International Journal of Mathematical Sciences and Applications
Vol. 1 No. 2 (May, 2011)

www.ijmsa.yolasite.com

PREDICTED SERVICE LIFE OF CHLORIDE TRANSPORT EQUATION USING FINITE DIFFERENCE SCHEME

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ABSTRACT

The concrete structures such as wharves, docks, breakwaters, dolphins ect, are exposed to salinity of seawater. The complex marine environment is a jumble of perpetual tidal, intermittent cycling of wetting and drying spells by high and low tide ebbs, wave splashing and pounding of chloride impregnated atmosphere. Ingress of moisture, carrying chloride ions, is central to all these phases of marine environment, though with varying degrees of penetration by the two transport processes are diffusion and convection. This paper illustrates the use of a general modeled of prediction of service life of reinforced concrete structure subjected to chloride induced corrosion. This model has been performed with different combinations of chloride concentrations, temperature variation and wetting/drying spell cycle durations. Used the finite different method for the solution of boundary and initial value problems for a one dimensional transport equation and calculated the prediction of service life of reinforced concrete in three different zones. This model by describing its capabilities the theoretical background upon which it is based, and the limitation on it's used. The refinement of a surface chloride build-up suggested a more realistic prediction of chloride profile and corrosion risk, has been presented in three different zones.

KEYWORDS

Diffusion, Convection, Permeability, Transport equation, Chloride, Service life

INTRODUCTION

The development of numerical techniques such as finite difference method has enabled engineers to solve extremely complex physical phenomena for a variety of boundary conditions and material properties. Transport of chloride ions in concrete is commonly modeled as through diffusion process only [1]. But, in reality all transport processes analogous to that of momentum and heat transfer play their role. Convective transport of chloride ions in concrete is the major pick up the pace source of rapid corrosion or reinforcing steel. The durability of concrete is put to assessment when it is exposed to the harsh marine environment saturated with chloride, a source of chloride ions penetration in concrete structures.

The paper has been organized as follows. In began described the transport equation and their coefficient terms followed by assumptions. Then next section described in finite difference scheme to apply in transport equation and hence calculated the predicted service life of reinforced concrete has been presented input parameter values in table 1. Since predicted service life of reinforced concrete has been use temperature in two ways (i) mean temperature and (ii) varying temperature. Details of results and discussion of results are presented in the following section. The paper concludes with a section on conclusion. The present study can be realistically used to predict the penetration of chloride ions into concrete structures under sea environments.

DESCRIPTION OF THE CHLORIDE TRANSPORT MODEL

The transportation of chloride ions into concrete is a complicated process which involves diffusion, capillary suction, permeation and convective flow through the pore system and micro cracking network, accompanied by physical adsorption and chemical binding. However the dominate mechanism, if chloride ingress through concrete are diffusion and permeation. Here, in the present study the two mechanisms are considered for modeling the chloride ingress through concrete. The chloride transport equation is given by [8]:

$$\frac{\partial C}{\partial t} = D.\frac{\partial^2 C}{\partial x^2} - V.\frac{\partial C}{\partial x} \tag{1}$$

Where

C – “free” chloride in solution at depth x after time t,

D – diffusion co-efficient

V – average linear velocity = $\frac{Q}{nA} = -\frac{k}{n}\frac{dh}{dx}$

Q – flow rate

A – cross – sectional area

k – permeability co-efficient (hydraulic conductivity)

h – hydraulic head

n – porosity

The left-hand side of Eq (1) indicates the rate of change in chloride concentration with time. Two different mechanisms are represented by the two terms on the right-hand side of the equation. The first term, the diffusion term, comes from Fick’s second law for one –dimensional, non-stationary flow in a semi – infinite medium. The second term describes the change in chloride concentration due to permeation.

DIFFUSION

Chloride transport by diffusion in saturated concrete occurs in the presence of a chloride concentration gradient created when at least one face is continuously exposed to chloride. The value of D depends on the age of concrete (t) as well as the exposure temperature (T) and can be modeled as [2]:

Beatriz Martin – Perez was described in diffusion term as

$$D(t,T) = D_{ref}.\left(\frac{t_{ref}}{t}\right)^{m}.\exp\left[\frac{U}{R}.\left(\frac{1}{T_{ref}} - \frac{1}{T}\right)\right]$$

where

D(t,T) – diffusion co-efficient at time t and temperature T
D_{ref} – diffusion co-efficient at some reference time t_{ref} and temperature T_{ref}
m – constant (depending on mix proportions)
U – activation energy of the diffusion process
R – gas constant and
T – exposure température (^{0}K)

The value of constant “m” is mainly dependent on the type of cement. The value of m can be taken as 0.2 for ordinary Portland cement concrete.

