$$\phi\left(\frac{1}{4}\begin{bmatrix} d(gxn+1,\ gxn) \\ +d(gyn+1,\ gyn) \\ +d(gzn+1,\ gzn) \\ +d(gwn+1,\ gwn)\end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(gxn,\ gxn-1) \\ +d(gyn,\ gyn-1) \\ +d(gzn,\ gzn-1) \\ +d(gwn,\ gwn-1)\end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(gxn,\ gxn-1) \\ +d(gyn,\ gyn-1) \\ +d(gzn,\ gzn-1) \\ +d(gwn,\ gwn-1)\end{bmatrix}\right) \qquad (2.2)$$

which, by the fact that $\psi \geq 0$, implies

$$\phi\left(\frac{1}{4}\begin{bmatrix} d(gxn+1,\ gxn) \\ +d(gyn+1,\ gyn) \\ +d(gzn+1,\ gzn) \\ +d(gwn+1,\ gwn)\end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(gxn,\ gxn-1) \\ +d(gyn,\ gyn-1) \\ +d(gzn,\ gzn-1) \\ +d(gwn,\ gwn-1)\end{bmatrix}\right)$$

this shows, by the monotony of ϕ, that the sequence $\{\delta n\}_{n=0}^{\infty}$ given by

$$\delta_n = \frac{1}{4}\begin{bmatrix} d(gxn+1,\ gxn) \\ +d(gyn+1,\ gyn) \\ +d(gzn+1,\ gzn) \\ +d(gwn+1,\ gwn)\end{bmatrix} \forall n \geq 0,$$

is non-increasing. Therefore, there exists some $\delta \geq 0$ such that

$$\lim_{n\to\infty} \delta_n = \frac{1}{4}\lim_{n\to\infty}\begin{bmatrix} d(gxn+1,\ gxn) \\ +d(gyn+1,\ gyn) \\ +d(gzn+1,\ gzn) \\ +d(gwn+1,\ gwn)\end{bmatrix} = \delta$$

We shall prove that $\delta = 0$. Assume that $\delta > 0$. Then by letting $n\to\infty$ in (2.2), by using (i_ϕ) and (i_ψ), we get

$$\phi(\delta) = \lim_{n\to\infty} \phi(\delta_{n+1})$$

$$\leq \lim_{n\to\infty} \phi(\delta_n) - \lim_{n\to\infty} \psi(\delta_n)$$

$$\leq \phi(\delta) - \lim_{\delta n \to \delta+} \psi(\delta n)$$

$$< \phi(\delta),$$

which is a contradiction. Thus $\delta = 0$ and hence

$$\lim_{n\to\infty} \delta_n = \frac{1}{4}\lim_{n\to\infty}\begin{bmatrix} d(gxn+1,\ gxn) \\ +d(gyn+1,\ gyn) \\ +d(gzn+1,\ gzn) \\ +d(gwn+1,\ gwn) \end{bmatrix} = 0 \qquad (2.3)$$

We now prove that $\{gxn\}_{n=0}^{\infty}$, $\{gyn\}_{n=0}^{\infty}$, $\{gzn\}_{n=0}^{\infty}$, $\{gwn\}_{n=0}^{\infty}$ are Cauchy sequences in (X, d). Suppose, to the contrary, that at least one of the sequences $\{gxn\}_{n=0}^{\infty}$, $\{gyn\}_{n=0}^{\infty}$, $\{gzn\}_{n=0}^{\infty}$, *and* $\{gwn\}_{n=0}^{\infty}$ is not a Cauchy sequence. Then there exists an $\varepsilon > 0$ for which we can find subsequences $\{gx_{n(k)}\}$, $\{gx_{m(k)}\}$ of $\{gxn\}_{n=0}^{\infty}$, $\{gy_{n(k)}\}$, $\{gy_{m(k)}\}$ of $\{gyn\}_{n=0}^{\infty}$, $\{gz_{n(k)}\}$, $\{gz_{m(k)}\}$ of $\{gzn\}_{n=0}^{\infty}$, and , $\{gw_{n(k)}\}$, $\{gw_{m(k)}\}$ of $\{gwn\}_{n=0}^{\infty}$, such that

$$\frac{1}{4}\begin{bmatrix} d(gxn(k),\ gxm(k)) \\ +d(gyn(k),\ gym(k)) \\ +d(gzn(k),\ gzm(k)) \\ +d(gwn(k),\ gwm(k)) \end{bmatrix} \geq \varepsilon,\ k = 1, 2, \ldots \qquad (2.4)$$

We can choose n(k) to be the smallest positive integer satisfying (2.4). Then

$$\frac{1}{4}\begin{bmatrix} d(gxn(k)-1,\ gxm(k)) \\ +d(gyn(k)-1,\ gym(k)) \\ +d(gzn(k)-1,\ gzm(k)) \\ +d(gwn(k)-1,\ gwm(k)) \end{bmatrix} < \varepsilon. \qquad (2.5)$$

