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Hypothetical six-dimensional reference frames

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Abstract

A commonly accepted version of space-time in four dimensions depends on an axiomatic assumption: there exists only one temporal coordinate measured on a line, like the spatial ones, although with imaginary values. Recalling logical, epistemological and physical argumentations developed in previous works, we explore a possible scenario where an event requires three time coordinates to be defined and a surface is a geometrical entity more effective than a line for representing time. The main consequences of a three-dimensional time (instead of monodimensional) measured on mutually orthogonal surfaces (instead of axes) are the introduction of new six-dimensional reference frames in flat and curved space-time and the reformulation of Relativity equations in 6D.

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1. Introduction

Among the manifold arguments [2-35] supporting the three-dimensional time hypothesis (3T for brevity), we focus uniquely on those supplied by the author in previous works about this topic [2-16].

There we analyzed the space-time structure, starting from two questions:

1) How many geometric dimensions does a single temporal coordinate have?

2) How many temporal coordinates are necessary to determine an event in the space-time continuum?

We found that each time coordinate shows a *bidimensional* nature, whereas *three* is the minimum number of temporal coordinates individuating an event.

In order to describe a so formulated SO(3,3) space-time, the best representation in Euclidean conditions would be given by three orthogonal axes (spatial directions) and three orthogonal surfaces (temporal orientations). Such 6D Cartesian-like reference frame, where the six coordinates are pairwise mutually perpendicular, should become 6D Gaussian-like in a curved space-time.

Being the space-time metric changed, all the equations of the standard 4D Relativity should be subject to a six-dimensional revision.

2. Bidimensionality of the temporal coordinate

2.1 Time as a surface from the dimensions of $\vec{g}$

In any acceleration the linear space measure x is related to a squared time measure t^2.

Gravity in its vectorial formulation on Earth (or on any celestial body) is a constant acceleration $\vec{g}$ and, according to the author's opinion, it is more than a mere coincidence.

Since gravity is a direct expression of the space-time structure, the ratio between *linear meters* and *squared seconds* in $\vec{g}$ $(m \cdot s^{-2})$ induces to treat the spatial coordinate as linear (i.e., described by an axis) and the temporal coordinate as squared (i.e., laying on a surface).

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We may represent the temporal surface as a square, whose side is the measured time t, perpendicular to the displacement vector $\vec{x}$ oriented towards the Earth as the gravity vector $\vec{g}$ (Fig. 1).

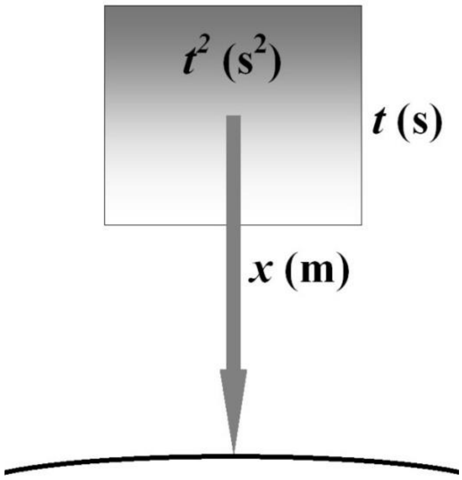

Figure 1. The gravitational acceleration $g = x/t^2$.

2.2 Time as a surface from its numerical "reality"

The interpretation of the negative term $-c^2\Delta t^2$ from squaring the imaginary temporal coordinate $ic\Delta t$ (with $i = \sqrt{-1}$) in the 4D space-time interval equation (with a space-like sign convention for the metric signature)

(1) $\quad \Delta\sigma^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2,$

presupposes a *dimensional homogeneity* between spatial and temporal coordinates, so that all of them are postulated to be *linear* (Fig. 2).

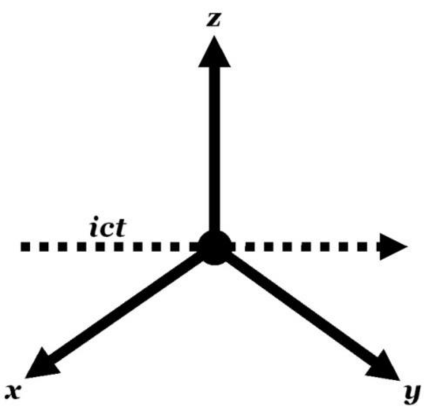

Figure 2. Classic "imaginary" interpretation of Δt.