PERMEATION

Permeation is the rate of fluid ingress driven by a pressure gradient. It is generally known that the permeability co-efficient varies with time, as a result of continued hydration of the cement paste, and with temperature, due to its influence on the viscosity and density of the penetrating fluid. In the present study, the time dependent permeability co-efficient is modeled as,

Kropp.J and H.K. Hilsdorf, was described in permeation term as

$$k\,(t, T) = \frac{k_{ref}}{Z}.\left(\frac{t_{ref}}{t}\right)^{n}$$

where

k – permeability co-efficient at time t and temperature T
k_{ref} – permeability co-efficient at some reference time t_{ref}
Z – viscosity temperature correction factor
n – porosity

The chloride transport model is capable of assimilation the following considerations into its calculations,

- initial diffusion value(D) for concrete,
- time (t) – dependent reduction of D,
- temperature (T) dependent variation of D,

- varying surface chloride concentration with time and
- Variation in temperature with time.
- Time (t) and temperature (T) are dependent variation of permeation.

The following assumptions are made in the present study,

- the analysis is one – dimensional,
- the concrete is homogeneous and isotropic (no cracking),
- the concrete is fully saturated in the modeling region
- The liquid carrying the chlorides is incompressible,
- Diffusion co-efficient is constant with depth,
- The concrete is under isothermal conditions, and
- There is an infinite supply of chloride to ingress into the concrete.

ESTIMATION OF SERVICE LIFE:

In the present study, the initiation of chloride induced corrosion of reinforcement in concrete is considered as the end of service life. Hence, the time to corrosion which defines the service life of the structural behavior, can be estimated as the time at which the chloride concentration at the level of reinforcement becomes equal to the critical chloride concentration (C_{cr}). The following seven are the input parameters,

- m, constant dependent on mix proportions, which controls the rate of reduction of diffusivity,
- T, absolute temperature of exposure for structure,
- C_{cr}, threshold chloride concentration or critical chloride level required to initiate corrosion of steel,
- D_{ref}, diffusion co-efficient at some reference time t_{ref} and reference temperature T_{ref},
- k_{ref}, permeability co-efficient at some reference time t_{ref},
- C_0, surface chloride concentration, and
- Thickness of cover.

IMPLIEMENTATION

Consider the ordinary differential equation (ODE), $u'(t) = f(u),\ u(t_0) = u_0$ (4)

The derivative can be approximated by

$$u'(t) \approx \frac{u(t+h) - u(t)}{h} \qquad (h\ small). \tag{5}$$

This leads to the Euler method [4]:

$$u_{n+1} = u_n + hf(u_n)$$

The solution of the ODE at time t = nh is approximated by u_n.

Chloride transport equation can be written as in finite different method form [6] and [5]

$$\frac{\partial}{\partial x}C(x,t) \approx \frac{1}{\Delta x}(C(x+\Delta x,t) - C(x,t))$$

$$\frac{\partial^2}{\partial x^2}C(x,t) \approx \frac{1}{\Delta x^2}(C(x-\Delta x,t)-2C(x,t) + C(x+\Delta x,t))$$

$$\frac{\partial}{\partial t}C(x,t) \approx \frac{1}{\Delta t}(C(x,t+\Delta t) - C(x,t))$$

Substitution in the transport equation Eq. 1

$$\frac{1}{\Delta t}(C(x,t+\Delta t) - C(x,t)) = D\left\lfloor \frac{1}{\Delta x^2}(C(x-\Delta x,t)-2C(x,t) + C(x+\Delta x,t))\right\rfloor - V\left[\frac{1}{\Delta x}(C(x+\Delta x,t) - C(x,t))\right]$$

$$C(x,t+\Delta t) = C(x,t) + \frac{D.\Delta t}{\Delta x^2}\lfloor (C(x-\Delta x,t)-2C(x,t) + C(x+\Delta x,t))\rfloor - \frac{V.\Delta t}{\Delta x}\left[(C(x+\Delta x,t) - C(x,t))\right]$$

The above equation can be solved by using the proper initial and boundary conditions

Initial condition: C (x, 0) = $\begin{Bmatrix} 0 & if & x > 0 \\ C_{cr}(0) & if & x = 0 \end{Bmatrix}$

Boundary condition: C (0, t) = Cs (t)
C (∞ , t) = 0

Then by knowing the initial and boundary condition, and having through of D and V on can determines in the chloride concentration at any depth x at any time t using Eq 6. The finite difference method explained above is implemented in MATLAB, which will be useful in determines the time to corrosion initiation reinforcement concrete structure member affected by chloride induced corrosion. An example is provided in the next section to illustrate the proposal method.