By (2.4), (2.5) and triangle inequality, we have

$$\varepsilon \leq r_k = \frac{1}{4}\begin{bmatrix} d(gxn(k),\ gxm(k)) \\ +d(gyn(k),\ gym(k)) \\ +d(gzn(k),\ gzm(k)) \\ +d(gwn(k),\ gwm(k)) \end{bmatrix}$$

$$\leq \frac{1}{4}\begin{bmatrix} d(gxn(k),\ gxn(k)-1) \\ +d(gyn(k),\ gyn(k)-1) \\ +d(gzn(k),\ gzn(k)-1) \\ +d(gwn(k),\ gwn(k)-1) \end{bmatrix} + \frac{1}{4}\begin{bmatrix} d(gxn(k)-1,\ gxm(k)) \\ +d(gyn(k)-1,\ gym(k)) \\ +d(gzn(k)-1,\ gzm(k)) \\ +d(gwn(k)-1,\ gwm(k)) \end{bmatrix}$$

$$< \frac{1}{4}\begin{bmatrix} d(gxn(k),\ gxn(k)-1) \\ +d(gyn(k),\ gyn(k)-1) \\ +d(gzn(k),\ gzn(k)-1) \\ +d(gwn(k),\ gwn(k)-1) \end{bmatrix} + \varepsilon.$$

Letting $k\to\infty$ in the above inequality and using (2.3), we get

$$\lim_{k\to\infty} r_k = \lim_{k\to\infty}\frac{1}{4}\begin{bmatrix} d(gxn(k),\ gxm(k)) \\ +d(gyn(k),\ gym(k)) \\ +d(gzn(k),\ gzm(k)) \\ +d(gwn(k),\ gwm(k)) \end{bmatrix} = \varepsilon. \qquad (2.6)$$

By the triangle inequality, we have

$$d(gx_{n(k)}, gx_{m(k)}) \leq \begin{bmatrix} d(gxn(k),\ gxn(k)+1) \\ +d(gxn(k)+1,\ gxm(k)+1) \\ +d\big(gxm(k)+1,\ gxm(k)\big) \end{bmatrix},$$

$$d(gy_{n(k)}, gy_{m(k)}) \leq \begin{bmatrix} d(gyn(k),\ gyn(k)+1) \\ +d(gyn(k)+1,\ gym(k)+1) \\ +d\big(gym(k)+1,\ gym(k)\big) \end{bmatrix}$$

$$d(gz_{n(k)}, gz_{m(k)}) \leq \begin{bmatrix} d(gzn(k),\ gzn(k)+1) \\ +d(gzn(k)+1,\ gzm(k)+1) \\ +d\big(gzm(k)+1,\ gzm(k)\big) \end{bmatrix}$$

$$d(gw_{n(k)}, gw_{m(k)}) \leq \begin{bmatrix} d(gwn(k),\ gwn(k)+1) \\ +d(gwn(k)+1,\ gwm(k)+1) \\ +d\big(gwm(k)+1,\ gwm(k)\big) \end{bmatrix}$$

This shows that

$$r_k \leq \delta_{n(k)} + \delta_{m(k)} + \frac{1}{4}\begin{bmatrix} d(gxn(k)+1,\ gxm(k)+1) \\ +d(gyn(k)+1,\ gym(k)+1) \\ +d(gzn(k)+1,\ gzm(k)+1) \\ +d(gwn(k)+1,\ gwm(k)+1) \end{bmatrix} \tag{2.7}$$

Now, since $gx_{n(k)+1} \in F(x_{n(k)}, y_{n(k)}, z_{n(k)}, w_{n(k)})$, $gx_{m(k)+1} \in F(x_{m(k)}, y_{m(k)}, z_{m(k)}, w_{m(k)})$, $gy_{n(k)+1} \in F(y_{n(k)}, z_{n(k)}, w_{n(k)}, x_{n(k)})$, $gy_{m(k)+1} \in F(y_{m(k)}, z_{m(k)}, w_{m(k)}, x_{m(k)})$, $gz_{n(k)+1} \in F(z_{n(k)}, w_{n(k)}, x_{n(k)}, y_{n(k)})$, $gz_{m(k)+1} \in F(z_{m(k)}, w_{m(k)}, x_{m(k)}, y_{m(k)})$, and $gw_{n(k)+1} \in F(w_{n(k)}, x_{n(k)}, y_{n(k)}, z_{n(k)})$, $gw_{m(k)+1} \in \in F(w_{m(k)}, x_{m(k)}, y_{m(k)}, z_{m(k)})$, and therefore by using triangle inequality, (2.1) and (i_ϕ), we get

