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ON L-FUZZY $\tilde{\omega}$-BASICALLY DISCONNECTED SPACES

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Abstract

In this paper L-fuzzy $\tilde{\omega}$-closed and L-fuzzy $\tilde{\omega}$-open sets are introduced. Also a new class of L-fuzzy topological space called L-fuzzy $\tilde{\omega}$-basically disconnected space is introduced. Several characterizations and some interesting properties are also given.

Keywords L-fuzzy $\tilde{\omega}$-closed, L-fuzzy $\tilde{\omega}$-open set, L-fuzzy ω-basically disconnected space, L-fuzzy $\tilde{\omega}$*-continuous map, L-fuzzy $\tilde{\omega}$*-irresolute, strong F_σ L-fuzzy $\tilde{\omega}$*-continuous map, lower (upper) L-fuzzy $\tilde{\omega}$*-continuous map.

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1. Introduction

The fuzzy concept has invaded almost all branches of Mathematics since the introduction of the concept by Zadeh[14]. Fuzzy sets have applications in many fields such as information [11] and control [12]. The theory of fuzzy topological spaces was introduced and developed by Chang [3] and since then various important notions in classical topology have been extended to fuzzy topological spaces. Rodabaugh [7] discussed normality and the L-fuzzy unit interval. He [8] also studied fuzzy addition in the L-fuzzy real line. Hoeche [6] studied the characterizations of L-topologies by L-valued neighbourhoods. An L-fuzzy normal spaces and Tietze extension theorem were discussed by Tomash Kubiak [14]. The concept of ω-open set was studied in [9]. The purpose of this paper is to introduce L-fuzzy $\tilde{\omega}$-closed, L-fuzzy $\tilde{\omega}$-open sets and a new class of L-fuzzy topological spaces called L-fuzzy $\tilde{\omega}$-basically disconnected space. In this connection several characterizations and some interesting properties are also given.

2. Preliminaries

Definition [1] Let (X, T) be a fuzzy topological space and λ be a fuzzy set in (X, T). λ is called a fuzzy G_δ-set if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in T$, $i \in I$.

Definition [1] Let (X, T) be a fuzzy topological space and λ be a fuzzy set in (X, T). λ is called a fuzzy F_σ-set if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$ where each $1 - \lambda_i \in T$, $i \in I$.

Definition [2] Throughout this paper $(L, \leq, ')$ stands for an infinitely distributive lattice with an order reversing involution. Such a lattice being complete has a least element 0 and a greatest element 1. Let X be a non-empty set. An L-fuzzy set in X is an element of the set L^X of all functions from X to L.

Definition [4] The L-fuzzy real line R(L)is the set of all monotone decreasing elements $\lambda \in L^R$ satisfying $\vee \{ \lambda(t)/t \in R \} = 1$ and $\wedge \{ \lambda(t)/t \in R \} = 0$, after the identification of $\lambda, \mu \in L^R$ iff $\lambda(t-) = \mu(t-)$ and $\lambda(t+) = \mu(t+)$ for all $t \in R$ where $\lambda(t-) = \wedge \{ \lambda(s)/s < t \}$ and $\lambda(t+) = \vee \{ \lambda(s)/s > t \}$. The natural L-fuzzy topology on R(L) is generated from the subbases $\{ L_t, R_t / t \in R \}$, where $L_t(\lambda) = \lambda(t-)'$ and $R_t(\lambda) = \lambda(t+)$.

The L-fuzzy unit interval I(L) **[5]** is a subset of R(L) such that $[\lambda] \in I(L)$ if $\lambda(t) = 1$ for $t < 0$ and $\lambda(t) = 0$ for $t > 1$. It is equipped with the subspace L-fuzzy topology.

Definition [14] If $A \in L^X$ is crisp, then (A, T_A) is an L-fuzzy topological space called a crisp subspace of (X, T), where $T_A = \{ U/A \mid U \in T \}$ is called the subspace L-fuzzy topology.

Definition [9] A subset of a topological space (X, T) is called ω-closed in (X, T) if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, T). A subset A is called ω-open in (X, T) if its complement, A^C is ω-closed.

Definition [13]

Let (X, T) be any fuzzy topological space. (X, T) is called fuzzy basically disconnected if the closure of every fuzzy open F_σ set is fuzzy open.

Definition [10] An L-fuzzy set λ of an L-fuzzy topological space (X, T) is called L-fuzzy ω-closed in (X, T) if L-cl (λ) ≤ μ whenever λ ≤ μ and μ is L-fuzzy semi-open in (X, T). The complement of L-fuzzy ω-closed set is L-fuzzy ω-open.

Definition [10] Let (X, T) be an L-fuzzy topological space. For any L-fuzzy set λ in (X,T), L-fuzzy ω-closure of λ (briefly, L-cl(λ)) is defined as

Lω-cl (λ) = ∧ { μ / μ ≥ λ and μ is L-fuzzy ω-closed }.