EXAMPLE

A reinforcement concrete bridge deck, located in a marine environment is considered in the present study. The service life of the bridge deck, which is made up of two different types of concrete namely typical bridge deck concrete (Case I) and High performance concrete (Case II) are estimated considered bridge deck to be exposed to different concentration condition. The effect of temperature variation is also included in the analysis by considering the daily variation in temperature for the different exposure condition. The input parameters considered are given in Table 1,

TABLE .1 INPUT PRAMETER

<table>
<tr><th>Parameters</th><th>Values</th></tr>
<tr><td>m</td><td>0.2</td></tr>
<tr><td>T</td><td>
<table>
<tr><th>Zones</th><th>Constant temperature</th><th>Varying temperature</th></tr>
<tr><td>Atmospheric zone</td><td>30^0C</td><td>30^0C + 5 sin(π/12 (t – 8))</td></tr>
<tr><td>Splash zone</td><td>25^0C</td><td>25^0C + 5sin(π/12 (t – 8))</td></tr>
<tr><td>Tidal zone</td><td>15^0C</td><td>15^0C + 5sin(π/12 (t – 8))</td></tr>
</table>
Where t is the time in hours.</td></tr>
<tr><td>C_{cr}</td><td>0.05% by weight of concrete [1]</td></tr>
<tr><td>D_{ref}</td><td>2.53 x 10^{-12} m^2/s (23^0C, 120 days) for Case (I) [1]
0.33 x 10^{-12} m^2/s (23^0C, 180 days) for Case (II) [1]</td></tr>
<tr><td>k_{ref}</td><td>1.00x 10^{-13}m/s, [1]</td></tr>
<tr><td>Thickness of cover</td><td>60 mm</td></tr>
<tr><td>C_0</td><td>It has been observed from field data that the surface chloride content tends to increase with time. C_{0_1} is assumed to be a linear function of the square root of the concrete age [2],

$$C_{0_1} = S\sqrt{t} \qquad \text{if} \quad t > 0 \text{ years} \qquad [\% \text{ of concrete weight}]$$

And $$C_{0_2} = \begin{Bmatrix} S\sqrt{t} & \text{if} \quad t < 10 \ \text{years} \\ 17758\,S & \text{if} \quad t \geq 10 \ \text{years} \end{Bmatrix}$$

Where S is surface chloride co-efficient ($s^{-1/2}$) dependent on the type of structure and zone of exposure and t is the exposure time (s).
<table>
<tr><th>Atmospheric zone</th><th>Splash zone</th><th>Tidal zone</th></tr>
<tr><td>S = $(5.57 \text{ x } 10^{-6}\ (s^{-1/2}))$</td><td>S= $(16.6 \text{ x } 10^{-6}(s^{-1/2}))$</td><td>S = $(23.5 \text{ x } 10^{-6}\ (s^{-1/2}))$</td></tr>
</table></td></tr>
</table>

The value of U, the activation energy of the diffusion process, from the diffusion co-efficient equation was taken to be 35000j/mol. The viscosity temperature correction factor, Z included in the permeability co-efficient equation was set to 1.0. A porosity n of 8% was used. A hydraulic gradient h of 25m/m was used when investigating the effects of pressure- driven flow. The value of the gas constant (R) is 8.314472 J K^{-1} mol^{-1}.

RESULTS AND DISCUSSION

EFFECT OF CONSTANT TEMPERATURE

The values of service life considering typical bridge deck concrete for the three exposure conditions assuming constant atmospheric temperature throughout the service life are given in Table 2. The value of constant temperature are taken as the mean values given in Table 1 for the respective exposure condition. The effect of permeability is neglected in this computation. From the results obtained, it is noted that the service life is highest for the atmospheric zone and the lowest for the splash zone. Splash zone gives a lower value of predicted service life than the tidal zone in spite of having a lower value of C_0 in splash zone. This is attributed to the higher value of mean temperature in the splash zone due to which the value of D in the splash zone is higher than in the tidal zone.

