

# On general multiple Eulerian integrals involving the multivariable A-function , a general class of polynomials and the A-function of one variable

F.Y. AYANT<sup>1</sup>

<sup>1</sup> Teacher in High School , France  
 E-mail : fredericayant@gmail.com

## ABSTRACT

The object of this paper is first, to evaluate a general multiple Eulerian integrals with general integrands involving the product of a multivariable A-function defined by Gautam et al [4], a general class of polynomials, the A-function of one variable and generalized hypergeometric function with general arguments. The second multiple Eulerian integral contain a general class of polynomials, the general polynomial set, the A-function of one variable and multivariable A-function with general arguments. Our integral formulas are interesting and unified nature.

Keywords :Multivariable H-function, class of polynomials, general polynomials set, multiple Eulerian integral, Aleph-function of one variable, multivariable A-function.

**2010 Mathematics Subject Classification :33C05, 33C60**

## 1. Introduction

In this paper, we evaluate two multiple Eulerian integrals involving the multivariable A-function defined by Gautam et al [4], the A-function of one variable defined by Gautam et al [3] and multivariable class of polynomials with general arguments. The multivariable A-function defined by Gautam et al [4] is a extension of the multivariable H-function defined by Srivastava et al [10]. We will use the contracted form.

The A-function is defined and represented in the following manner.

$$A(z_1, \dots, z_r) = A_{p,q;p_1,q_1;\dots;p_r,q_r}^{m,n;m_1,n_1;\dots;m_r,n_r} \left( \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix} \middle| \begin{matrix} (a_j; A_j^{(1)}, \dots, A_j^{(r)})_{1,p} : \\ \\ \\ (b_j; B_j^{(1)}, \dots, B_j^{(r)})_{1,q} : \end{matrix} \right. \\ \left. \begin{matrix} (c_j^{(1)}, C_j^{(1)})_{1,p_1}; \dots; (c_j^{(r)}, C_j^{(r)})_{1,p_r} \\ \\ (d_j^{(1)}, D_j^{(1)})_{1,q_1}; \dots; (d_j^{(r)}, D_j^{(r)})_{1,q_r} \end{matrix} \right) \quad (1.1)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \phi(s_1, \dots, s_r) \prod_{i=1}^r \theta_i(s_i) z_i^{s_i} ds_1 \dots ds_r \quad (1.2)$$

where  $\phi(s_1, \dots, s_r), \theta_i(s_i), i = 1, \dots, r$  are given by :

$$\phi(s_1, \dots, s_r) = \frac{\prod_{j=1}^m \Gamma(b_j - \sum_{i=1}^r B_j^{(i)} s_i) \prod_{j=1}^n \Gamma(1 - a_j + \sum_{i=1}^r A_j^{(i)} s_j)}{\prod_{j=n+1}^p \Gamma(a_j - \sum_{i=1}^r A_j^{(i)} s_j) \prod_{j=m+1}^q \Gamma(1 - b_j + \sum_{i=1}^r B_j^{(i)} s_j)} \quad (1.3)$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma(1 - c_j^{(i)} + C_j^{(i)} s_i) \prod_{j=1}^{m_i} \Gamma(d_j^{(i)} - D_j^{(i)} s_i)}{\prod_{j=n_i+1}^{p_i} \Gamma(c_j^{(i)} - C_j^{(i)} s_i) \prod_{j=m_i+1}^{q_i} \Gamma(1 - d_j^{(i)} + D_j^{(i)} s_i)} \quad (1.4)$$

Here  $m, n, p, m_i, n_i, p_i, c_i \in \mathbb{N}^*; i = 1, \dots, r; a_j, b_j, c_j^{(i)}, d_j^{(i)}, A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i)z_k| < \frac{1}{2}\eta_k\pi, \xi^* = 0, \eta_i > 0 \quad (1.5)$$

$$\Omega_i = \prod_{j=1}^p \{A_j^{(i)}\}^{A_j^{(i)}} \prod_{j=1}^q \{B_j^{(i)}\}^{-B_j^{(i)}} \prod_{j=1}^{q_i} \{D_j^{(i)}\}^{D_j^{(i)}} \prod_{j=1}^{p_i} \{C_j^{(i)}\}^{-C_j^{(i)}}; i = 1, \dots, r \quad (1.6)$$

$$\xi_i^* = Im\left(\sum_{j=1}^p A_j^{(i)} - \sum_{j=1}^q B_j^{(i)} + \sum_{j=1}^{q_i} D_j^{(i)} - \sum_{j=1}^{p_i} C_j^{(i)}\right); i = 1, \dots, r \quad (1.7)$$

$$\eta_i = Re\left(\sum_{j=1}^n A_j^{(i)} - \sum_{j=n+1}^p A_j^{(i)} + \sum_{j=1}^m B_j^{(i)} - \sum_{j=m+1}^q B_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\right) \text{ with} \\ i = 1, \dots, r \quad (1.8)$$