$$\phi\left(\frac{1}{4}\begin{bmatrix} d(gxn(k)+1,\ gxm(k)+1) \\ +d(gyn(k)+1,\ gym(k)+1) \\ +d(gzn(k)+1,\ gzm(k)+1) \\ +d(gwn(k)+1,\ gwm(k)+1) \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} H(F(xn(k)\ , yn(k), zn(k), wn(k)), F(xm(k), ym(k), zm(k), wm(k)) \\ +H(F(yn(k), zn(k), wn(k), xn(k)), F(ym(k), zm(k), wm(k), xm(k)) \\ +H(F(zn(k), wn(k), xn(k), yn(k)), F(zm(k), wm(k), xm(k), ym(k)) \\ +H(F\big(wn(k), xn(k), yn(k), zn(k)\big), F(wm(k), xm(k), ym(k), zm(k)) \\ \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(gxn(k),\ gxm(k)) \\ +d(gyn(k),\ gym(k)) \\ +d(gzn(k),\ gzm(k)) \\ +d(gwn(k),\ gwm(k)) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(gxn(k),\ gxm(k)) \\ +d(gyn(k),\ gym(k)) \\ +d(gzn(k),\ gzm(k)) \\ +d(gwn(k),\ gwm(k)) \end{bmatrix}\right)$$

$\leq \phi\,(r_k) - \psi\,(r_k)$.

Thus

$$\phi\left(\frac{1}{4}\begin{bmatrix} d(gxn(k)+1,\ gxm(k)+1) \\ +d(gyn(k)+1,\ gym(k)+1) \\ +d(gzn(k)+1,\ gzm(k)+1) \\ +d(gwn(k)+1,\ gwm(k)+1) \end{bmatrix}\right) \qquad (2.8)$$

$\leq \phi\,(r_k) - \psi\,(r_k)$.

On the other hand, by (2.7) and (iii_ϕ), we get

$$\phi(r_k) \leq \phi(\delta_{n(k)}) + \phi(\delta_{m(k)}) + \phi\left(\frac{1}{4}\begin{bmatrix} d(gxn(k)+1,\ gxm(k)+1) \\ +d(gyn(k)+1,\ gym(k)+1) \\ +d(gzn(k)+1,\ gzm(k)+1) \\ +d(gwn(k)+1,\ gwm(k)+1) \end{bmatrix}\right). \qquad (2.9)$$

By (2.8) and (2.9), we get

$$\phi(r_k) \leq \phi(\delta_{n(k)}) + \phi(\delta_{m(k)}) + \phi\,(r_k) - \psi\,(r_k). \qquad (2.10)$$

By (2.8) and (2.9), we get

Letting $k\to\infty$ in (2.10), by using (2.3), (2.6), (i_ϕ), (ii_ϕ) and (i_ψ), we get

$\phi(\varepsilon) \leq \phi(0) + \phi(0) + \phi(\varepsilon) - \lim_{k\to\infty} \psi(r_k)$

$\leq \phi(\varepsilon) - \lim_{rk\to\varepsilon+} \psi(r_k)$

$< \phi(\varepsilon)$,

which is a contradiction. This shows that $\{gxn\}_{n=0}^{\infty}$, $\{gyn\}_{n=0}^{\infty}$, $\{gzn\}_{n=0}^{\infty}$, $\{gwn\}_{n=0}^{\infty}$ are indeed Cauchy sequences in g(X). Since g(X) is complete, there exist x, y, z, w ∈ X such that

$$\lim_{n\to\infty} gx_n = gx,\ \lim_{n\to\infty} gy_n = gy,\ \lim_{n\to\infty} gz_n = gz,\ \lim_{n\to\infty} gw_n = gw. \qquad (2.11)$$

Now, since $gx_{n+1} \in F(x_n, y_n, z_n, w_n)$, $gy_{n+1} \in F(y_n, z_n, w_n, x_n)$, $gz_{n+1} \in F(z_n, w_n, x_n, y_n)$ and $gw_{n+1} \in F(w_n, x_n, y_n, z_n)$ Therefore by using condition (2.1) and (i_ϕ), we get

$$\phi\left(\frac{1}{4}\begin{bmatrix} D(gxn+1,\ F(x,\ y,\ z,\ w)) \\ +D(gyn+1,\ F(y,\ z,\ w,\ x)) \\ +D(gzn+1,\ F(z,\ w,\ x,\ y)) \\ +D(gwn+1,\ F(w,\ x,\ y,\ z)) \end{bmatrix}\right)$$

$$\phi\left(\frac{1}{4}\begin{bmatrix} H(F(xn,\ yn,\ zn,\ wn),\ F(x,\ y,\ z,\ w)) \\ +H(F(yn,\ zn,\ wn,\ xn), F(y,\ z,\ w,\ x)) \\ +H(F(zn,\ wn,\ xn,\ yn), F(z,\ w,\ x,\ y)) \\ +H(F(wn,\ xn,\ yn,\ zn),), F(w,\ x,\ y,\ z)) \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(gxn,\ gx) \\ +d(gyn,\ gy) \\ +d(gzn,\ gz) \\ +d(gwn,\ gw) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(gxn,\ gx) \\ +d(gyn,\ gy) \\ +d(gzn,\ gz) \\ +d(gwn,\ gw) \end{bmatrix}\right)$$