Table 2: Services lives for different exposure conditions in years (typical bridge deck concrete, constant temperature, permeability is not considered)

ZONE	SURFACE CHLORIDE CONCENTRATION	
	C_{0_1}	C_{0_2}
ATMOSPHERIC ZONE	44.31	97.97
SPLASH ZONE	23.86	25.47
TIDAL ZONE	30.03	32.43

The values of service life considering high performance concrete for the three exposure condition assuming constant atmospheric temperature throughout the service life are given in Table 3. The value of constant temperature are taken as the mean values given in Table 1 for the respective exposure condition. The effect of permeability is neglected in this computation. It is noted that as expected, the use of high performance concrete increase, the values of service life in all the three zones. This is because of the lower value of D for the high performance concrete. As in the typical bridge deck concrete, it is noted that the service life is highest in the atmospheric zone and is the lowest in the splash zone. It is also noted that the change in C_0 has a significant effect on the service life.

Table 3: Services lives for different exposure conditions in years (High performance concrete, constant temperature, permeability is not considered)

ZONE	SURFACE CHLORIDE CONCENTRATION	
	C_{0_1}	C_{0_2}
ATMOSPHERIC ZONE	232.48	1047.60
SPLASH ZONE	153.74	233.47
TIDAL ZONE	191.65	312.68

The values of service life considering typical bridge deck concrete with permeability for the three exposure condition constant atmospheric temperature throughout the service life are given in Table 4. The value of constant temperature are taken as the mean values given in Table 1 for the respective exposure condition. From the results obtained, it is noted that the service life is highest for the atmospheric zone and the lowest for the splash zone. The values of service life of typical bridge deck concrete with permeability are to be low with comparing typical bridge deck concrete without permeability.

Table 4: Services lives for different exposure conditions in years (typical bridge deck concrete, constant temperature, permeability)

ZONE	SURFACE CHLORIDE CONCENTRATION	
	C_{0_1}	C_{0_2}
ATMOSPHERIC ZONE	40.36	95.32
SPLASH ZONE	21.79	22.82
TIDAL ZONE	28.07	29.79

EFFECT OF VARYING TEMPERATURE

The values of service life considering typical bridge deck concrete for the three exposure condition assuming varying atmospheric temperature throughout the service life as given in Table 5. The value of varying temperature is taken as given in Table 1 for the respective exposure condition. The effect of permeability is neglected in this computation. From the results obtained, it is noted that the service life is highest for the atmospheric zone and the lowest for the splash zone. Comparing the table 2 with table 5, it is noted that the service life of splash zone increases 3% in constant temperature. Hence atmospheric zone, tidal zone are negligible.

Table 5: Services lives for different exposure conditions in years (typical bridge deck concrete, varying temperature, permeability is not considered)

ZONE	SURFACE CHLORIDE CONCENTRATION	
	C_{0_1}	C_{0_2}
ATMOSPHERIC ZONE	43.03	96.12
SPLASH ZONE	22.87	24.80
TIDAL ZONE	29.49	31.87

The values of service life considering high performance concrete for the three exposure condition considered assuming varying atmospheric temperature throughout the service life are given in Table 6. The value of varying temperature is taken as given in Table 1 for the respective exposure condition. The effect of permeability is neglected in this computation. It is noted that an expected, the use of high performance concrete increase the value of service life in all the three zones. This is because of the lower value of D for the high performance concrete. As in the typical bridge deck concrete, it is noted that the service life is highest in the atmospheric zone and is the lowest in the splash zone. It is also noted that the change in C_0 has a significant effect on the service life. By comparing table 3 and table 6, it can be noted that there is a small difference in the service life of atmospheric zone, splash and tidal zone and are negligible.

Table 6: Services lives for different exposure conditions in years (High performance concrete, varying temperature, permeability is not considered)

	SURFACE CHLORIDE CONCENTRATION

ZONE	C_{0_1}	C_{0_2}
ATMOSPHERIC ZONE	230.49	1043.78
SPLASH ZONE	152.32	231.55
TIDAL ZONE	188.15	309.84

The values of service life considering typical bridge deck concrete with permeability for the three exposure condition assuming varying atmospheric temperature throughout the service life are given in Table 7. The value of varying temperature are taken as the mean values given in Table 1 for the respective exposure condition. From the results obtained, it is noted that the service life is highest for the atmospheric zone and the lowest for the splash zone. Splash zone gives a lower value service life then the tidal zone in spite of having a lower value of C_s in splash zone. The values service lives of typical bridge deck concrete with permeability are to be low with comparing typical bridge deck concrete without permeability. Comparing table 4 and table 7, it can be noted that the service life of splash zone, atmospheric zone, and tidal zone are increasing 4%, 3%, 2% respectively where constant temperature in reinforced concrete.