The inclusion of all the variables in the set of real numbers leads to another conclusion: in order to be *real* (i.e., no imaginary value allowed) a temporal coordinate must be a surface with a negative orientation

(2) $\quad \Delta S = -c^2\Delta t^2.$

According to time measures on surfaces (Fig. 3), $\Delta\sigma^2$ (not $\Delta\sigma$) is the minimal irreducible physical reality.

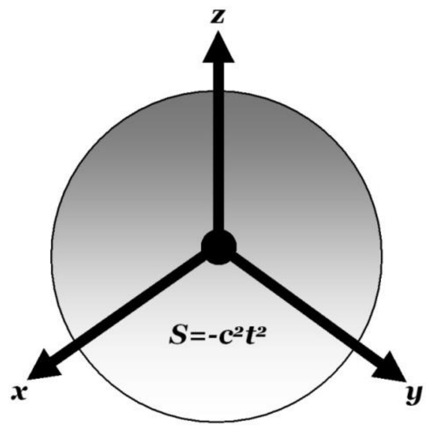

Figure 3. Alternative "real" interpretation of Δt^2.

2.3 Time as a surface from the triangle of velocities

The locally quasi-Euclidean space-time permits to add the orthogonal trajectories of a light ray and its emitting source in uniform rectilinear motion (Fig. 4) via the Pythagorean theorem:

(3) $\quad l^2 = l_0^2 + d^2.$

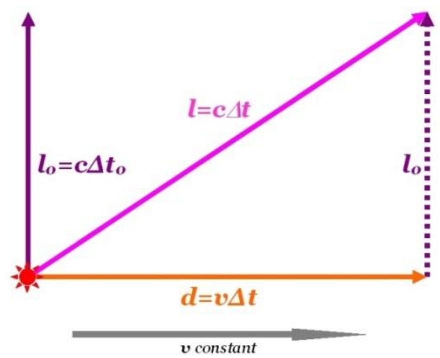

Figure 4. Relativistic triangle of velocities.

The subsequent measure of time does not depend on the specific direction of l_0 but upon its *orientation*, always perpendicular to the vector velocity $\vec{v}$:

(4) $(c\Delta t)^2 = (c\Delta t_0)^2 + (v\Delta t)^2,$

(5) $\Delta t = \Delta t_0/\sqrt{1-(v/c)^2} = \gamma\Delta t_0\ .$

Therefore, what the same measures of Δt have in common is a *surface* (Figs. 5 and 6).

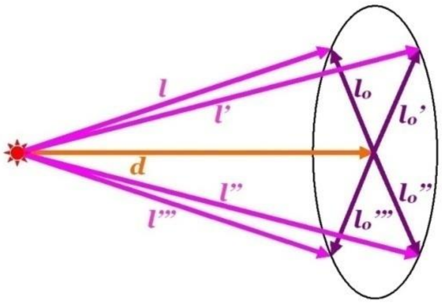

Figure 5. Triangles of velocities sharing an orientation.

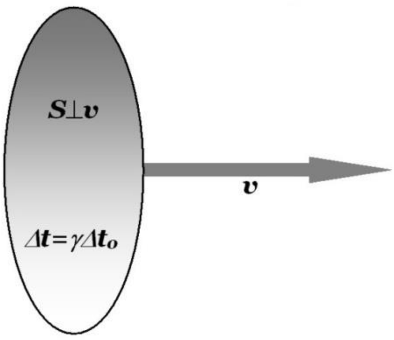

Figure 6. The temporal measures' common surface.

3. Why should time have three coordinates?

3.1 The 3T from the reciprocity principle

The *acausal* and *precausal* paradoxes in Quantum Mechanics show that *cause* and *effect* are not inescapable in the physical description [1].

The Reciprocity Principle [3] expresses the invariance following the permutation between subject (cause) and direct object (effect) within a well-formulated proposition, with a perfect logic symmetry and temporal reversibility in the physical description.

Based on reciprocity, the Fitzgerald contraction is interpretable both in the conventional way, i.e., the *speed* of the body generates the *length's contraction* in the movement direction and in the opposite, i.e., the *length's contraction* in a certain direction generates the *speed* of the body.