Letting $n\to\infty$ in the above inequality, by using (2.11), (i_ϕ), (ii_ϕ) and (i_ψ), we obtain

$$\phi\left(\frac{1}{4}\begin{bmatrix} D(gx, F(x,y,z,w)) \\ +D(gy, F(y,z,w,x)) \\ +D(gz, F(z,w,x,y)) \end{bmatrix}\right) \leq \phi(0) - 0 = 0 - 0 = 0,$$

which, by (i_ϕ) and (ii_ϕ), implies

$$\begin{bmatrix} D(gx,\ F(x,\ y\ z,\ w)), \\ D(gy,\ F(y,\ z,\ w,\ x)), \\ D(gz,\ F(z,\ w,\ x,\ y)), \\ D(gw,\ F(w,\ x,\ y,\ z)) \end{bmatrix} = 0$$

it follows that

$gx \in F(x, y, z, w)$, $gy \in F(y, z, w, x)$, $gz \in F(z, w, x, y)$, $gw \in F(w, x, y, z)$,

that is, (x, y, z, w) is a quadruple coincidence point of F and g. Hence $C\{F, g\}$ is nonempty.

Suppose now that (a) holds. Assume that for some $(x, y, z, w) \in C\{F, g\}$,

$$\lim_{n\to\infty} g^n x=p,\ \lim_{n\to\infty} g^n y=q,\ \lim_{n\to\infty} g^n z=r,\ \lim_{n\to\infty} g^n w=s \quad (2.12)$$

where $p, q, r, s \in X$. Since g is continuous at p, q, r, s. We have, by (2.12), that p, q, r and s are fixed points of g, that is,

$$gp=p,\ gq=q,\ gr=r,\ gs=s. \quad (2.13)$$

As F and g are w-compatible, so

$(g^n x, g^n y, g^n z, g^n w) \in C\{F,g\}$, for all $n\geq 1$,

that is, for all $n\geq 1$,

$g^n x \in F(g^{n-1}x, g^{n-1}y, g^{n-1}z, g^{n-1}w)$,

$g^n y \in F(g^{n-1}y, g^{n-1}z, g^{n-1}w, g^{n-1}x)$,

$g^n z \in F(g^{n-1}z, g^{n-1}w, g^{n-1}x, g^{n-1}y)$,

$g^n w \in F(g^{n-1}w, g^{n-1}x, g^{n-1}y, g^{n-1}z,)$. (2.14)

Now, by using (2.1), (2.14) and (i_ϕ), we obtain

$$\phi\left(\frac{1}{4}\begin{bmatrix} D(g^n x,\ F(p,\ q,\ r,\ s)) \\ +D(g^n y,\ F(q,\ r,\ s,\ p)) \\ +D(g^n z,\ F(r,\ s,\ p,\ q)) \\ +D(g^n w,\ F(s,\ p,\ q,\ r)) \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} H(F(g^{n-1}x,\ g^{n-1}y,\ g^{n-1}z,\ g^{n-1}w),\ F(p,\ q,\ r,\ s))) \\ +H(F(g^{n-1}y,\ g^{n-1}z\ g^{n-1}w,\ g^{n-1}x), F(q,\ r,\ s,\ p)) \\ +H(F(g^{n-1}z,\ g^{n-1}w,\ g^{n-1}x,\ g^{n-1}y)F(r,\ s,\ p,\ q)) \\ +H(F(g^{n-1}w,\ g^{n-1}x,\ g^{n-1}y,\ g^{n-1}z)F(s,\ p,\ q,\ r)) \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(g^n x,\ gp) \\ +d(g^n y,\ gq) \\ +d(g^n z,\ gr) \\ +d(g^n z,\ gs) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(g^n x,\ gp) \\ +d(g^n y,\ gq) \\ +d(g^n z,\ gr) \\ +d(g^n z,\ gs) \end{bmatrix}\right).$$

On taking limit as $n\to\infty$ in the above inequality, by using (2.12), (2.13), (i_ϕ), (ii_ϕ) and (i_ψ), we get

$$\phi\left(\frac{1}{4}\begin{bmatrix} D(gp,\ F(p,\ q,\ r,\ s)) \\ +D(gq,\ F(q,\ r,\ s,\ p)) \\ +D(gr,\ F(r,\ s,\ p,\ q)) \\ +D(gs,\ F(s, p,\ q,\ r)) \end{bmatrix}\right) \leq \phi(0) - 0 = 0 - 0 = 0.$$

which shows, by (i_ϕ) and (ii_ϕ), that

$D(gp, F(p, q, r, s)) = 0,$

$D(gq, F(q, r, s, p)) = 0,$

$D(gr, F(r, s, p, q)) = 0,$

$D(gs, F(s, p, q, r)) = 0,$

which implies that

$gp \in F(p, q, r, s)$, $gq \in F(q, r, s, p)$,

$gr \in F(r, s, p, q)$, $gs \in F(s, p, q, r)$. (2.15)

Now, from (2.13) and (2.15), we have

$p = gp \in F(p, q, r, s)$, $q = gq \in F(q, r, s, p)$,

$r = gr \in F(r, s, p, q)$, $s = gs \in F(s, p, q, r)$,

that is, (p, q, r, s) is a common quadruple fixed point of F and g.

