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Study of Instability of Streaming Rivlin-Ericksen Fluid in Porous Medium

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Abstract

The present paper deals with the instability of streaming Rivlin-Ericksen fluids with Kelvin-Helmholtz (KH) in the presence of a two dimensional magnetic fields through porous medium. The fluids are assumed to be incompressible with same or different densities. By applying normal mode technique to be linearized perturbation equations of motion and boundary conditions lead to deriving two more general simultaneous equations of damping terms with magnetic fields coefficients. It results that both viscosity and porosity suppressed the stability while streaming motion had a destabilized due to different densities in the presence of horizontal magnetic field. The stability criteria are examined theoretically and numerically are obtained. Dual roles are observed for the fluid velocity and the porosity in the stability criteria.

Key words: Kelvin-Helmholtz (KH), magnetic field, Instability, porous medium, different densities fluids.

1. INTRODUCTION

The behaviour of surface waves propagating between two Rivlin–Ericksen elastico-viscous fluids with different densities is examined. The investigation is made in the presence of a vertical electric field and a relative horizontal magnetic fields. The influence of both surface tension and gravity force is taken into account. Due to the inclusion of streaming flow a mathematical simplification is considered. The visco-elastic contribution is demonstrated in the boundary conditions. From this point of view the approximation equations of motion are solved in the absence of visco-elastic effects. The solutions of the linearized equations of motion under nonlinear boundary conditions lead to derivation of a nonlinear equation governing the interfacial displacement and having damping terms with complex coefficients.

This equation is accomplished by utilizing the cubic nonlinearity. The use of the Gardner–Morikawa transformation yields a simplified linear dispersion relation so that the periodic solution for the linear form is utilized. The perturbation analysis, in the light of the multiple scales in both space and time, leads to imposing the well-known nonlinear Schrödinger equation having complex coefficients. The stability criteria are discussed theoretically and illustrated graphically in which stability diagrams are obtained. Regions of stability and instability are identified for the electric fields versus the wave number for the wave strain of the disturbance. Numerical calculations showed that the ratio of the dielectric constant plays a dual role in the stability criteria. The visco-elasticity coefficient plays two different roles. A stabilizing influence is observed through the linear scope and a destabilizing role in the nonlinear stability. The stability of stratified Rivlin-Ericksen fluid in porous medium in the presence of suspended particles and magnetic field has been investigated. Upon application of normal mode technique, the dispersion relation was obtained. Also, it was found that the system is stable for $\beta < 0$ and unstable for $\beta > 0$ under certain conditions. The problem of instability of the Rivlin–Ericksen elasto-viscous fluid in a porous medium is considered in the presence of uniform rotation, suspended particles and variable gravity field. The rotation, gravity field, suspended particles and visco-elasticity introduce oscillatory modes. It is found that the principle of the exchange of stabilities is valid, provided that some condition is fulfilled. In a stationary convection, suspended particles are found to have destabilizing effect on the system due to different densities, while rotation has stabilizing effect on the system under certain conditions. The effect of rotation, suspended particles, and medium permeability have also been shown graphically. The theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under conditions of varying hydrodynamic and hydromagnetic stability have been treated in detail by chandrasekhar [l]. lapwood [2] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by wooding [3]. sharma and kumar [4] have studied the thermal instability of an Oldroydian viscoelastic fluid in porous medium and also have considered the effect of uniform rotation on the instability. With a growing importance of non-Newtonian fluids in modern technology and industries, the investigations into such fluids are desirable. There are many elasto-viscous fluids that can be characterised neither by Maxwell's constitutive relations nor by Oldroyd's constitutive relations. One such class of viscoelastic fluids is the Rivlin– Ericksen's fluid. Rivlin and Ericksen [5] have proposed a theoretical model for such visco-elastic fluid. This and the other class of polymers are used for manufacturing parts of space-crafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastics, engineering equipments, etc. Recently, polymers are also used in agriculture, communication appliances and in biomedical applications. When fluid permeated a porous material, the gross effect is represented by Darcy's law. As a result of this macroscopic law, the usual viscous terms in the equations of Rivlin–Ericksen's elasto-viscous fluid motion is replaced with $-\frac{1}{n_1}\left(\mu+\mu'\frac{\partial \vec{q}}{\partial t}\right)$ where μ and μ' are the viscosity and visco-elasticity in the instability of the Rivlin–Ericksen fluid, n_1 is the medium permeability and $q\|$ is the Darcian (filter) velocity of the fluid. The Kelvin-Helmholtz instability arising at the interface separating two superposed, viscous, electrically conducting fluids through a porous medium in the presence of a uniform two dimensional horizontal magnetic field. The stability motion was also assumed to be uniform two dimensional horizontal. By applying the normal mode technique to the linearized perturbation equations, the dispersion relation was derived. The stability analysis was carried out for fluids of high kinematic viscosities. It was found that both viscosity and porosity suppressed the stability, while streaming motion had a destabilizing influence due to different densities. The Kelvin-Helmholtz discontinuity arising at the plane interface between two superposed streaming fluids is of prime importance in various astrophysical, geophysical, and laboratory situations. The Kelvin-Helmholtz instability arises in such situations as when air is blown over mercury, highly ionized hot plasma is surrounded by a slightly cold gas, or a meteor enters the earth's atmosphere.

