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A STUDY ON FUZZY REGULAR SEMI α-OPEN SETS

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ABSTRACT

This paper gives a brief survey on the properties and the characterizations of fuzzy regular semi α-connected spaces and fuzzy regular super semi α-connected spaces with respect to fuzzy regular semi α-open sets.

Key words

fuzzy regular semi α-open sets, fuzzy regular semi α-continuous functions, fuzzy regular semi α-connected spaces and fuzzy regular super semi α-connected spaces.

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1.INTRODUCTION AND PRELIMINARIES

The concept of fuzzy set was introduced by Zadeh [5] in his classical paper. Fuzzy sets have applications in many fields such as information and control. The concept of fuzzy topological spaces was introduced and developed by Chang [3]. The concepts of fuzzy pre-closed sets, fuzzy semi-closed sets and α-open sets were introduced by Bin Shahna [2] and the concept of fuzzy regular closed was introduced by Azad [1]. The concept of fuzzy semi-connected spaces was introduced by Uma, Roja and Balasubramanian [4].

DEFINITION 1.1 [3] A fuzzy topology on a set X is a collection δ of fuzzy sets in X satisfying:

(i) $0 \in \delta$ and $1 \in \delta$, (ii) if μ and ν belong to δ, then so does $\mu \wedge \nu$, and (iii) if μ_i belongs to δ for each i $\in$I, then so does $\vee_{i \in I} \mu_i$.

If δ is a fuzzy topology on X, then the pair (X, δ) is called a **fuzzy topological space**.

Members of $\delta$are called open fuzzy sets. Fuzzy sets of the form$1 - \mu$, where $\mu$is an open fuzzy set, are called closed fuzzy sets.

DEFINITION 1.2[1]Any fuzzy set $\lambda \in I^X$ in a fuzzy topological space (X, T) is said to be a **fuzzy semi-closed set** if $\lambda \geq$int (cl (λ)). Its complement is said to be a **fuzzy semi-open set.**

DEFINITION 1.3[2]Any fuzzy set $\lambda \in I^X$ in a fuzzy topological space (X, T) is said to be a **fuzzy α-closed set** if $\lambda \geq$ cl (int (cl(λ))). Its complement is said to be a **fuzzy α-open set.**

2. ON FUZZY REGULAR SEMI α-OPEN SETS

This section deals with the interrelation between fuzzy regular semi α-open sets and fuzzy regular pre α-open sets.**DEFINITION 2.2**Let (X, T) be a fuzzy topological space. Any fuzzy set $\lambda \in I^X$ is said to be a **fuzzy regular semi α-open set** if λ=Fsint (Fαcl(λ). The complement of a fuzzy regular semi α-open set is said to be a **fuzzy regular semi α-closed set.**

DEFINITION 2.4 Let (X, T) be a fuzzy topological space. For a fuzzy set λ of X, the **fuzzy regular semi α-closure** of λ and the **fuzzy regular semi-interior** of λ are defined respectively, as follows:

Frsαcl λ=$\wedge$\{ :$\mu \geq \lambda$, μ is fuzzy regular semi α-closed \} and

Frsαintλ=$\vee$\{ :$\mu \leq \lambda$, μ is fuzzy regular semi α-open \}

REMARK 2.3

The notions of fuzzy regular semi α-open sets and fuzzy regular pre α-open sets are of independent.

EXAMPLE 2.1 Every fuzzy regular semi α-open set need not be fuzzy regular pre α-open.

Let X ={ a, b } and λ_1, $\lambda_2 \in I^X$ be defined as λ_1 (a) = 0.2, λ_1 (b) = 0.3; λ_2 (a) = 0.4, λ_2(b) = 0.5.Define the fuzzy topology T on X as T ={ 0, 1, λ_1, λ_2}. Clearly (X, T) is a fuzzy topological space. Any fuzzy set $\lambda \in I^X$ be defined as λ (a) = 0.6, λ (b) = 0.5. Then, **λ is fuzzy regular semi α-open.** But λ is not fuzzy regular pre α-open set. Therefore, **every fuzzy regular semi α-open set need not be fuzzy regular pre α-open.**

EXAMPLE 2.2 Every fuzzy regular pre α-open set need not be fuzzy regular semi α-open.