Suppose now that (b) holds. Assume that for some $(x, y, z, w) \in C\{F, g\}$, g is F-weakly commuting, that is, $g^2x \in F(gx, gy, gz, gw)$, $g^2y \in F(gy, gz, gw, gx)$, $g^2z \in F(gz, gw, gx, gy)$, $g^2w \in F(gw, gx, gy, gz,)$ and $g^2x = gx$, $g^2y = gy$, $g^2z = gz$, $g^2w = gw$.. Thus, $gx = g^2x \in F(gx, gy, gz, gw)$, $gy = g^2y \in F(gy, gz, gw, gx)$, $gz = g^2z \in F(gz, gw, gx, gy)$, $gw = g^2w \in F(gw, gx, gy, gz)$that is, (gx, gy, gz, gw) is a common quadruple fixed point of F and g.

Suppose now that (c) holds. Assume that for some $(x, y, z, w) \in C\{F, g\}$ and for some $p, q, r, s \in X$,

$$\lim_{n\to\infty} g^n p = x,\ \lim_{n\to\infty} g^n q = y,\ \lim_{n\to\infty} g^n r = z,\ \lim_{n\to\infty} g^n s = w. \tag{2.16}$$

Since g is continuous at x, y, z, w. We have, by (2.16), that x, y, z, w are fixed points of g, that is,

$$gx = x,\ gy = y,\ gz = z,\ gw = w. \tag{2.17}$$

Since (x, y, z, w) ∈ C{F, g}, therefore by (2.17), we obtain

$x = gx \in F(x, y, z, w)$,

$y = gy \in F(y, z, w, x)$,

$z = gz \in F(z, w, x, y)$,

$w = gw \in F(w, x, y, z)$,

that is, (x, y, z, w) is a common quadruple fixed point of F and g.

Finally, suppose that (d) holds. Let g(C{F, g})={(x, x, x, x)}. Then {x}={gx}=F(x, x, x, x). Hence (x, x, x, x) is a common quadruple fixed point of F and g.

Put g=I (the identity mapping) in Theorem 2.1, we get the following result:

Corollary 2.2. Let (X, d) be a complete metric space. Assume F:X×X×X×X→CB(X) be a mapping satisfying

$$\phi\left(\frac{1}{4}\begin{bmatrix} H(F(x, y, z, w), F(p, q, r, s)) \\ +H(F(y, z, w, x), F(q, r, s, p)) \\ +H(F(z, w, x, y), F(r, s, p, q)) \\ +H(F(w, x, y, z), F(s, p, q, r)) \end{bmatrix}\right)$$

$$\phi\left(\frac{1}{4}\begin{bmatrix} d(x, p) \\ +d(y, q) \\ +d(z, r) \\ +d(w, s) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(x, p) \\ +d(y, q) \\ +d(z, r) \\ +d(w, s) \end{bmatrix}\right),$$

for all x, y, z, w , p, q, r, s ∈ X, where $\phi \in \Phi$ and $\psi \in \Psi$. Then F has a quadruple fixed point.

Corollary 2.3. Let (X, d) be a metric space. Assume F:X×X× X×X→CB(X) and g:X→X be two mappings satisfying

$$\begin{bmatrix} H(F(x, y, z, w), F(p, q, r, s)) \\ +H(F(y, z, w, x), F(q, r, s, p)) \\ +H(F(z, w, x, y), F(r, s, p, q)) \\ +H(F(w, x, y, z), F(s, p, q, r)) \end{bmatrix}$$

$$\leq \begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix} - 4\,\psi\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right),$$

for all x, y, z, w, p, q, r, s ∈ X, where $\psi \in \Psi$. Furthermore assume that F(X×X× X×X) ⊆ g(X) and g(X) is a complete subset of X. Then F and g have a quadruple coincidence point. Moreover, F and g have a common quadruple fixed point, if one of the following conditions holds:

(a) F and g are w-compatible. $\lim_{n\to\infty} g^n x = p$, $\lim_{n\to\infty} g^n y = q$, $\lim_{n\to\infty} g^n z = r$, $\lim_{n\to\infty} g^n w = s$ for some (x, y, z, w) ∈ C{F, g} and for some p, q, r, s ∈ X and g is continuous at p, q, r, s.