3. Characterizations and properties of L-fuzzy $\tilde{\omega}$-basically disconnected spaces.

In this section a new class of set called L-fuzzy $\tilde{\omega}$-closed set and thereby a new class of space called L-fuzzy $\tilde{\omega}$-basically disconnected space is introduced. Some interesting properties and characterizations are also discussed.

Definition 3.1 Let (X, T) be any L-fuzzy topological space and λ be any L-fuzzy set in (X, T). λ is called

(a) an L-fuzzy G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ where each λ_i is L-fuzzy open.

(b) an L-fuzzy F_σ set if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$ where each ($1 - \lambda_i$) is L-fuzzy open.

Definition 3.2 Let λ be any L-fuzzy set in the L-fuzzy topological space (X,T). Then we define

L-int (λ) = ∨ { μ / μ ≤ λ and μ is L-fuzzy open }

L-cl (λ) = ∧ { μ / μ ≥ λ and μ is L-fuzzy closed }.

Definition 3.3 Let λ be any L-fuzzy set in the L-fuzzy topological space (X, T). λ is called L-fuzzy semi-open if λ ≤ L-cl (L-int (λ)).

Definition 3.4 An L-fuzzy set λ of an L-fuzzy topological space (X, T) is called L-fuzzy $\tilde{\omega}$-closed in (X, T) if L-ωcl (λ) ≤ μ whenever λ ≤ μ and μ is L-fuzzy semi-open in (X, T). The complement of L-fuzzy $\tilde{\omega}$-closed set is L-fuzzy $\tilde{\omega}$-open.

Note 3.1

a) Let (X, T) be an L-fuzzy topological space. An L-fuzzy set λ in (X, T) which is both L-fuzzy $\tilde{\omega}$-open and L-fuzzy F_σ is denoted by L-fuzzy $\tilde{\omega}$-open F_σ.

b) Let (X, T) be an L-fuzzy topological space. An L-fuzzy set λ in (X, T) which is both L-fuzzy $\tilde{\omega}$-closed and L-fuzzy G_δ is denoted by L-fuzzy $\tilde{\omega}$-closed G_δ.

Notation 3.1 An L-fuzzy set λ which is both L-fuzzy $\tilde{\omega}$-open F_σ and L-fuzzy $\tilde{\omega}$-closed G_δ is denoted by L-fuzzy $\tilde{\omega}$-COGF.

Definition 3.5 Let (X, T) be an L-fuzzy topological space. For any L-fuzzy set λ in (X, T), L-fuzzy $\tilde{\omega}$*-closure of λ (briefly, L$\tilde{\omega}$*-cl (λ) is defined as

L$\tilde{\omega}$*-cl (λ) = ∧ { μ : μ ≥ λ and μ is L-fuzzy $\tilde{\omega}$-closed G_δ }.

Definition 3.6 Let (X, T) be an L-fuzzy topological space. For any L-fuzzy set λ in (X, T), L-fuzzy $\tilde{\omega}$*-interior of λ (briefly, L$\tilde{\omega}$*-int (λ)) is defined as L$\tilde{\omega}$*-int (λ) = ∨ { μ : μ ≤ λ and μ is L-fuzzy $\tilde{\omega}$-open F_σ }.

Remark 3.1 Let (X, T) be an L-fuzzy topological space. For any L-fuzzy set λ in (X, T)

a) $1 - L\tilde{\omega}^*\text{-int}(\lambda) = L\tilde{\omega}^*\text{-cl}(1-\lambda)$.
b) $1 - L\tilde{\omega}^*\text{-cl}(\lambda) = L\tilde{\omega}^*\text{-int}(1-\lambda)$.

Definition 3.7 Let (X, T) and (Y, S) be any two L-fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is called L-fuzzy $\tilde{\omega}^*$-continuous if $f^{-1}(\lambda)$ is L-fuzzy $\tilde{\omega}$-closed G_δ in (X,T) for every L-fuzzy closed and L-fuzzy G_δ set λ in (Y, S).

Definition 3.8 Let (X, T) and (Y, S) be any two L-fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is called L-fuzzy $\tilde{\omega}^*$-irresolute if the inverse image of every L-fuzzy $\tilde{\omega}$-open F_σ set in (Y, S) is L-fuzzy $\tilde{\omega}$-open F_σ in (X, T).

Definition 3.9 Let (X, T) and (Y, S) be any two L-fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is said to be L-fuzzy $\tilde{\omega}^*$-open if the image of every L-fuzzy $\tilde{\omega}$-open F_σ set in (X, T) is L-fuzzy $\tilde{\omega}$-open F_σ in (Y, S).