Let

$$X = m_1, n_1; \dots; m_r, n_r; Y = p'_1, q'_1; \dots; p'_s, q'_s \quad (1.9)$$

$$A = (a_j; A_j^{(1)}, \dots, A_j^{(r)})_{1,p} \quad ; B = (b_j; B_j^{(1)}, \dots, B_j^{(r)})_{1,q} \quad (1.10)$$

$$C = (c_j^{(1)}, C_j^{(1)})_{1,p_1}; \dots; (c_j^{(s)}, C_j^{(r)})_{1,p_r}; D = (d_j^{(1)}, D_j^{(1)})_{1,q_1}; \dots; (d_j^{(r)}, D_j^{(r)})_{1,q_r} \quad (1.11)$$

the contracted form is

$$A(z_1, \dots, z_r) = A_{p,q;Y}^{m,n;X} \left( \begin{array}{c|c} z_1 & A : C \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & B : D \\ z_r & \end{array} \right) \quad (1.12)$$

The A- function , introduced by Gautam et al [3], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$A(z) = A_{p,q}^{m,n} \left( z \left| \begin{array}{l} (a_j, \alpha_i)_{n,n+1}, (a_j, \alpha_i)_p \\ (b_j, \beta_j)_{m,m+1}, (b_j, \beta_j)_q \end{array} \right. \right) = \frac{1}{2\pi\omega} \int_L \Omega_{p,q}^{m,n}(s) z^{-s} ds \quad (1.7)$$

for all  $z$  different to 0 and

$$\Omega_{p,q}^{m,n}(s) = \frac{\prod_{j=1}^m \Gamma(b_j - \beta_j s) \prod_{j=1}^n \Gamma(1 - a_j + \alpha_j s)}{\prod_{j=n+1}^p \Gamma(a_j - \alpha_j s) \prod_{j=m+1}^q \Gamma(1 - b_j + \beta_j s)} \quad (1.8)$$

When the poles of  $\Gamma(b_j - \beta_j s), j = 1, \dots, m$ , are simples the integral (1.7) can be evaluate with the help of the Residue Theorem. We obtain

$$A(z) = \sum_{G=1}^m \sum_{g=0}^{\infty} \frac{(-)^g \Omega_{p,q}^{m,n}(s)}{B_G g!} z^s \quad (1.9)$$

with  $s = \eta_{G,g} = \frac{b_G + g}{B_G}, p < q, |z| < 1$  and  $\Omega_{p,q}^{m,n}(s)$  is given in (1.8)

The generalized polynomials defined by Srivastava [7], is given in the following manner :

$$S_{N_1, \dots, N_u}^{M_1, \dots, M_u} [y_1, \dots, y_u] = \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_u=0}^{[N_u/M_u]} \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_u)_{M_u K_u}}{K_u!} A[N_1, K_1; \dots; N_u, K_u] y_1^{K_1} \dots y_u^{K_u} \quad (1.10)$$

Where  $M_1, \dots, M_u$  are arbitrary positive integers and the coefficients  $A[N_1, K_1; \dots; N_u, K_u]$  are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_u = \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_u)_{M_u K_u}}{K_u!} A[N_1, K_1; \dots; N_u, K_u] \quad (1.11)$$

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

## 2. Sequence of functions

Agarwal and Chaubey [1], Salim [6] and several others have studied a general sequence of functions. In the present document we shall study the following useful series formula for a general sequence of functions.

$$R_n^{\alpha, \beta} [x; E, F, g, h; p, q; \gamma; \delta; e^{-sx^r}] = \sum_{w, v, u, t, e, k_1, k_2} \psi(w, v, u, t, e, k_1, k_2) x^R \quad (2.1)$$

$$\text{where } \sum_{w, v, u, t, e, k_1, k_2} = \sum_{w=0}^{\infty} \sum_{v=0}^n \sum_{u=0}^v \sum_{t=0}^n \sum_{c=0}^t \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \quad (2.2)$$

and the infinite series on the right side (2.1) is absolutely convergent,  $R = ln + qv + pt + rw + k_1 r + k_2 q$  (2.3)

$$\text{and } \psi(w, v, u, t, e, k_1, k_2) = \frac{(-)^{t+w+k_2} (-v)_u (-t)_e (\alpha)_t l^n}{w! v! u! t! e! K_n k_1! k_2!} \frac{s^{w+k_1} F^{\gamma n-t}}{(1 - \alpha - t)_e} (\alpha - \gamma n)_e$$

$$(-\beta - \delta n)_v g^{v+k_2} h^{\delta n-v-k_2} (v - \delta n)_{k_2} E^t \left( \frac{pe + rw + \lambda + qn}{l} \right)_n \quad (2.4)$$

where  $K_n$  is a sequence of constants. This function will note  $R_n^{\alpha, \beta} [x]$

By suitably specializing the parameters involving in (2.1), a general sequence of function reduced to generalized polynomials set studied by Raizada [1], a class of polynomials introduced by Fujiwara [2] and several others authors.