Similarly, in the Euclidean space-time of the Special Relativity (flat) we can say both that the *speed* of the body causes the *time dilation* and that, on the contrary, the *time dilation* causes the *speed* of the body. If time were not three-dimensional, the second interpretation would not be possible; in fact, without a direction identifying Δt, a temporal dilation could not be associated with a specific vector velocity $\vec{v}$.

3.2 The 3T from algebraic symmetry

Since in the space-time interval equation (with a space-like sign convention for the metric signature)

(6) $\Delta\sigma^2 = \Delta s^2 - c^2\Delta t^2$

the spatial term is

(7) $\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2,$

then the temporal component should be

(8) $\Delta t^2 = \Delta t_1^2 + \Delta t_2^2 + \Delta t_3^2$

by algebraic symmetry; it would mean to substitute the ordinary SO(1,3) with the SO(3,3) symmetry group.

3.3 The 3T from an ideal experiment

Through an ideal test with a laser diode-photodiode device in uniform circular motion (Fig. 7), we obtained [2] temporal measures different on each orientation.

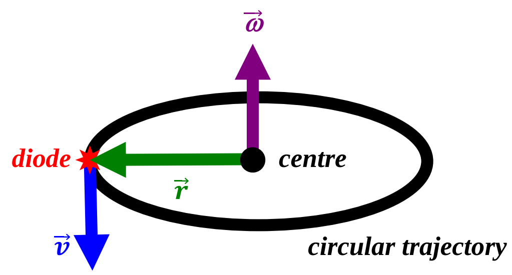

Figure 7. Diode in uniform circular motion $\vec{v} = \vec{\omega} \times \vec{r}$.

We calculated five kinds of time measure:

1) The time at rest Δt_0.

2) The inertial time Δt_i, measured in the uniform rectilinear motion and, in general, when the laser ray is emitted in any direction perpendicular to the motion.

3) The tangential time $\Delta\tau$, measured on the orientation perpendicular to the tangential velocity $\vec{v}$ ($\Delta\tau \perp \vec{v}$), whose value is coincident with the inertial time.

4) The angular time $\Delta\theta$, measured on the orientation perpendicular to the angular velocity $\vec{\omega}$ ($\Delta\theta\perp\vec{\omega}$), whose value grows with the diode-photodiode distance and coincides with Δt_i just at an intermediate position.

5) The radial time $\Delta\rho$, measured on the orientation perpendicular to the radius of curvature $\vec{r}$ ($\Delta\rho\perp\vec{r}$), whose value decreases asymptotically towards Δt_i with the diode-photodiode distance.

4. Six dimensional reference frames

4.1 Instantaneous reference frame in 6D

The three instantaneous vectors $\vec{v},\vec{\omega},\vec{r}$ and their perpendicular temporal planes (respectively τ,θ,ρ) altogether constitute an instantaneous reference frame $v\omega r\tau\theta\rho$ (Fig. 8) on whose orientations the three times (tangential, angular, radial) are measured.

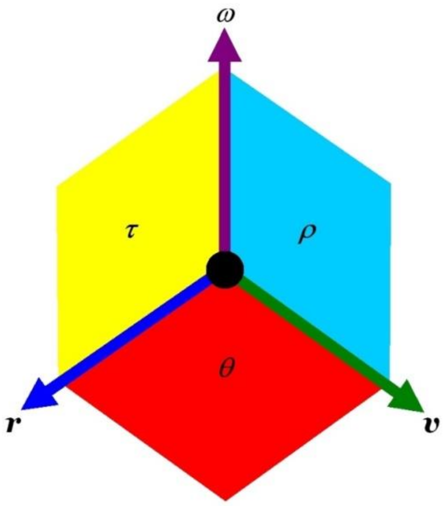

Figure 8. 6D instantaneous reference frame.