A comprehensive account of investigations of these problems in hydrodynamics and hydromagnetics was given by Chandrasekhar[6] in his monograph. These problems of instabilities in hydrodynamic and hydromagnetic configurations continue to attract the attention of researchers due to their importance in actual physical situations. The problems of nonstreaming, superposed instability (Rayleigh-Taylor instability) and Kelvin-Helmholtz instability have been investigated by several researchers from different points of view. Jukes[7] investigated the problem by incorporating finite electrical conductivity effects and concluded that this inclusion introduced new and unexpected solutions. Gerwin [8] studied the stability problem of nonconducting, streaming gas flowing over an incompressible conducting liquid. The influence of viscosity on the stability of the plane interface separating two incompressible, superposed fluids of uniform densities was studied by Bhatia [9], who concluded that viscosity has a stabilizing influence. A comprehensive account of various hydrodynamic stability problems was also given by Joseph [10] and by Drazin and Reid [11]. Several researchers have examined the Kelvin- Helmholtz instability in superposed fluids in hydrodynamic, hydromagnetic, and plasma regimes from different points of views.

Sengar [12] analyzed the stability of two superposed gravitating streams in a uniform, vertical magnetic field in the presence of effects of magnetic resistivity. He found that magnetic resistivity had a destabilizing effect on the system. Mehta and Bhatia [13] studied the Kelvin-Helmholtz instability of two viscous, superposed plasmas in the presence of finite Larmor radius (FLR) effects and showed that both viscosity and FLR effects suppressed the instability. D´avalos-Orozco and Aguilar-Rosas [14][15] and D´avalos-Orozco [16] examined the effects of a general rotation and a horizontal magnetic field on the stability of superposed inviscid fluids. El-Sayeed[17]