### 3. Integral representation of generalized hypergeometric function

The following generalized hypergeometric function in terms of multiple contour integrals is also required [9 ,page 39 eq .30]

$$\frac{\prod_{j=1}^P \Gamma(A_j)}{\prod_{j=1}^Q \Gamma(B_j)} {}_P F_Q [(A_P); (B_Q); -(x_1 + \cdots + x_r)]$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \frac{\prod_{j=1}^P \Gamma(A_j + s_1 + \cdots + s_r)}{\prod_{j=1}^Q \Gamma(B_j + s_1 + \cdots + s_r)} \Gamma(-s_1) \cdots \Gamma(-s_r) x_1^{s_1} \cdots x_r^{s_r} ds_1 \cdots ds_r \quad (3.1)$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of  $\Gamma(A_j + s_1 + \cdots + s_r)$  are separated from those of  $\Gamma(-s_j), j = 1, \cdots, r$ . The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of  $\Gamma(-s_j), j = 1, \cdots, r$

The equivalent form of Eulerian beta integral is :

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} dt = (b-a)^{\alpha+\beta} B(\alpha, \beta) (Re(\alpha) > 0, Re(\beta) > 0, a \neq b) \quad (3.2)$$

### 4. First integral

We note :

$$X = m_1, n_1; \cdots; m_r, n_r; 1, 0; \cdots; 1, 0; 1, 0; \cdots; 1, 0 \quad (4.1)$$

$$Y = p_1, q_1; \cdots; p_r, q_r; 0, 1; \cdots; 0, 1; 0, 1; \cdots; 0, 1 \quad (4.2)$$

$$A = (a_j; A_j^{(1)}, \cdots, A_j^{(r)})_{1,p} ; B = (b_j; B_j^{(1)}, \cdots, B_j^{(r)})_{1,q} \quad (4.3)$$

$$C = (c_j^{(1)}, \gamma_j^{(1)})_{1,p_1}; \cdots; (c_j^{(r)}, \gamma_j^{(r)})_{1,p_r}; (1, 0); \cdots; (1, 0); (1, 0); \cdots; (1, 0) \quad (4.4)$$

$$D = (d_j^{(1)}, \bar{\delta}_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, \bar{\delta}_j^{(r)})_{1,q_r}; (0, 1); \cdots; (0, 1); (0, 1); \cdots; (0, 1) \quad (4.5)$$

$$A^* = [1 + \sigma'_i - \eta_{G,g} c'_i - \sum_{j=1}^u K_j \xi_j^{(1,j)}; \rho_i^{(1,1)}, \cdots, \rho_i^{(1,r)}, \tau_i^{(1,1)}, \cdots, \tau_i^{(1,l)}, 1, 0, \cdots, 0]_{1,s}, \cdots,$$

$$[1 + \sigma_i^{(T)} - \eta_{G,g} c_i^{(T)} - \sum_{j=1}^u K_j \xi_j^{(T,j)}; \rho_i^{(T,1)}, \cdots, \rho_i^{(T,r)}, \tau_i^{(T,1)}, \cdots, \tau_i^{(T,l)}, 0, \cdots, 0, 1]_{1,s},$$

$$[1 - A_j; 0, \cdots, 0, 1, \cdots, 1, 0, \cdots, 0]_{1,P},$$

$$[1 - \alpha_i - \eta_{G,g} a_i - \sum_{j=1}^u K_j \alpha_i^{(j)}; \delta'_i, \cdots, \delta_i^{(r)}, \mu'_i, \cdots, \mu_i^{(l)}, 1, \cdots, 1, 0, \cdots, 0]_{1,s},$$

W-items (T-W)-items

$$[1 - \beta_i - \eta_{G,g} b_i - \sum_{j=1}^u K_j \beta_i^{(j)}; \eta'_i, \dots, \eta_i^{(r)}, \theta'_i, \dots, \theta_i^{(l)}, 0, \dots, 0, 1, \dots, 1]_{1,s} \quad (4.6)$$

W-items      (T-W)-items

$$B^* = [1 + \sigma'_i - \eta_{G,g} c'_i - \sum_{j=1}^u K_j \xi_i^{(1,j)}; \rho_i^{(1,1)}, \dots, \rho_i^{(1,r)}, \tau_i^{(1,1)}, \dots, \tau_i^{(1,l)}, 0, \dots, 0]_{1,s}, \dots,$$

$$[1 + \sigma_i^{(T)} - \eta_{G,g} c_i^{(T)} - \sum_{j=1}^u K_j \xi_i^{(T,j)}; \rho_i^{(T,1)}, \dots, \rho_i^{(T,r)}, \tau_i^{(T,1)}, \dots, \tau_i^{(T,l)}, 0, \dots, 0]_{1,s},$$

$$[1 - B_j; 0, \dots, 0, 1, \dots, 1, 0, \dots, 0]_{1,Q},$$

$$[1 - \alpha_i - \beta_i - \eta_{G,g} (a_i + b_i) - \sum_{j=1}^u K_j (\alpha_i^{(j)} + \beta_i^{(j)}); (\delta'_i + \eta'_i), \dots, (\delta_i^{(r)} + \eta_i^{(r)})$$

$$(\mu'_i + \theta'_i), \dots, (\mu_i^{(l)} + \theta_i^{(l)}), 1, \dots, 1]_{1,s} \quad (4.7)$$