(b) g is F-weakly commuting for some (x, y, z, w) ∈ C{F, g} and gx, gy, gz, gw are fixed points of g, that is, $g^2x = gx$, $g^2y = gy$, $g^2z = gz$, $g^2w = gw$.

(c) g is continuous at x, y, z, w. $\lim_{n\to\infty} g^n p = x$, $\lim_{n\to\infty} g^n q = y$, $\lim_{n\to\infty} g^n r = z$, $\lim_{n\to\infty} g^n s = w$ for some (x, y, z, w) ∈ C{F, g} and for some p, q, r, s ∈ X.

(d) g(C{F, g}) is a singleton subset of C{F, g}.

Proof. If ψ ∈ Ψ, then for all r > 0, rψ ∈ Ψ. Now divide (2.18) by 4 and take ϕ(t) = (1/2)t, t∈[0, ∞), then condition (2.18) reduces to (2.1) with $\psi_1=(1/2)\psi$ and hence by Theorem 2.1 we get Corollary 2.3.

Put g = I (the identity mapping) in Corollary 2.3, we get the following result:

Corollary 2.4. Let (X, d) be a complete metric space. Assume F:X×X×X×X→CB(X) be a mapping satisfying

$$\begin{bmatrix} H(F(x,\ y,\ z,\ w),\ F(p,\ q,\ r,\ s)) \\ +H(F(y,\ z,\ w,\ x),\ F(q,\ r,\ s,\ p)) \\ +H(F(z,\ w,\ x,\ y),\ F(r,\ s,\ p,\ q)) \\ +H(F(w,\ x,\ y,\ z),\ F(s,\ p,\ q,\ r)) \end{bmatrix}$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(x,\ p) \\ +d(y,\ q) \\ +d(z,\ r) \\ +d(w,\ s) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(x,\ p) \\ +d(y,\ q) \\ +d(z,\ r) \\ +d(w,\ s) \end{bmatrix}\right),$$

for all x, y, z, w, p, q, r, s ∈ X, where ϕ ∈ Φ and ψ ∈ Ψ. Then F has a quadruple fixed point.

Example 2.1. Suppose that X = [0, 1], equipped with the metric d:X×X→[0, +∞) defined as d(x, y) = max{x, y} and d(x, x) = 0 for all x, y, ∈X. Let $F:X^4\to CB(X)$ be defined as

$$F(x, y, z, w) = \begin{cases} \{0\}, for\ x,y,z,w = 1 \\ 0, \frac{x+y+z+w}{16}, for\ x,y,z,w \in [0,1). \end{cases}$$

and g:X→X be defined as

gx = (x/2) for all x ∈ X.

Define ϕ : [0, +∞)→[0, +∞) by

ϕ(t)=(t/2), for all t>0,

and ψ:[0, +∞)→ [0, +∞) by

$$\psi(t) = \begin{cases} \frac{t}{4}, for\ t \neq 1 \\ 0,\ \ for\ t = 1 \end{cases}$$

Now, for all x, y, z, w, p, q, r, s ∈ X with x, y, z, w, p, q, r, s ∈ [0, 1), we have

Case (a). If x+y+z+w = p+q+r+s, then

$$\phi\left(\frac{1}{4}\begin{bmatrix} H(F(x, y, z, w), F(p, q, r, s)) \\ +H(F(y, z, w, x), F(q, r, s, p)) \\ +H(F(z, w, x, y), F(r, s, p, q)) \\ +H(F(w, x, y, z), F(s, p, q, r)) \end{bmatrix}\right)$$

$$= \frac{1}{8}\begin{bmatrix} H(F(x, y, z, w), F(p, q, r, s)) \\ +H(F(y, z, w, x), F(q, r, s, p)) \\ +H(F(z, w, x, y), F(r, s, p, q)) \\ +H(F(w, x, y, z), F(s, p, q, r)) \end{bmatrix}$$

$$= \frac{1}{8}\left[\frac{p+q+r+s}{16} + \frac{q+r+s+p}{16} + \frac{r+s+p+q}{16} + \frac{s+p+q+r}{16}\right]$$

$$= \frac{1}{8}\left[\frac{p+q+r+s}{4}\right]$$

$$\leq \frac{1}{4}\left[\frac{max\{\frac{x}{2}, \frac{p}{2}\}+max\{\frac{y}{2}, \frac{q}{2}\}+max\{\frac{z}{2}, \frac{r}{2}\}+max\{\frac{w}{2}, \frac{s}{2}\}}{4}\right]$$

$$\leq \frac{1}{4}\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right).$$