4.2 Cartesian-like reference in a 6D flat space-time

For any point on a continuous trajectory (Fig. 9), the measures in the instantaneous reference $v\omega r\tau\theta\rho$ (Fig. 8)

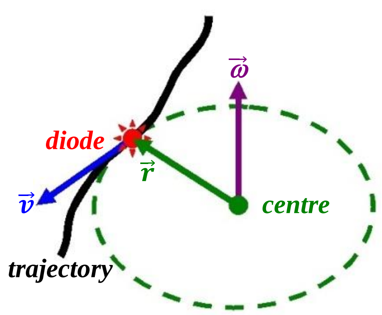

Figure 9. A diode along a continuous trajectory any.

are univocally projected on a fixed 6D Cartesian-like reference $xyzt_xt_yt_z$ (Fig. 10) where the three spatial axes x,y,z and the three temporal orientations t_x,t_y,t_z are mutually orthogonal and at rest. Each 6D event $E(x,y,z,t_x,t_y,t_z)$ is representable without ambiguity.

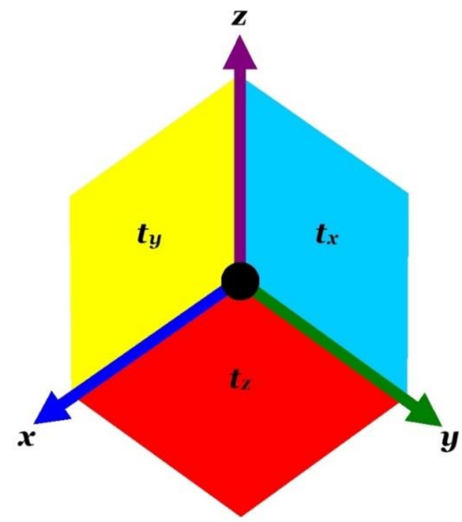

Figure 10. 6D Cartesian-like reference frame.

4.3 Gaussian-like reference in a 6D curved space-time

Assuming a space-time continuum quasi-Euclidean at local level, any 6D Cartesian-like reference $xyzt_xt_yt_z$ is connectable to a 6D Gaussian-like reference (Fig. 11)

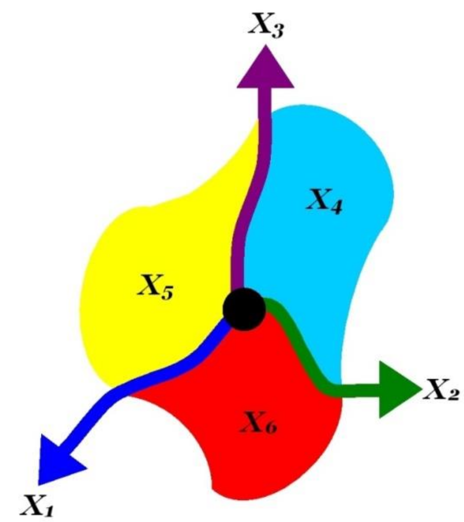

Figure 11. 6D Gaussian-like reference frame.

$X_1X_2\,X_3X_4\,X_5X_6$ with the three spatial lines X_1, X_2, X_3 not necessarily neither rectilinear nor mutually orthogonal and the three temporal surfaces X_4, X_5, X_6 not necessarily neither plane nor reciprocally orthogonal.

The only condition is that each event in a curved 6D space-time locally almost flat $E(X_1, X_2, X_3, X_4, X_5, X_6)$ must be univocally individuated.

The six-dimensional binary quadratic form is:

(9) $d\sigma^2 = g_{\mu\nu} dX_\mu dX_\nu$ with $\mu, \nu = 1, 2, 3, 4, 5, 6$.

4.4 A meaningless opposite reference frame?

The symmetrical 6D Cartesian-like reference frame would consist of three temporal axes t_x, t_y, t_z and three spatial orientations x, y, z mutually orthogonal and at rest; each event $E(t_x, t_y, t_z, x, y, z)$ would be unique, i.e., representable without ambiguity.

Such 6D reference (Fig. 12) seems worthy of study, especially in relation with an alternative *time-like* sign convention for the metric signature of $\Delta\sigma^2$.

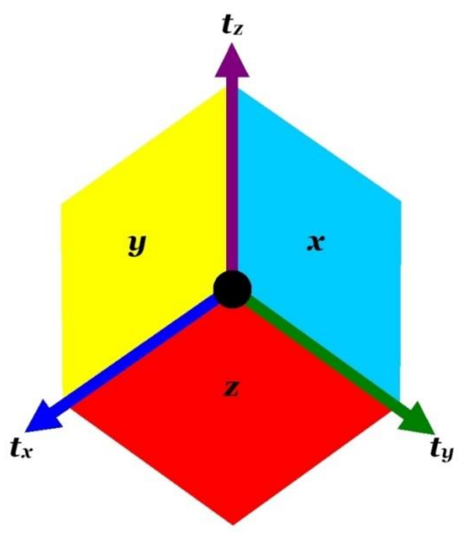

Figure 12. Inverted 6D Cartesian-like reference frame.