Proposition 3.1 Let (X, T) and (Y, S) be any two L-fuzzy topological spaces. Then $f : (X, T) \to (Y, S)$ is L-fuzzy $\tilde{\omega}^*$-irresolute iff $f(L\tilde{\omega}^*\text{-cl}(\lambda)) \leq L\tilde{\omega}^*\text{-cl}(f(\lambda))$, for every L-fuzzy set λ in (Y, S).

Proposition 3.2 Let (X, T) and (Y, S) be any two L-fuzzy topological spaces and let $f : (X, T) \to (Y, S)$ be an L-fuzzy $\tilde{\omega}^*$-open surjective function. Then $f^{-1}(L\tilde{\omega}^*\text{-cl}(\lambda)) \leq L\tilde{\omega}^*\text{-cl}(f^{-1}(\lambda))$, for each L-fuzzy set λ in (Y, S).

Definition 3.10 Let (X, T) be any L-fuzzy topological space. (X, T) is called L-fuzzy $\tilde{\omega}$-basically disconnected if the L-fuzzy $\tilde{\omega}^*$-closure of every L-fuzzy $\tilde{\omega}$-open F_σ set is L-fuzzy $\tilde{\omega}$-open F_σ.

Proposition 3.3 For an L-fuzzy topological space (X, T) the following statements are equivalent:

(a) (X, T) is an L-fuzzy $\tilde{\omega}$-basically disconnected space.
(b) For each L-fuzzy $\tilde{\omega}$-closed G_δ set λ, $L\tilde{\omega}^*\text{-int}(\lambda)$ is L-fuzzy $\tilde{\omega}$-closed G_δ.
(c) For each L-fuzzy $\tilde{\omega}$-open F_σ set λ, $L\tilde{\omega}^*\text{-cl}(\lambda) + L\tilde{\omega}^*\text{-cl}(1 - L\tilde{\omega}^*\text{-cl}(\lambda)) = 1$.
(d) For every pair of L-fuzzy $\tilde{\omega}$-open F_σ sets λ and μ such that $L\tilde{\omega}^*\text{-cl}(\lambda)+\mu = 1$, we have $L\tilde{\omega}^*\text{-cl}(\lambda) + L\tilde{\omega}^*\text{-cl}(\mu) = 1$.

Proposition 3.4 Let (X, T) be any L-fuzzy ω-basically disconnected space and (Y, S) be any L- fuzzy topological space. Let $f : (X, T) \to (Y, S)$ be
L-fuzzy $\tilde{\omega}^*$-irresolute, L-fuzzy $\tilde{\omega}^*$-open and surjective function. Then (Y, S) is L-fuzzy $\tilde{\omega}$-basically disconnected.

Definition 3.11 Let $\{ (X_\alpha, T_\alpha) / \alpha \in \Delta \}$ be a family of disjoint L-fuzzy topological spaces. Let $X = \bigcup_{\alpha\in\Delta} X_\alpha$. Define $T = \{ \lambda \in L^X / \lambda / X_\alpha$ is L-fuzzy $\tilde{\omega}$-open F_σ in $(X_\alpha, T_\alpha) \}$. Then (X, T) is an L-fuzzy topological space called the L-fuzzy topological sum of $\{ (X_\alpha, T_\alpha) / \alpha \in \Delta \}$.

Proposition 3.5 Let $\{ (X_\alpha, T_\alpha) / \alpha \in \Delta \}$ be a family of disjoint L-fuzzy $\tilde{\omega}$-basically disconnected spaces and let (X, T) be their L-fuzzy topological sum. Then (X, T) is L-fuzzy $\tilde{\omega}$-basically disconnected.

Definition 3.12 Let (X, T) be an L-fuzzy topological space. A mapping $f : X \to R(L)$ is called lower (resp. upper) L-fuzzy $\tilde{\omega}^*$-continuous if $f^{-1}(R_t)$ (resp. $f^{-1}(L_t)$) is L-fuzzy $\tilde{\omega}$-open F_σ (resp. L-fuzzy $\tilde{\omega}$-open F_σ / L-fuzzy $\tilde{\omega}$-closed G_δ), for each $t \in R$.

Proposition 3.6 Let (X, T) be an L-fuzzy topological space. Then (X, T) is L-fuzzy $\tilde{\omega}$-basically disconnected iff for all L-fuzzy $\tilde{\omega}$-open F_σ set λ and an L-fuzzy $\tilde{\omega}$-closed G_δ set μ such that $\lambda \leq \mu$, $L\tilde{\omega}^*\text{-cl}(\lambda) \leq L\tilde{\omega}^*\text{-int}(\mu)$.