Case (b). If x+y+z+w ≠ p+q+r+s with x+y+z+w < p+q+r+s, then

$$\phi\left(\frac{1}{4}\begin{bmatrix} H(F(x, y, z, w)), F(p, q, r, s)) \\ +H(F(y, z, w, x), F(q, r, s, p)) \\ +H(F(z, w, x, y), F(r, s, p, q)) \\ +H(F(w, x, y, z), F(s, p, q, r)) \end{bmatrix}\right)$$

$$= \frac{1}{8}\begin{bmatrix} H(F(x, y, z, w)), F(p, q, r, s)) \\ +H(F(y, z, w, x), F(q, r, s, p)) \\ +H(F(z, w, x, y), F(r, s, p, q)) \\ +H(F(w, x, y, z), F(s, p, q, r)) \end{bmatrix}$$

$$= \frac{1}{8}\left[\frac{p+q+r+s}{16} + \frac{q+r+s+p}{16} + \frac{r+s+p+q}{16} + \frac{s+p+q+r}{16}\right]$$

$$= \frac{1}{8}\left[\frac{p+q+r+s}{4}\right]$$

$$\leq \frac{1}{4}\left[\frac{max\{\frac{x}{2}, \frac{p}{2}\}+max\{\frac{y}{2}, \frac{q}{2}\}+max\{\frac{z}{2}, \frac{r}{2}\}+max\{\frac{w}{2}, \frac{s}{2}\}}{4}\right]$$

$$\leq \frac{1}{4}\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right).$$

Similarly, we obtain the same result for $p+q+r+s < x+y+z+w$. Thus the contractive condition (2.1) is satisfied for all $x, y, z, w, p, q, r, s \in X$ with $x, y, z, w, p, q, r, s \in [0, 1)$. Again, for all $x, y, z, w, p, q, r, s \in X$ with $x, y, z, w \in [0, 1)$ and $p, q, r, s = 1$, we have

$$\phi\left(\frac{1}{4}\begin{bmatrix} H(F(x, y, z, w)), F(p, q, r, s)) \\ +H(F(y, z, w, x), F(q, r, s, p)) \\ +H(F(z, w, x, y), F(r, s, p, q)) \\ +H(F(w, x, y, z), F(s, p, q, r)) \end{bmatrix}\right)$$

$$= \frac{1}{8}\begin{bmatrix} H(F(x, y, z, w)), F(p, q, r, s)) \\ +H(F(y, z, w, x), F(q, r, s, p)) \\ +H(F(z, w, x, y), F(r, s, p, q)) \\ +H(F(w, x, y, z), F(s, p, q, r)) \end{bmatrix}$$

$$= \frac{1}{8}\left[\frac{x+y+z+w}{16} + \frac{y+z+w+x}{16} + \frac{z+w+x+y}{16} + \frac{w+x+y+z}{16}\right]$$

$$= \frac{1}{8}\left[\frac{x+y+z+w}{4}\right]$$

$$\leq \frac{1}{4}\left[\frac{max\{\frac{x}{2}, \frac{p}{2}\}+max\{\frac{y}{2}, \frac{q}{2}\}+max\{\frac{z}{2}, \frac{r}{2}\}+max\{\frac{w}{2}, \frac{s}{2}\}}{4}\right]$$

$$\leq \frac{1}{4}\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right)$$

$$\leq \phi\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right) - \psi\left(\frac{1}{4}\begin{bmatrix} d(gx, gp) \\ +d(gy, gq) \\ +d(gz, gr) \\ +d(gw, gs) \end{bmatrix}\right).$$

Thus the contractive condition (2.1) is satisfied for all $x, y, z, w, p, q, r, s \in X$ with $x, y, z, w \in [0, 1)$ and $p, q, r, s = 1$. Similarly, we can see that the contractive condition (2.1) is satisfied for all $x, y, z, w, p, q, r, s \in X$ with $x, y, z, w, p, q, r, s = 1$. Hence, the hybrid pair $\{F, g\}$ satisfies the contractive condition (2.1), for all $x, y, z, w, p, q, r, s \in X$. In addition, all the other conditions of Theorem 2.1 are satisfied and $z = (0, 0, 0, 0)$ is a common quadruple fixed point of hybrid pair $\{F, g\}$.

Remark 2.1. We improve, extend and generalize the result of Jain, Tas, Kumar and Gupta [15] in the following sense:

(i) We prove our result in the settings of multivalued mapping and for hybrid pair of mappings.

(ii) To prove our result we consider non complete metric space and the space is also not partially ordered.

(iii) The multivalued mapping $F:X^4 \to CB(X)$ is discontinuous and not satisfying mixed g-monotone property.

(iv) The function $\psi:[0, +\infty) \to [0, +\infty)$ involved in our theorem and example is discontinuous.

References:

[1] M. Abbas, L. Ciric, B. Damjanovic and M. A. Khan, Coupled coincidence point and common fixed point theorems for hybrid pair of mappings, Fixed Point Theory Appl. doi:10.1186/1687-1812-2012-4 (2012).

[2] S. M. Alsulami and A. Alotaibi, Tripled coincidence points for monotone operators in partially ordered metric spaces, International Mathematical Forum 7 (37) (2012), 1811-1824.