examined the effect of viscosity and finite ion Larmor radius on the hydrodynamic transverse instability problem.
An excellent reappraisal of the Kelvin-Helmholtz problem was given by Benjamin and Bridges [18], who gave the Hamiltonian formulation of the classic Kelvin-Helmholtz problem in hydrodynamics. Allah [19] investigated the effects of magnetic field, heat, and mass transfer on the Kelvin-Helmholtz instability of superposed fluids. El-Ansary et al. [20] studied the effects of rotation on the hydrodynamic stability of three layers. Meignin et al. [21] and Watson et al. [22] studied the Kelvin-Helmholtz instability in a Hele-Shaw cell and a weakly ionized medium, respectively. In recent years, investigations of the flow of fluids through porous media have become an important topic due to the recovery of crude oil from the pores of reservoir rocks. McDonnell [23] pointed out the physical properties of comets; meteorites and interplanetary dust strongly suggest the significance of the effects of porosity in the astrophysical context.
Several researchers Sharma and Kumar [4], and Khan and Bhatia [24] have studied the effects of the permeability of a porous medium on different problems in hydrodynamic and hydromagnetic stability in view of the importance of such studies in rocks and heavy oil recovery.
In the absence of a magnetic field, Allah [25] investigated the stability of superposed Newtonian fluids through a porous medium in the presence of the effects of surface tension, while Kumar and Lal [26] recently studied the stability in two superposed Rivlin-Ericksen visco-elastic fluids through a porous medium. Kumar et al. [26] studied the instability of a rotating, superposed Walters B' viscoelastic fluid through a porous medium. More recently, Kumar et al. [27] investigated the effect of viscosity on stratified, superposed, non-Newtonian fluids. In all of the above mentioned studies of flow and stability in Newtonian and non-Newtonian fluids through a porous medium, the effects of streaming motions were not included. It would be of interest to examine the instability in superposed, streaming, viscous, electrically conducting fluids through a porous medium in the presence of a magnetic field. One can study the problem of the stability of the horizontal layer of stratified fluids and the stability of 2 superposed fluids, whether streaming or not, in the presence of a horizontal or a vertical magnetic field. Khan and Bhatia (2001) studied the stability of 2 nonstreaming, superposed, viscoelastic fluids in a horizontal magnetic field. In this chapter, we have examined the stability of a planar interface separating 2 streaming, electrically conducting, viscous fluids in a horizontal magnetic field through a porous medium. For a uniform vertical magnetic field, Bhatia and Sharma [28] studied the problem of Kelvin-Helmholtz instability in superposed viscous fluids through a porous medium. Khan and patni [29] examined the instability of the fluids of same densities through porous medium.

2. MATHEMATICAL FORMULATION

We considered the motion of an incompressible, viscous, infinitely electrically conducting fluid of uniform viscosity, moving with a uniform horizontal velocity $\vec{u}$ has the horizontal component u_x and vertical component u_y through a porous medium in the presence of uniform two dimensional magnetic field $\vec{H} = (H_x, H_y, 0)$ .The relevant linearized perturbation equations are:

$$\frac{\rho}{\varepsilon}\frac{\partial \vec{u}}{\partial t} + \frac{\rho}{\varepsilon}(\vec{u}.\nabla)\vec{u} = -\nabla \delta p + \vec{g}\delta\rho + (\nabla \times \vec{h}) \times \vec{H} + \frac{\mu}{\varepsilon}\nabla^2 \vec{u} - \frac{\mu}{Q}\vec{u} \qquad (1)$$

$$\varepsilon \frac{\partial}{\partial t}(\delta\rho) + (\vec{u}.\nabla)\delta\rho + (\vec{u}.\nabla)\rho = 0 \qquad (2)$$

$$\varepsilon \frac{\partial \vec{h}}{\partial t} + (\vec{u}.\nabla)\vec{h} = (\vec{H}.\nabla)\vec{u} \qquad (3)$$

$$\nabla.\vec{h} = 0 \qquad (4)$$

$$\nabla.\vec{u} = 0 \qquad (5)$$

Where $\vec{h}(h_x, h_y, h_z), \delta\rho$ and δp are the perturbations respectively in magnetic field $\vec{H}$, density ρ and pressure p resulting from the disturbance, the Darcian velocity $\vec{u}(u, v, w)$ to the system and μ is the coefficient of viscosity, $\vec{g}$ is the acceleration due to gravity, Q is the permeability of porous medium and ε is the medium of porosity. Wesupposed that the solution of these quantites can be expressed as $e^{ik_x x + ik_y y + nt}$, (6)

where k_x and k_y are the horizontal wave numbers and n is the rate at which the system departs away from equilibrium. Eliminating some of the variables, we get the following equation in *w*:

$$\frac{n}{\varepsilon}[\rho k^2 w - D(\rho D w)] - \frac{gk^2}{n\varepsilon}(D\rho)w - \frac{(\vec{k}.\vec{H})^2}{n\varepsilon}[D^2 - k^2]w + \frac{\mu}{\varepsilon}[D^2 - k^2]^2 w - \frac{\mu}{Q}[D^2 - k^2]w = 0 \qquad (7)$$