5. Dealing with a six-dimensional space-time

5.1 Beyond the concept of vector in 6D

If time were not a monodimensional scalar, the vector velocity

(10) $\vec{v} = \frac{\Delta\vec{s}}{\Delta t}$

should be replaced by another operator, but which?

For finite differences, a logical solution would be:

(11) $\vec{v} = \frac{\Delta x}{\Delta t_x}\hat{x} + \frac{\Delta y}{\Delta t_y}\hat{y} + \frac{\Delta z}{\Delta t_z}\hat{z}$

and, recalling the Eq. 7, a differential equation could be:

(12) $\vec{v} = \frac{\partial s}{\partial t_x}\hat{x} + \frac{\partial s}{\partial t_y}\hat{y} + \frac{\partial s}{\partial t_z}\hat{z}$.

Unfortunately, both the Eqs. 11 and 12 would not solve the geometrical problem of the incommensurability between the vectorial numerators (axes) and the oriented denominators (surfaces); a mathematical-physical tool merging Euclidean vectors and orientations is auspicated.

We might, however, measure the squared modulus of velocity by keeping the spatial contributions (numerator) separate from the temporal contributions (denominator):

(13) $v^2 = \frac{\Delta s^2}{\Delta t^2} = \frac{\Delta x^2 + \Delta y^2 + \Delta z^2}{\Delta t_x^2 + \Delta t_y^2 + \Delta t_z^2}$.

5.2 Particles apparently faster than light in 4D

In a $(4+n)$D space-time, with at least one time extra-dimension u, we can suppose a particle p moving along the temporal dimensions u and t at:

(14) $v^2 = \frac{\Delta x^2 + \Delta y^2 + \Delta z^2}{\Delta t^2 + \Delta u^2} < c^2$.

A normal ($v < c$) motion in 6D (Fig. 13) could appear faster than light in the ordinary 4D space-time (Fig. 14) if the extra time dimensions were hidden to the observer and we had a very small variation $\Delta t = \varepsilon$:

(15) $v^2 = \frac{\Delta x^2 + \Delta y^2 + \Delta z^2}{\varepsilon^2} > c^2$.

For infinitesimal variations $\Delta t \to 0$, the velocity could appear even *infinite* in 4D.

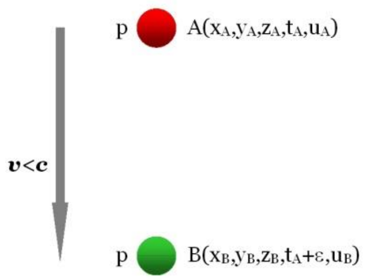

Figure 13. A particle *p* slower than light in $(4+n)$D.

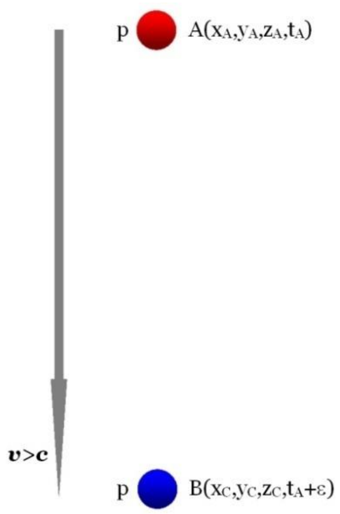

Figure 14. A particle *p* appearing faster than light in 4D.

5.3 Particles apparently multilocated in 4D

In a $(4+n)$D space-time (with at least one time extra-dimension u) we can suppose (Fig. 15) a particle p moving along the temporal dimension u, keeping t invariant ($\Delta t = 0$), at:

$$(16) \quad v^2 = \frac{\Delta x^2 + \Delta y^2 + \Delta z^2}{\Delta u^2} < c^2.$$

If the extra time dimensions were somehow hidden, a normal ($v < c$) motion in 6D could be interpreted as appearance of a particle in many places, at the same time t, in the ordinary 4D space-time (Fig. 16).