Remark 3.2 Let (X, T) be an L-fuzzy $\tilde{\omega}$-basically disconnected space. Let $\{ \lambda_i, 1-\mu_i / i \in N \}$ be a collection such that λ_i's, are L-fuzzy $\tilde{\omega}$-open F_σ and μ_i's are L-fuzzy $\tilde{\omega}$-closed G_δ and let

λ, μ are L-fuzzy $\tilde{\omega}$-COGF. If $\lambda_i \le \lambda \le \mu_j$ and $\lambda_i \le \mu \le \mu_j$ for all i, j $\in$ N, then there exists an L-fuzzy $\tilde{\omega}$-COGF set γ such that $L\tilde{\omega}^*$-cl (λ_i) $\le \gamma \le L\tilde{\omega}^*$-int (μ_j), for all i, j $\in$ N.

Proposition 3.7 Let (X, T) be an L-fuzzy ω-basically disconnected space. Let $\{\lambda_r\}_{r\in Q}$ and $\{\mu_r\}_{r\in Q}$ be monotone increasing collections of L-fuzzy $\tilde{\omega}$-open F_σ sets and L-fuzzy $\tilde{\omega}$-closed G_δ sets of (X, T) and suppose that $\lambda_{q_1} \le \mu_{q_2}$ whenever $q_1 < q_2$ (Q is the set of all rational numbers). Then there exists a monotone increasing collection $\{\gamma_r\}_{r\in Q}$ of L-fuzzy $\tilde{\omega}$-COGF sets of (X, T) such that $L\tilde{\omega}^*$-cl (λ_{q_1}) $\le$ (γ_{q_2}) and $\gamma_{q_1} \le L\tilde{\omega}^*$-int (μ_{q_2}) whenever $q_1 < q_2$.

Proposition 3.8 Let (X, T) be any L-fuzzy topological space; let $\lambda \in L^X$ and let f : X $\to$ R(L) be such that

$$f(x)(t) = \begin{cases} 1 & \text{if } t < 0 \\ \lambda(x) & \text{if } 0 \le t \le 1 \\ 0 & \text{if } t > 0 \end{cases}$$

for all x $\in$ X. Then f is lower (resp. upper) L-fuzzy $\tilde{\omega}^*$-continuous iff λ is L-fuzzy $\tilde{\omega}$-open F_σ (resp. L-fuzzy $\tilde{\omega}$-open F_σ / L-fuzzy $\tilde{\omega}$-closed G_δ).

Definition 3.13 The characteristic function of $\lambda \in L^X$ is the map χ_λ : X $\to$ I(L) defined by χ_λ (x) = (λ(x)), x $\in$ X.

Proposition 3.9 Let (X, T) be an L-fuzzy topological space and let $\lambda \in L^X$. Then χ_λ is lower (resp.upper) L-fuzzy $\tilde{\omega}^*$-continuous iff λ is L-fuzzy $\tilde{\omega}$-open F_σ (resp. L-fuzzy $\tilde{\omega}$-open F_σ/ L-fuzzy $\tilde{\omega}$-closed G_δ).

Definition 3.14 Let (X, T) and (Y,S) be any two L-fuzzy topological spaces. A mapping f : (X, T)$\to$(Y, S) is called strong F_σ L-fuzzy $\tilde{\omega}^*$-continuous if $f^{-1}(\lambda)$ is L-fuzzy $\tilde{\omega}$-COGF set of (X, T), for every L-fuzzy $\tilde{\omega}$-open F_σ set λ of (Y, S).

Proposition 3.10 Let (X, T) be an L-fuzzy topological space. Then the following statements are equivalent :

(a) (X, T) is an L-fuzzy $\tilde{\omega}$-basically disconnected space.

(b) If g, h : X $\to$ R(L) where g is lower L-fuzzy $\tilde{\omega}^*$-continuous, h is upper L fuzzy $\tilde{\omega}^*$-continuous, then there exists f $\in$ C_{F_σ} $L\omega\tilde{\omega}$ (X) such that g $\le$ f $\le$ h. [C_{F_σ} $L\tilde{\omega}$ (X) = collection of all strong F_σ L-fuzzy $\tilde{\omega}^*$-continuous function on X with values in R (L)].

(c) If λ is L-fuzzy $\tilde{\omega}$-closed G_δ and μ is L-fuzzy $\tilde{\omega}$-open F_σ sets such that $\mu \le \lambda$, then there exists a strong F_σ L-fuzzy $\tilde{\omega}^*$-continuous function f : X$\to$ I(L) such that $\mu \le (1-L_1)f \le R_o f \le \lambda$.

Proposition 3.11 Let (X, T) be an L-fuzzy $\tilde{\omega}$-basically disconnected space and let A $\subset$ X be such that χ_A is L-fuzzy $\tilde{\omega}^*$-open. Let f : (A, T/A) $\longrightarrow$ I(L) be strong F_σ L-fuzzy $\tilde{\omega}^*$-continuous. Then f has a strong F_σ L-fuzzy $\tilde{\omega}^*$-continuous extension over (X, T).

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