Eq. (7) holds for all ρ (*z*), Consider the case in which two superposed fluids, occupying the regions $z > 0$ and $z < 0$, are separated by a horizontal boundary at $z = 0$. In the two regions of constant ρ, equation (6) becomes:

$$w_1 = A_1 n_1 e^{kz} + B_1 n_1 e^{M_1 z}, z < 0 \qquad \text{(a)}$$

$$w_2 = A_2 n_2 e^{-kz} + B_2 n_2 e^{-M_2 z}, z > 0 \qquad \text{(b)}$$

where A_1, B_1, A_2, B_2 are constants of integration and M_1, M_2 are respectively the square roots of M for the two regions. The expressions determining M_1 and M_2 are

$$M_1 = \sqrt{k^2 + \frac{n_1^2}{v_1}\left(1 + \frac{(\vec{k}.\vec{H})^2}{n_1^2 \rho} + \frac{v_1 \varepsilon}{Q n_1}\right)}$$

$$M_2 = \sqrt{k^2 + \frac{n_2^2}{v_2}\left(1 + \frac{(\vec{k}.\vec{H})^2}{n_2^2 \rho} + \frac{v_2 \varepsilon}{Q n_2}\right)}$$

It was assumed that M_1, M_2 were so defined that their real parts were positive.

The solutions for Eqs. must satisfy certain boundary conditions. These conditions require that at the interface $z = 0$,

w, Dw and $\mu(D^2 + k^2)w$ must be continuous. These conditions ensure the requirement of the continuity of the normal component of velocity, its derivative, and the tangential stress at the interface. By integrating Eq. (7) across the interface, $z = 0$, we obtain the relations.

On eliminating the constants A1, B1, A2, and B2 and evaluating the determinant of the given matrix of the coefficients, we obtained the following characteristic equation.

3. NUMERICAL CALCULATION

The dispersion relation given by Equation is quite complicated, particularly as the coefficient *Ei* values involve several parameters. It is thus not feasible to analyze the dispersion relation analytically. We therefore solved it numerically, for different values of the parameters, for an unstable arrangement of superposed fluids, i.e. a top-heavy configuration and the same. We were interested in the qualitative behaviour of the various parameters on the instability of the configuration. The dispersion relation was first non-dimensionalized by measuring *n* and the parameters in terms of $\sqrt{g}$ .For an unstable system, we must have

$\alpha_1 < \alpha_2 \ (\alpha_1 + \alpha_2 = 1)$.

The numerical calculations presented above, although carried out for a few representative values of the physical parameters involved, reveal the tendencies of the nature of the physical effects on the instability of the superposed porous streaming fluids.

4. CONCLUSION

we can say that both viscosity and porosity suppress the instability of streaming superposed fluids of different densities . The streaming motion tends to further destabilize the unstable arrangement of the superposed fluids, if upper layer fluid has more density than lower layer fluid.

5. REFERENCES

[1]Chandrasekhar S., *Hydrodynamic and Hydromagnetic Stability*, Dover Publications, New York, 1981.

[2] Lapwood E.R., *Convection of a fluid in a porous medium*, Proc. Camb. Phil. Soc., 1948, 44, 508.

[3] Wooding R.A., *Rayleigh instability of a thermal boundary layer in flow through a porous medium*, J. Fluid Mech., 1960, 9, 183.

[4] Sharma R.C., Kumar P., *Thermal instability of an Oldroydian viscoelastic fluid in porous medium*, Engng. Trans, Poland, 1996, 14, 99.

[5] Rivlin R.S., Ericksen J.L., *Stress–deformation relations for isotropic material*, J. Rational Mech. Anal., 1955, 4, 323.

[6]Chandrasekhar S., "Hydrodynamic and Hydromagnetic Stability", Clarendon Press, Oxford, 1961.

[7]Jukes, J.D., "Micro-Instabilities in Magnetically Confined, Inhomogeneous Plasma", Phy. Fluids, 7, 1468-1474, 1964.

[8] Gerwin, R.A., "Hydromagnetic Surface Wave in a Conducting Liquid Surrounded by a Flowing Gas", Phys. Fluids, 11, 1699-1708, 1968.

[9] Bhatia, P.K., "Rayleigh-Taylor Instability of Two Viscous Superposed Conducting Fluids", Nuovo Cimento, 19, 161-168, 1974.

[10] Joseph, D.D., "Stability of Fluid Motion II", Springer-Verlag, New York, 1976.