Let X ={ a, b } and λ_1, $\lambda_2 \in I^X$ be defined as λ_1 (a) = 0.2, λ_1 (b) = 0.3; λ_2 (a) = 0.4, λ_2(b) = 0.5.Define the fuzzy topology T on X as T ={ 0, 1, λ_1, λ_2}. Clearly (X, T) is a fuzzy topological space. Any fuzzy set $\lambda \in I^X$ be defined as λ (a) = 0.4, λ (b) = 0.5. Then, **λ is fuzzy regular pre α-open.** But λ is not fuzzy regular semi α-open set.Therefore, **every fuzzy regular pre α-open set need not be fuzzy regular semi α-open.**

3. ON FUZZY REGULAR SEMI α-CONTINUOUS FUNCTIONS

DEFINITION 3.1 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function f : (X, T) ⟶ (Y, S) is said to be a **fuzzy regular semiα-continuous function** if for each fuzzy open set $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is fuzzy regular semi α-open.

DEFINITION 3.2 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function f : (X, T) ⟶ (Y, S) is said to be a **fuzzy regular pre α-continuous function** if for each fuzzy open set $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is fuzzy regular pre α-open.

REMARK 3.1

The notions of fuzzy regular semi α-continuity and fuzzy regular pre α-continuity are independent.

EXAMPLE 3.1 Every fuzzy regular semi α-continuous function need not be a fuzzy fuzzy regular pre α-continuous function.

Let X = { a, b } and $\lambda_1, \lambda_2 \in I^X$ be defined as λ_1 (a) = 0.2, λ_1 (b) = 0.3; λ_2 (a) = 0.4, λ_2(b) = 0.5.Define the fuzzy topology T on X as T ={ 0, 1, λ_1, λ_2}. Define the fuzzy topology S on Y as S = { 0, 1, λ }, where $\lambda \in I^Y$ be defined as λ (a) = 0.6, λ (b) = 0.5. Clearly (X, T) and (Y, S) are fuzzy topological spaces. Let f : (X, T) ⟶ (Y, S) be an identity function. Now,$f^{-1}(\lambda)$ = (0.6, 0.5) is fuzzy regular semi α-open but not fuzzy regular pre α-open in (X, T). Thus f is a fuzzy regular semi α-continuous function but not fuzzy regular pre α-continuous. Hence **every fuzzy regular semi α-continuous function need not be fuzzy regular pre α-continuous.**

EXAMPLE 3.2 Every fuzzy regular pre α-continuous function need not be fuzzy fuzzy regular semi α-continuous function.

Let X = { a, b } and $\lambda_1, \lambda_2 \in I^X$ be defined as λ_1 (a) = 0.2, λ_1 (b) = 0.3; λ_2 (a) = 0.4, λ_2(b) = 0.5.Define the fuzzy topology T on X as T ={ 0, 1, λ_1, λ_2}. Define the fuzzy topology S on Y as S = { 0, 1, λ }, where $\lambda \in I^Y$ be defined asλ (a) = 0.4, λ (b) = 0.5. Clearly (X, T) and (Y, S) are fuzzy topological spaces. Let$f$: (X, T) ⟶ (Y, S) be an identity function. Now, $f^{-1}(\lambda)$ = (0.4, 0.5) is fuzzy regular pre α-open but not fuzzy regular semi α-open in (X, T). Thus f is a fuzzy regular pre α-continuous function but not fuzzy regular semi α-continuous. Hence **every fuzzy regular pre α-continuous function need not be fuzzy regular semi α-continuous.**

4. FUZZY REGULR SEMI α-CONNECTED SPACES

In this section the properties and the characterizations of fuzzy regular semi α-connected spaces and fuzzy regular super semi α-connected spaces are discussed.

DEFINITION 4.1 Any fuzzy topological space (X, T) is said to be a **fuzzy regular semi α-connected**iff (X, T) has no proper fuzzy sets $\lambda_1$and $\lambda_2$which are fuzzy regular semi α-open such that $\lambda_1 + \lambda_2 = 1$.

PROPOSITION 4.1 For a fuzzy topological space (X, T) the following statements are equivalent:

(a) (X, T) is fuzzy regular semi α-connected.

(b) There exist no fuzzy regular semi α-open sets λ_1, $\lambda_2 \in I^X$ where $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$ such that $\lambda_1 + \lambda_2 = 1$.