We have the following multiple Eulerian integral and we obtain the I-function of  $(r + l + T)$ -variables

$$\int_{u_1}^{v_1} \cdots \int_{u_s}^{v_s} \prod_{i=1}^s \left[ (x_i - u_i)^{\alpha_i - 1} (v_i - x_i)^{\beta_i - 1} \prod_{j=1}^T (u_i^{(j)} x_i + v_i^{(j)})^{\sigma_i^{(j)}} \right]$$

$$A_{p,q}^{m,n} \left[ z \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{a_i} (v_i - x_i)^{b_i}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{c_i^{(j)}}} \right] \right] S_{N_1, \dots, N_u}^{M_1, \dots, M_u} \left( \begin{matrix} y_1 \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\alpha'_i} (v_i - x_i)^{\beta'_i}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\xi_i^{(j,1)}}} \right] \\ \vdots \\ y_u \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\alpha_i^{(u)}} (v_i - x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\xi_i^{(j,u)}}} \right] \end{matrix} \right)$$

$$A_{p,q,Y}^{m,n:X} \left( \begin{matrix} z_1 \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\delta'_i} (v_i - x_i)^{\eta'_i}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\rho_i^{(j,1)}}} \right] \\ \vdots \\ z_r \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\delta_i^{(r)}} (v_i - x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\rho_i^{(j,r)}}} \right] \end{matrix} \right)$$

$${}_p F_Q \left[ (A_P); (B_Q); - \sum_{k=1}^l g_k \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\mu_i^{(k)}} (v_i - x_i)^{\theta_i^{(r)}}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\tau_i^{(j,k)}}} \right] \right] dx_1 \cdots dx_s$$

$$= \frac{\prod_{j=1}^Q \Gamma(B_j)}{\prod_{j=1}^P \Gamma(A_j)} \prod_{i=1}^s \left[ (v_i - u_i)^{\alpha_i + \beta_i - 1} \prod_{j=1}^W \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{\sigma_i^{(j)}} \prod_{j=W+1}^T \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{\sigma_i^{(j)}} \right]$$

$$\sum_{G=1}^m \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} a_u \frac{(-)^g \Omega_{p,q}^{m,n}(\eta_{G,g})}{B_G g!} y_1^{K_1} \cdots y_u^{K_u} \prod_{i=1}^s \left[ (v_i - u_i)^{\eta_{G,g}(a_i + b_i) + \sum_{j=1}^u K_j (\alpha_i^{(j)} + \beta_i^{(j)})} \right]$$

$$z^{\eta_{G,g}} \prod_{i=1}^s \left[ \prod_{j=1}^W \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\eta_{G,g} c_i^{(j)} - \sum_{l=1}^u K_l \xi_i^{(j,l)}} \prod_{j=W+1}^T \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\eta_{G,g} c_i^{(j)} - \sum_{l=1}^u K_l \xi_i^{(j,l)}} \right]$$

$$A_{p+sT+P+2s;q+sT+Q+s;Y}^{m,n+sT+P+2s;X} \left( \begin{array}{c|c} \begin{matrix} z_1 w_1 \\ \vdots \\ \vdots \\ z_r w_r \\ g_1 W_1 \\ \vdots \\ \vdots \\ g_l W_l \\ G_1 \\ \vdots \\ \vdots \\ G_T \end{matrix} & \begin{matrix} A ; A^* : C \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ B ; B^* : D \end{matrix} \end{array} \right) \quad (4.8)$$

Where

$$w_m = \prod_{i=1}^s \left[ (v_i - u_i)^{\delta_i^{(m)} + \eta_i^{(m)}} \prod_{j=1}^W \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\rho_i^{(j,m)}} \prod_{j=W+1}^T \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\rho_i^{(j,m)}} \right], m = 1, \dots, r \quad (4.9)$$

$$W_k = \prod_{i=1}^s \left[ (v_i - u_i)^{\mu_i^{(k)} + \theta_i^{(k)}} \prod_{j=1}^W \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\tau_i^{(j,k)}} \prod_{j=W+1}^T \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\tau_i^{(j,k)}} \right], k = 1, \dots, l \quad (4.10)$$

$$G_j = \prod_{i=1}^s \left[ \frac{(v_i - u_i) U_i^{(j)}}{u_i U_i^{(j)} + V_i^{(j)}} \right], j = 1, \dots, W \quad (4.11)$$

$$G_j = - \prod_{i=1}^s \left[ \frac{(v_i - u_i) U_i^{(j)}}{u_i U_i^{(j)} + V_i^{(j)}} \right], j = W + 1, \dots, T \quad (4.12)$$