[11] Drazin, P.G., and Reid, W.H., "Hydrodynamic Stability", University Press, Cambridge, UK, 1981.

[12] Sengar, R.S., "Stability of Two Superposed Gravitating Streams in a Uniform, Vertical Magnetic Field", Proc. Natl. Acad. Sci. India, 54, 438-443, 1984.

[13] Mehta, V. and Bhatia, P.K., "Kelvin-Helmholtz Instability of Two Viscous Superposed Rotating and Conducting Fluids", Astrophysics and Space Sci., 141, 151-158, 1988.

[14] D´avalos-Orozco, L.A. and Aguilar-Rosas, J.E., "Rayleigh-Taylor Instability of a Continuously Stratified Fluid Under a General Rotation Field", Physics of Fluids A: Fluid Dynamics, 1, 1192-1199, 1989a.

[15] D´avalos-Orozco, L.A. and Aguilar-Rosas, J.E., "Rayleigh-Taylor Instability of a Continuously Stratified Magnetofluid Under a General Rotation Field", Physics of Fluids A: Fluid Dynamics, 1, 1600-1602, 1989b.

[16] D´avalos-Orozco L.A., "Rayleigh-Taylor Instability of a Two-Fluid Layer Under a General Rotation Field and a Horizontal Magnetic Field", Astrophysics and Space Science, 243, 291-313, 1996.

[17] El-Sayeed, M.F., "Hydromagnetic Transverse Instability of Two Highly Viscous Fluid-Particle Flows with Finite Ion Larmor Radius Corrections", Eur. Phys. J. D, 23, 391-403, 2003.

[18] Benjamin, T.B. and Bridges, T.J., "Reappraisal of the Kelvin-Helmholtz Problem Part 1, Hamiltonian Structure", J. Fluid Mech., 333, 301-325, 1997.

[19] Allah, M.H.O., "Rayleigh-Taylor Stability in the Presence of Rotation", Astrophysics and Space Science, 175, 149-155, 1991.

[20] El-Ansary, N.F., Hoshoudy, G.A., Abd-Elrad, A.S. and Ayyad, A.H.A., "Effects of Surface Tension and Rotation on the Rayleigh-Taylor Instability", Phys. Chem. Chem. Phys., 4, 1464-1470, 2002.

[21] Meignin, L., Ern, P., Gondret, P. and Rabaud, M., "Gap Size Effect for the Kelvin-Helmholtz Instability in a Hele-Shaw Cell", Phys. Rev. E, 64, 026308-026314, 2001.

[22] Watson, C., Zweibel, E.G., Heitsch, F. and Churchwell, E., "Kelvin-Helmholtz Instability in aWeakly Ionized Medium", Astrophys. J., 608, 274-281, 2004.

[23] McDonnell, J.A.M., “Cosmic Dust”, John Wiley and Sons, Toronto, 330, 1978.
[24] Khan, A. and Bhatia, P.K., “Stability of a Finitely Conducting Compressible Fluid through Porous Medium”, Ganita Sandesh, 17, 35-42, 2003.
[25] Allah, M.H.O., “Rayleigh-Taylor Instability with Surface Tension, Porous Media, Rigid Planes and Exponential Densities”, Indian J. Pure and Applied Maths, 33, 1391-1404, 2002.
[26] Kumar, P., Lal, R. and Singh, G.J., “MHD Instability of Rotating Superposed Walters B’ Viscoelastic Fluids through a Porous Medium”, J. Porous Media, 9, 463-468, 2006.
[27] Kumar, P., Lal, R. and Singh, M., “Hydrodynamic and Hydromagnetic Stability of Two Stratified Rivlin-Ericksen Elastico-Viscous Superposed Fluids”, Int. J. Appl. Mech. and Eng., 12, 645-853, 2007.
[28] Bhatia, P.K. and Sharma, A., “Kelvin-Helmholtz Instability of Two Viscous Superposed Conducting Fluids”, Proc. Natl. Acad. Sci., 73, 497-520, 2003.
[29] Khan, A. and Patni.P, “Instability of Superposed fluids Porous Medium”, Turkish J. Eng.Env.Sci. 34, 59-68, 2010.