(c) There exist no fuzzy regular semi α-closed sets λ_1, $\lambda_2 \in I^X$ where $\lambda_1 \neq 1$ and $\lambda_2 \neq 1$ such that $\lambda_1 + \lambda_2 = 1$.

(d) (X, T) contains no fuzzy set $\lambda \neq 0,1$ such that λ is both fuzzy regular semi α-open and fuzzy regular semi α-closed.

Proof: (a) $\Rightarrow$ (b).

Assume that (a) is true. Then (b) follows from the Definition 4.1.

(b) $\Rightarrow$(c). Assume that (b) is true. Suppose that there exist fuzzy regular semi α-closed sets $\lambda_1 \neq 1$ and $\lambda_2 \neq 1$ such that $\lambda_1 + \lambda_2 = 1$. Then, $1 - \lambda_1 \neq 1 - 1 \neq 0$ is a non-zero fuzzy regular semi α-open set. Similarly, $1 - \lambda_2$is a non zero fuzzy regular semi α-open set. Now, ($1 - \lambda_1$) + ($1 - \lambda_2$)= 2 – ($\lambda_1 + \lambda_2$) = 2 – 1 = 1, which is a contradiction. Hence (c).

(c) $\Rightarrow$(d). Assume that (c) is true. Suppose that (X, T) contains a fuzzy set $\lambda \neq 0,1$which is both fuzzy regular semi α-open and fuzzy regular semi α-closed. Then ($1 - \lambda$) is a proper fuzzy regular semi α-closed set and a fuzzy regular semi α-open set. Also, by assumption λ is fuzzy regular semi α-closed. Thus, ($1 - \lambda$) + λ = 1, which is a contradiction to (c). Hence (d).

(d) $\Rightarrow$(a). Assume that (d) is true. Suppose that (X, T) is not fuzzy regular semi α-connected. Then (X, T) has proper fuzzy sets λ_1 and λ_2, which are fuzzy regular semi α-open such that $\lambda_1 + \lambda_2 = 1$. Now, $\lambda_1 + \lambda_2 = 1$ implies that $\lambda_1 = 1 - \lambda_2$. Thus, λ_1 is both fuzzy regular semiα-closed and $\lambda_1 \neq 1$ as λ_2 is a non-zero fuzzy regular semi α-open set. Clearly, $\lambda_1 \neq 0, 1$ is in (X, T), which is a contradiction. Therefore (a).

DEFINITION 4.2 Let (X, T) be any fuzzy topological space. Then (X, T) is called **fuzzy regular super semi α-connected** if it has no proper fuzzy regular semi α-open set.

PROPOSITION 4.2 If (X, T) is any fuzzy topological space, then (a) $\Rightarrow$ (b) and (b) $\Rightarrow$(c), where

(a) (X, T) is a fuzzy regular super semi α-connected space.

(b) If $0 \neq \lambda$ is a fuzzy regular semi α-open set, then Frsαcl (λ) =1.

(c) If $1 \neq \lambda$ is a fuzzy regular semi α-closed set, then Frsαint (λ)=0.

DEFINITION 4.3 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function f : (X, T) $\rightarrow$ (Y, S) is called fuzzy regular semi α-irresolute if $f^{-1}(\lambda) \in I^X$ is fuzzy regular semi α-open for every fuzzy regular semiα-open set $\lambda \in I^Y$.

PROPOSITION 4.3 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let f : (X, T) $\rightarrow$ (Y, S) be a bijective fuzzy regular semi α-irresolute function. If (X, T) is fuzzy regular semi α-connected then(Y, S) is also fuzzy regular semi α-connected.

Proof: Suppose that (Y, S) is not fuzzy regular semi α-connected. Then there are non-zero fuzzy regular semi α-open sets λ_1, $\lambda_2 \in I^Y$ such that $\lambda_1 + \lambda_2 = 1$. Since f is bijective and fuzzy regular semi α-irresolute, $f^{-1}(\lambda_1)$, $f^{-1}(\lambda_2) \in I^X$ are non-zero fuzzy regular semi α-open sets such that $f^{-1}(\lambda_1) + f^{-1}(\lambda_2) = 1$. This is a contradiction to the fact that (X, T) is a fuzzy regular semi α-connected space. Hence (Y, S) is a fuzzy regular semi α-connected space.

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