Provided that :

**(A)**  $0 \leq W \leq T; u_i, v_i \in \mathbb{R}; \min\{a_i, b_i, c_i^{(j)}, \alpha_i^{(k)}, \beta_i^{(k)}, \xi_i^{(j,k)}\} > 0, i = 1, \dots, s; k = 1, \dots, u; j = 1, \dots, T$

**(B)**  $\min\{\delta_i^{(t)}, \eta_i^{(t)}, \rho_i^{(j,t)}, \mu_i^{(k)}, \theta_i^{(k)}, \tau_i^{(j,k)}\} \geq 0; j = 1, \dots, T; i = 1, \dots, s; k = 1, \dots, l; t = 1, \dots, r$

$$(C) \sigma_i^{(j)} \in \mathbb{R}, U_i^{(j)}, V_i^{(j)} \in \mathbb{C}, z_t, g_k \in \mathbb{C}; j = 1, \dots, s; t = 1, \dots, r; k = 1, \dots, l; t = 1, \dots, r$$

$$(D) \max \left[ \frac{(v_i - u_i)U_i^{(j)}}{u_i U_i^{(j)} + V_i^{(j)}} \right] < 1, i = 1, \dots, s; j = 1, \dots, W \text{ and}$$

$$\max \left[ \frac{(v_i - u_i)U_i^{(j)}}{u_i U_i^{(j)} + V_i^{(j)}} \right] < 1, i = 1, \dots, s; j = W + 1, \dots, T$$

$$(E) \operatorname{Re} \left[ \alpha_i + a_i \min_{1 \leq j \leq M} \frac{b_j}{B_j} + \sum_{t=1}^r \delta_i^{(t)} \min_{1 \leq j \leq m^{(i)}} \frac{d_j^{(t)}}{D_j^{(t)}} \right] > 0;$$

$$\operatorname{Re} \left[ \beta_i + b_i \min_{1 \leq j \leq M} \frac{b_j}{B_j} + \sum_{t=1}^r \eta_i^{(t)} \min_{1 \leq j \leq m^{(i)}} \frac{d_j^{(t)}}{D_j^{(t)}} \right] > 0; i = 1, \dots, s$$

$$(F) |\arg(\Omega_i)z_k| < \frac{1}{2}\eta_i\pi, \xi^* = 0, \eta_i > 0$$

$$\Omega_i = \prod_{j=1}^p \{A_j^{(i)}\} A_j^{(i)} \prod_{j=1}^q \{B_j^{(i)}\}^{-B_j^{(i)}} \prod_{j=1}^{q_i} \{D_j^{(i)}\}^{D_j^{(i)}} \prod_{j=1}^{p_i} \{C_j^{(i)}\}^{-C_j^{(i)}}; i = 1, \dots, r$$

$$\xi_i^* = \operatorname{Im} \left( \sum_{j=1}^p A_j^{(i)} - \sum_{j=1}^q B_j^{(i)} + \sum_{j=1}^{q_i} D_j^{(i)} - \sum_{j=1}^{p_i} C_j^{(i)} \right); i = 1, \dots, r$$

$$\eta_i = \operatorname{Re} \left( \sum_{j=1}^n A_j^{(i)} - \sum_{j=n+1}^p A_j^{(i)} + \sum_{j=1}^m B_j^{(i)} - \sum_{j=m+1}^q B_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)} \right)$$

$$-\delta_i^{(k)} - \eta_i^{(k)} - \sum_{j=1}^T \rho_i^{(j,k)} > 0$$

$$(G) \left| \arg \left( z_i \prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{-\rho_i^{(j,k)}} \right) \right| < \frac{1}{2} \eta_i \pi$$

(I) The series occuring on the right-hand side of (4.8) are absolutely and uniformly convergent.

(J)  $P \leq Q + 1$ . The equality holds, when , in addition,

$$\text{either } P > Q \text{ and } \sum_{k=1}^l \left| g_k \left( \prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{-\tau_i^{(j,k)}} \right) \right|^{\frac{1}{Q-P}} < 1 \quad (u_i \leq x_i \leq v_i; i = 1, \dots, s)$$

$$\text{or } P \leq Q \text{ and } \max_{1 \leq k \leq l} \left[ \left| \left( g_k \prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{-\tau_i^{(j,k)}} \right) \right| \right] < 1 \quad (u_i \leq x_i \leq v_i; i = 1, \dots, s)$$