Table 7: Services lives for different exposure conditions in years (typical bridge deck concrete, permeability, and varying temperature)

ZONE	SURFACE CHLORIDE CONCENTRATION	
	C_{0_1}	C_{0_2}
ATMOSPHERIC ZONE	39.02	93.02
SPLASH ZONE	19.92	21.09
TIDAL ZONE	27.98	28.44

The service life for the typical bridge deck considers effects of temperature are shown in Fig: 1, comparing the effect of Mean and varying temperature as above, it can be concluding that:

1. Service life for the reinforced concrete is more at mean temperature than for varying temperature, for both permeable concrete.
2. If permeability of the concrete is considered, the predicted service life decreases.
3. Concrete in the splash zones are affected by chloride ingress more than that in tidal zones.
4. For mean temperature the predicted service life effect remains high whereas it is low at varying temperature.

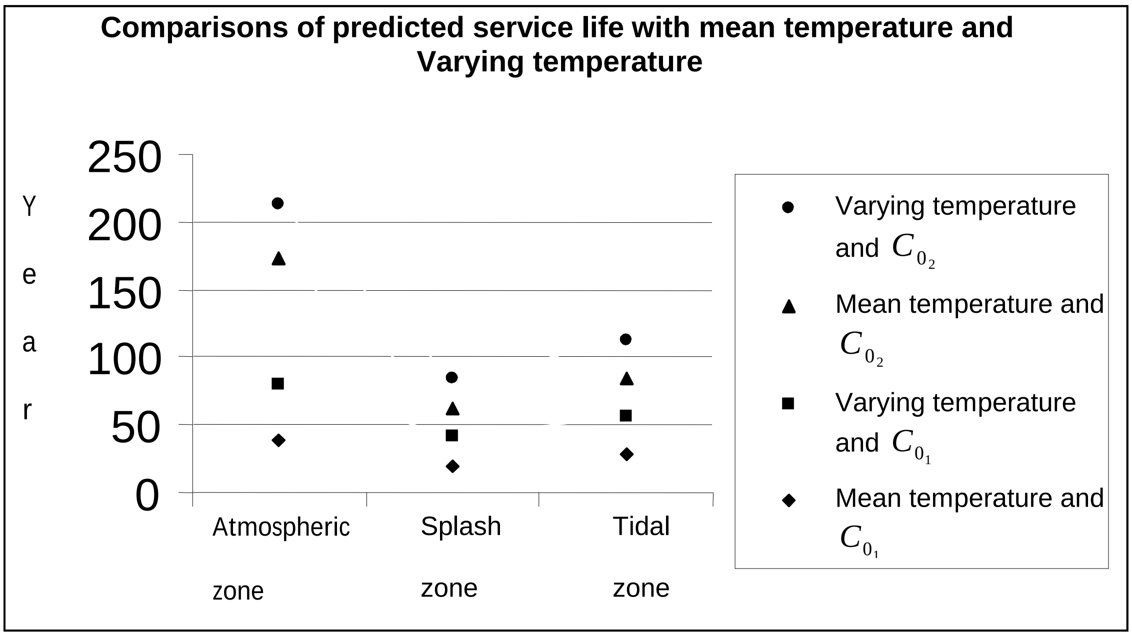

Fig.1: Comparison of Table 4 and Table7.

CONCLUSION

The conclusions from the present study are follows.

1. The present study indicates that the service life of reinforced concrete an atmospheric zone, splash zone and tidal zone are dependence on temperature.
2. Form which the service life for the atmospheric zone is higher than for the other zones, which is expected.
3. The service life for the Splash zone is relatively lesser than the service life for the concrete in the tidal zones.
4. The proposed model may be efficiently used to predict the chloride penetration of actual structures of to marine environments.

The service life of the concrete in the zones considered for this study are in the following order.,

Splash zone < Tidal zone < Atmospheric zone

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