[3] H. Aydi, E. Karapinar and M. Postolache, Tripled coincidence point theorems for weak ϕ-contractions in partially ordered metric spaces, Fixed Point Theory and Applications 44 (2012).

[4] V. Berinde, Coupled fixed point theorems for ϕ-contractive mixed monotone mappings in partially ordered metric spaces. Nonlinear Analysis 75 (2012), 3218-3228.

[5] V. Berinde and M. Borcut, Tripled fixed point theorems for contractive type mappings in partially ordered metric spaces, Nonlinear Anal. Theory, Methods and Applications 74 (15) (2011), 4889-4897.

[6] V. Berinde and M. Borcut, Tripled coincidence theorems of contractive type mappings in partially ordered metric spaces, Applied Mathematics and Computation 218 (10), 5929-5936.

[7] T. G. Bhaskar and V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Anal. 65 (7) (2006), 1379-1393.

[8] P. Charoensawan, Tripled fixed points theorems of ϕ-contractive mixed monotone operators on partially ordered metric spaces, Applied Mathematical Sciences 6 (105) (2012), 5229 - 5239.

[9] B. Deshpande and A. Handa, Nonlinear mixed monotone-generalized contractions on partially ordered modified intuitionistic fuzzy metric spaces with application to integral equations, Afr. Mat. DOI 10.1007/s13370-013-0204-0.

[10] B. Deshpande and A. Handa, Application of coupled fixed point technique in solving integral equations on modified intuitionistic fuzzy metric spaces, Advances in Fuzzy Systems Volume 2014, Article ID 348069, 11 pages.

[11] B. Deshpande, S. Sharma and A. Handa, Tripled fixed point theorem for hybrid pair of mappings under generalized nonlinear contraction, J. Korean Soc. Math. Educ. Ser. B: Pure Appl. Math., 21 (1) (2014), 23-38.

[12] B. Deshpande and A. Handa, Quadruple fixed point theorems for multivalued mappings under ϕ-ψ contraction on partially ordered metric spaces, accepted for publication in SEA Bull. Math.

[13] B. Deshpande and A. Handa, Generalized Mizoguchi-Takahashi contraction in consideration of common tripled fixed point theorem for hybrid pair of mappings, accepted for publication in Malaya Journal of Matematik.

[14] H. S. Ding, L. Li and S. Radenovic, Coupled coincidence point theorems for generalized nonlinear contraction in partially ordered metric spaces, Fixed Point Theory and Applications 2012, 2012:96.

[15] M. Jain, K. Tas, S. Kumar and N. Gupta, Coupled common fixed point results involving a ϕ-ψ contractive condition for mixed g-monotone operators in partially ordered metric spaces, Journal of Inequalities and Applications 2012, 2012:285.

[16] E. Karapinar and N. V. Luong, Quadruple fixed point theorems for nonlinear contractions, Computer & Mathematics with Applications, 64 (6) (2012), 1839-1848.

[17] V. Lakshmikantham and L. Ciric, Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces, Nonlinear Analysis: Theory, Method and Applications, 70 (12) (2009), 4341-4349.

[18] X. Liu, Quadruple fixed point theorems in partially ordered metric spaces with mixed g-monotone property, Fixed Point Theory and Applications 2013, 2013:147.

[19] W. Long, S. Shukla and S. Radenovic, Some coupled coincidence and common fixed point results for hybrid pair of mappings in 0-complete partial metric spaces, Fixed Point Theory and Applications 2013, 2013:145.

[20] N. V. Luong and N. X. Thuan, Coupled fixed points in partially ordered metric spaces and application, Non-linear Anal. 74 (2011), 983-992.

[21] Z. Mustafa, Mixed g-monotone property and quadruple fixed point theorems in partially ordered G-metric spaces using (phi-psi) contractions, Fixed Point Theory and Applications 2012, 2012:199.

[22] Z. Mustafa, H. Aydi and E. Karapinar, Mixed g-monotone property and quadruple fixed point theorems in partially ordered metric spaces, Mustafa et al. Fixed Point Theory and Applications 2012, 2012:71.

[23] Z. Mustafa, W. A. Shatanawi and E. Karapinar, Quadruple fixed point theorems under nonlinear contractive conditions in partially ordered metric spaces, J. Appl. Math. 2012, Article Id: 951912.

[24] S. H. Rasouli and M. Bahrampour, On Quadruple fixed point theorems in partially ordered metric spaces, International journal of mathematics and statistics, 13 (1) (2013).

[25] B. Samet, E. Karapinar, H. Aydi and V. C. Rajic, Discussion on some coupled fixed point theorems, Fixed Point Theory and Applications 2013, 2013:50.

[26] W. Sintunavarat, P. Kumam and Y. J. Cho, Coupled fixed point theorems for nonlinear contractions without mixed monotone property, Sintunavarat et al. Fixed Point Theory and Applications 2012, 2012:170.