**Proof**

To establish the formula (4.8), we first use series representation (1.9) and (1.10) for  $A(z)$  and  $S_{N_1, \dots, N_u}^{M_1, \dots, M_u}[\cdot]$  respectively, we use contour integral representation with the help of (1.1) for the multivariable A-function occurring in its left-hand side and use the contour integral representation with the help of (3.1) for the generalized hypergeometric function  ${}_pF_Q(\cdot)$ . Changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). Now we write :

$$\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} = \prod_{j=1}^W (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} \prod_{j=W+1}^T (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}}$$

$$\text{where } K_i^{(j)} = \eta_i^{(j)} - \eta_{G,g} c_i^{(j)} - \sum_{l=1}^u K_l \xi_i^{(j,l)} - \sum_{t=1}^r \rho_i^{(j,t)} \zeta_t - \sum_{k=1}^l \tau_i^{(j,k)} \zeta_k \quad i = 1, \dots, s; j = 1, \dots, T \quad (4.13)$$

and express the factor occuring in R.H.S. Of (4.8) in terms of following Mellin-Barnes contour integral with the help of the result [8, page 18, eq.(2.6.4)]

$$\frac{1}{(2\pi\omega)^W} \int_{L'_1} \cdots \int_{L'_W} \prod_{j=1}^W \left[ \Gamma(-\zeta'_j) \Gamma(-K_i^{(j)} + \zeta'_j) \prod_{j=1}^W (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} \right] = \prod_{j=1}^W \left[ \frac{(U_i^{(j)} u_i + V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})} \right]$$

$$\prod_{j=1}^W \left[ \frac{(U_i^{(j)} (x_i - u_i))}{(u_i U_i^{(j)} + V_i^{(j)})} \right]^{\zeta'_j} d\zeta'_1 \cdots d\zeta'_W \quad (4.14)$$

and

$$\frac{1}{(2\pi\omega)^{T-W}} \int_{L'_{W+1}} \cdots \int_{L'_T} \prod_{j=W+1}^T \left[ \Gamma(-\zeta'_j) \Gamma(-K_i^{(j)} + \zeta'_j) \prod_{j=W+1}^T (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} \right] = \prod_{j=W+1}^T \left[ \frac{(U_i^{(j)} v_i + V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})} \right]$$

$$\prod_{j=W+1}^T \left[ -\frac{(U_i^{(j)} (v_i - x_i))}{(v_i U_i^{(j)} + V_i^{(j)})} \right]^{\zeta'_j} d\zeta'_{W+1} \cdots d\zeta'_T \quad (4.15)$$

We apply the Fubini's theorem for multiple integral. Finally evaluating the innermost  $\mathbf{x}$ -integral with the help of (3.2) and reinterpreting the multiple Mellin-Barnes contour integral in terms of multivariable A-function of  $(r + l + T)$ -variables, we obtain the formula (4.8).

## 5. Second integral

We note :

$$X = m_1, n_1; \cdots; m_r, n_r; 1, 0; \cdots; 1, 0 \quad (5.1)$$

$$Y = p_1, q_1; \cdots; p_r, q_r; 0, 1; \cdots; 0, 1 \quad (5.2)$$

$$A = (a_j; A_j^{(1)}, \cdots, A_j^{(r)})_{1,p} \quad ; B = (b_j; B_j^{(1)}, \cdots, B_j^{(r)})_{1,q} \quad (5.3)$$

$$C = (c_j^{(1)}, \gamma_j^{(1)})_{1,p_1}; \cdots; (c_j^{(r)}, \gamma_j^{(r)})_{1,p_r}; (1, 0); \cdots; (1, 0) \quad (5.4)$$

$$D = (d_j^{(1)}, \bar{\delta}_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, \bar{\delta}_j^{(r)})_{1,q_r}; (0, 1); \cdots; (0, 1) \quad (5.5)$$

$$A^* = [1 + \sigma'_i - \theta'_i R - \eta_{G,g} c'_i - \sum_{j=1}^u K_j \xi_i^{(1,j)}; \rho_i^{(1,1)}, \cdots, \rho_i^{(1,r)}, 1, 0, \cdots, 0]_{1,s}, \cdots,$$

$$[1 + \sigma_i^{(T)} - \theta_i^{(T)} R - \eta_{G,g} c_i^{(T)} - \sum_{j=1}^u K_j \xi_i^{(T,j)}; \rho_i^{(T,1)}, \cdots, \rho_i^{(T,r)}, 0, \cdots, 0, 1]_{1,s},$$

$$[1 - \alpha_i - \zeta_i R - \eta_{G,g} a_i - \sum_{j=1}^u K_j \alpha_i^{(j)}; \delta'_i, \cdots, \delta_i^{(r)}, 1, \cdots, 1, 0, \cdots, 0]_{1,s},$$

W-items (T-W)-items



$$[1 - \beta_i - \lambda_i R - \eta_{G,g} b_i - \sum_{j=1}^u K_j \beta_i^{(j)}; \eta'_i, \dots, \eta_i^{(r)}, 0, \dots, 0, 1, \dots, 1]_{1,s} \quad (5.6)$$