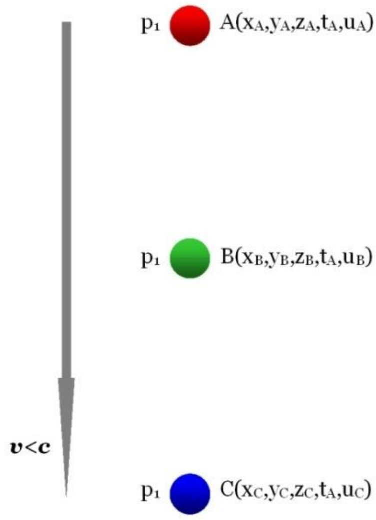

Figure 15. A particle p_1 slower than light in $(4+n)$D.

Figure 16. A particle p_1 appearing multilocated in 4D.

5.4 Extension of Lorentz transformation in 6D

In a 6D space-time, we should six-dimensionally extend the Lorentz transformations by adding two further equations: $t_y^{'} = t_y$, $t_z^{'} = t_z$. Let v_x be the constant velocity along the spatial x-axis (Fig. 17).

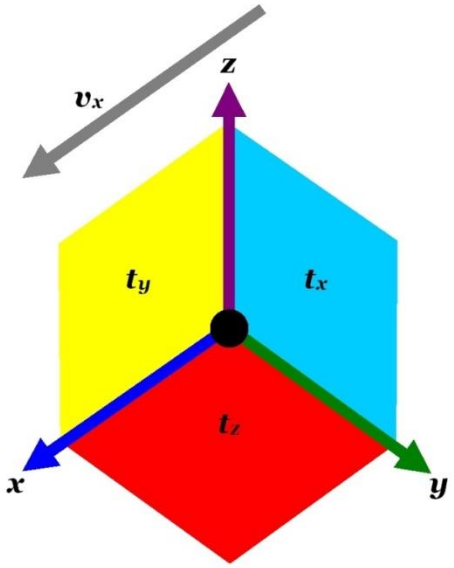

Figure 17. Velocity v_x oriented along the x-axis.

Denoting

(17) $\beta_x = v_x / c$

and

(18) $\gamma_x = 1/\sqrt{1-\beta_x^2}$

the hypothetical 6D Lorentz transformations would be:

$$(19)\quad \begin{cases} x' = \gamma_x(x - v_x t_x) \\ y' = y \\ z' = z \\ t'_x = \gamma_x(t_x - x\frac{\beta_x}{c}) \\ t'_y = t_y \\ t'_z = t_z \end{cases}$$

5.5 Extension of the invariant squared interval in 6D

Denote K and K' two inertial reference frames with parallel and equioriented coordinate axes.

Let us employ the Eq. (19) where v_x, constant and parallel to the spatial x-axis, is the relative velocity of K' respect to K.

In K the event "1" is $E_1(x_1, y_1, z_1, t_{x1}, t_{y1}, t_{z1})$ and the event "2" is $E_2(x_2, y_2, z_2, t_{x2}, t_{y2}, t_{z2})$.

In K' the event "1" is $E'_1(x'_1, y'_1, z'_1, t'_{x1}, t'_{y1}, t'_{z1})$ and the event "2" is $E'_2(x'_2, y'_2, z'_2, t'_{x2}, t'_{y2}, t'_{z2})$.

The event "1" in K' is transformed as:

$$(20)\quad \begin{cases} x'_1 = \gamma_x(x_1 - v_x t_{x1}) \\ y'_1 = y_1 \\ z'_1 = z_1 \\ t'_{x1} = \gamma_x(t_{x1} - x_1\frac{\beta_x}{c}) \\ t'_{y1} = t_{y1} \\ t'_{z1} = t_{z1} \end{cases}$$

The event "2" in K' is transformed as:

$$(21)\quad \begin{cases} x'_2 = \gamma_x(x_2 - v_x t_{x2}) \\ y'_2 = y_2 \\ z'_2 = z_2 \\ t'_{x2} = \gamma_x(t_{x2} - x_2\frac{\beta_x}{c}) \\ t'_{y2} = t_{y2} \\ t'_{z2} = t_{z2} \end{cases}$$