W-items    (T-W)-items

$$B^* = [1 + \sigma'_i - \theta'_i R - \eta_{G,g} c'_i - \sum_{j=1}^u K_j \xi_i^{(1,j)}; \rho_i^{(1,1)}, \dots, \rho_i^{(1,r)}, 0, \dots, 0]_{1,s}, \dots,$$

$$[1 + \sigma_i^{(T)} - \theta_i^{(T)} R - \eta_{G,g} c_i^{(T)} - \sum_{j=1}^u K_j \xi_i^{(T,j)}; \rho_i^{(T,1)}, \dots, \rho_i^{(T,r)}, 0, \dots, 0]_{1,s},$$

$$[1 - \alpha_i - \beta_i (\zeta_i + \lambda_i) R - \eta_{G,g} (a_i + b_i) - \sum_{j=1}^u K_j (\alpha_i^{(j)} + \beta_i^{(j)}); (\delta'_i + \eta'_i), \dots, (\delta_i^{(r)} + \eta_i^{(r)})$$

$$(\mu'_i + \theta'_i), \dots, (\mu_i^{(l)} + \theta_i^{(l)}), 1, \dots, 1]_{1,s} \quad (5.7)$$

We have the following multiple Eulerian integral

$$\int_{u_1}^{v_1} \cdots \int_{u_s}^{v_s} \prod_{i=1}^s \left[ (x_i - u_i)^{\alpha_i - 1} (v_i - x_i)^{\beta_i - 1} \prod_{j=1}^T (u_i^{(j)} x_i + v_i^{(j)})^{\sigma_i^{(j)}} \right]$$

$$A_{p,q}^{m,n} \left[ z \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\alpha_i} (v_i - x_i)^{\beta_i}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{c_i^{(j)}}} \right] \right] R_n^{\alpha,\beta} \left[ Z \prod_{j=1}^s \left[ \frac{(x_i - u_i)^{\zeta_i} (v_i - x_i)^{\lambda_i}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\theta_i^{(j)}}} \right] \right]$$

$$S_{N_1, \dots, N_u}^{M_1, \dots, M_u} \left( \begin{array}{c} y_1 \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\alpha'_i} (v_i - x_i)^{\beta'_i}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\xi_i^{(j,1)}}} \right] \\ \vdots \\ y_u \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\alpha_i^{(u)}} (v_i - x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\xi_i^{(j,u)}}} \right] \end{array} \right)$$

$$A_{p,q:Y}^{m,n:X} \left( \begin{array}{c} z_1 \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\delta'_i} (v_i - x_i)^{\eta'_i}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\rho_i^{(j,1)}}} \right] \\ \vdots \\ z_r \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\delta_i^{(r)}} (v_i - x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\rho_i^{(j,r)}}} \right] \end{array} \right) dx_1 \cdots dx_s$$

$$= \prod_{i=1}^s \left[ (v_i - u_i)^{\alpha_i + \beta_i - 1} \prod_{j=1}^W \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{\sigma_i^{(j)}} \prod_{j=W+1}^T \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{\sigma_i^{(j)}} \right]$$

$$\sum_{G=1}^m \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} a_u \frac{(-)^g \Omega_{p,q}^{m,n}(\eta_{G,g})}{B_G g!} y_1^{K_1} \cdots y_u^{K_u}$$

$$\sum_{w,v,u,t,e,k_1,k_2} \psi'(w,v,u,t,e,k_1,k_2) Z^R \prod_{i=1}^s \left[ (v_i - u_i)^{\eta_{G,g}(a_i+b_i) + \sum_{j=1}^u K_j(\alpha_i^{(j)} + \beta_i^{(j)})} \right]$$

$$\prod_{i=1}^s \left[ \prod_{j=1}^W \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\eta_{G,g} c_i^{(j)} - \sum_{l=1}^u K_l \xi_i^{(j,l)}} \prod_{j=W+1}^T \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\eta_{G,g} c_i^{(j)} - \sum_{l=1}^u K_l \xi_i^{(j,l)}} \right]$$

$$A_{p+n+sT+2s;X}^{m,n+sT+2s;q+sT+s;Y} \left( \begin{array}{c|c} \begin{matrix} z_1 w_1 \\ \vdots \\ z_r w_r \\ \\ G_1 \\ \vdots \\ G_T \end{matrix} & \begin{matrix} A; A^* : C \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ B; B^* : D \end{matrix} \end{array} \right) \quad (5.8)$$

where

$$\psi'(w,v,u,t,e,k_1,k_2) = \frac{\psi(w,v,u,t,e,k_1,k_2) \prod_{i=1}^s (v_i - u_i)^{(\zeta_i + \lambda_i)R}}{\prod_{i=1}^s \left[ \prod_{j=1}^W (u_i U_i^{(j)} + V_i^{(j)})^{\theta_i^{(j)} R} \prod_{j=W+1}^T (u_i U_i^{(j)} + V_i^{(j)})^{\theta_i^{(j)} R} \right]} \quad (5.9)$$

$\psi(w,v,u,t,e,k_1,k_2)$  and  $R$  are given by (2.4) and (2.3) respectively.