Assuming the space-like sign convention for the metric signature, the six-dimensional squared distance (interval) between the two events E_2 and E_1, in K, is:

$$(22)\quad \Delta\sigma^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2[(t_{x2} - t_{x1})^2 + (t_{y2} - t_{y1})^2 + (t_{z2} - t_{z1})^2].$$

Similarly, the six-dimensional squared distance (interval) between the two events E'_2 and E'_1, in K', is:

$$(23)\quad \Delta\sigma'^2 = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2 - c^2[(t'_{x2} - t'_{x1})^2 + (t'_{y2} - t'_{y1})^2 + (t'_{z2} - t'_{z1})^2].$$

According to the Eqs. (20) and (21), it is easy to reduce the Eq. (23) to (22) verifying that:

(24) $\Delta\sigma'^2 = \Delta\sigma^2$.

Thus, the six-dimensional squared interval

$$(25)\quad \Delta\sigma^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2(\Delta t_x^2 + \Delta t_y^2 + \Delta t_z^2)$$

is invariant to 6D reference frames.

5.6 Extension of the General Relativity in 6D

If space and time are both three-dimensional, then we have $\mu, \nu = 1,2,3,4,5,6$ not only for the line element and its metric tensor $g_{\mu\nu}$ (Eq. 9) but for all the other relativistic tensors: Ricci $R_{\mu\nu}$, Einstein $G_{\mu\nu}$, source $T_{\mu\nu}$.

In particular, the six-dimensional field equations $G_{\mu\nu} = kT_{\mu\nu}$ are 36 (reducible to 21 in case of symmetry) and the 6D source tensor $T_{\mu\nu}$ consists of four quadrants (Fig. 18) each containing nine components (Fig. 19).

Energy density	Energy flux

Momentum density	**Momentum flux**

Figure 18. The four quadrants of the 6D source tensor.

T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}
T_{21}	T_{22}	T_{23}	T_{24}	T_{25}	T_{26}
T_{31}	T_{32}	T_{33}	T_{34}	T_{35}	T_{36}
T_{41}	T_{42}	T_{43}	T_{44}	T_{45}	T_{46}
T_{51}	T_{52}	T_{53}	T_{54}	T_{55}	T_{56}
T_{61}	T_{62}	T_{63}	T_{64}	T_{65}	T_{66}

Figure 19. The 36 components of the 6D source tensor.

6. Conclusions

We motivate that, differently from a linear space coordinate, each time coordinate is 2-dimensional (Δt^2) requiring a surface to be measured (namely the one perpendicular to motion), with three observations:

1) the dimensions ($m \cdot s^{-2}$) of gravity $\vec{g}$, showing a link between a linear space and a squared time;

2) the exclusion of the imaginary value $ic\Delta t$, needing a negatively oriented temporal surface $\Delta S = -c^2 \Delta t^2$;

3) the time measures in the relativistic triangle of velocities, sharing the perpendicular orientation.

We motivate that the temporal coordinates necessary to define a single event in the space-time continuum are three (3T), each one locally oriented perpendicularly to a space axis, through a three-fold argumentation:

1) the Reciprocity Principle (cause-effect permutation) applied to Lorentz transformations, leading to the 3T;

2) the algebraic symmetry between space and time, involving the group SO(3,3);

3) an ideal laser diode-photodiode device in uniform circular motion, revealing a 3D-oriented time.

We introduce new reference frames (instantaneous, Cartesian-like and Gaussian-like) for an event in the 6D space-time, characterized by three spatial axes (vectors) and three temporal surfaces (orientations).

Afterward, we six-dimensionally extend the Lorentz transformations and the relativistic equations.

The 6D model is not homogeneous with respect to the representation of space and time; the hyphened term "space-time" is used throughout the paper to remark it. The diversity between the spatial and temporal components in our hypothetical 6D chronotope raises the open question of finding a mathematical-physical entity, different from multivectors or spinors, to describe the 6D velocity.

We suggest to investigate also the possible physical meaning of an inverted 6D Cartesian-like reference frame, i.e., with three temporal axes and three spatial surfaces, starting from a time-like (instead of space-like) sign convention for the metric signature of $\Delta\sigma^2$.

Acknowledgments

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