$$w_l = \prod_{i=1}^s \left[ (v_i - u_i)^{\delta_i^{(l)} + \eta_i^{(l)}} \prod_{j=1}^W \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\rho_i^{(j,l)}} \prod_{j=W+1}^T \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\rho_i^{(j,l)}} \right], l = 1, \dots, r \quad (5.10)$$

$$G_j = \prod_{i=1}^s \left[ \frac{(v_i - u_i) U_i^{(j)}}{u_i U_i^{(j)} + V_i^{(j)}} \right], j = 1, \dots, W \quad (5.11)$$

$$G_j = - \prod_{i=1}^s \left[ \frac{(v_i - u_i) U_i^{(j)}}{u_i U_i^{(j)} + V_i^{(j)}} \right], j = W + 1, \dots, T \quad (5.12)$$

Provided that :

$$\mathbf{(A)} \min\{a_i, b_i, c_i^{(j)}, \alpha_i^{(k)}, \beta_i^{(k)}, \xi_i^{(j,k)}\} > 0, i = 1, \dots, s; k = 1, \dots, u; j = 1, \dots, T; 0 \leq W \leq T; u_i, v_i \in \mathbb{R}$$

$$\textbf{(B)} \min\{\zeta_i, \lambda_i, \theta_i^{(j)}, \delta_i^{(l)}, \eta_i^{(l)}, \rho_i^{(j,l)}\} \geq 0; j = 1, \dots, T; i = 1, \dots, s; l = 1, \dots, r$$

$$\textbf{(C)} \operatorname{Re}(\alpha_i, \beta_i, v_i^{(j)}) > 0 (i = 1, \dots, s; j = 1, \dots, T); |\tau| < 1$$

$$\textbf{(D)} \max \left[ \frac{(v_i - u_i)U_i^{(j)}}{u_i U_i^{(j)} + V_i^{(j)}} \right] < 1, i = 1, \dots, s; j = 1, \dots, W \text{ and}$$

$$\max \left[ \frac{(v_i - u_i)U_i^{(j)}}{u_i U_i^{(j)} + V_i^{(j)}} \right] < 1, i = 1, \dots, s; j = W + 1, \dots, T$$

$$\textbf{(E)} \operatorname{Re} \left[ \alpha_i + R\zeta_i + a_i \min_{1 \leq j \leq M} \frac{b_j}{B_j} + \sum_{t=1}^r \delta_i^{(t)} \min_{1 \leq j \leq m^{(i)}} \frac{d_j^{(t)}}{D_j^{(t)}} \right] > 0;$$

$$\operatorname{Re} \left[ \beta_i + R\lambda_i + b_i \min_{1 \leq j \leq M} \frac{b_j}{B_j} + \sum_{t=1}^r \eta_i^{(t)} \min_{1 \leq j \leq m^{(i)}} \frac{d_j^{(t)}}{D_j^{(t)}} \right] > 0; i = 1, \dots, s$$

$$\textbf{(F)} \Omega_i = \prod_{j=1}^p \{A_j^{(i)}\}^{A_j^{(i)}} \prod_{j=1}^q \{B_j^{(i)}\}^{-B_j^{(i)}} \prod_{j=1}^{q_i} \{D_j^{(i)}\}^{D_j^{(i)}} \prod_{j=1}^{p_i} \{C_j^{(i)}\}^{-C_j^{(i)}}; i = 1, \dots, r$$

$$\xi_i^* = \operatorname{Im} \left( \sum_{j=1}^p A_j^{(i)} - \sum_{j=1}^q B_j^{(i)} + \sum_{j=1}^{q_i} D_j^{(i)} - \sum_{j=1}^{p_i} C_j^{(i)} \right); i = 1, \dots, r$$

$$\eta_i = \operatorname{Re} \left( \sum_{j=1}^n A_j^{(i)} - \sum_{j=n+1}^p A_j^{(i)} + \sum_{j=1}^m B_j^{(i)} - \sum_{j=m+1}^q B_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)} \right)$$

$$-\delta_i^{(k)} - \eta_i^{(k)} - \sum_{j=1}^T \rho_i^{(j,k)} > 0$$

$$\textbf{(G)} \left| \arg \left( z_i \prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{-\rho_i^{(j,k)}} \right) \right| < \frac{1}{2} \eta_i \pi$$

**(H)** The series occuring on the right-hand side of (5.8) are absolutely and uniformly convergent

### Proof

To establish the formula (5.8), we first use series representation (1.8), (1.9) and (3.1) for ,  $A(z), S_{N_1, \dots, N_u}^{M_1, \dots, M_u}[\cdot]$  and  $R_n^{\alpha, \beta}[\cdot]$  respectively and the contour integral representation with the help of (1.2) for the multivariable A-function occuring in its left-hand side. Changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) . Now, we write:

$$\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} = \prod_{j=1}^W (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} \prod_{j=W+1}^T (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} \quad (5.13)$$

$$\text{where } K_i^{(j)} = \eta_i^{(j)} - \eta_{G,g} c_i^{(j)} - R\theta_i^{(j)} - \sum_{l=1}^u L_l \xi_i^{(j,l)} - \sum_{t=1}^r \rho_i^{(j,t)} \zeta_t; i = 1, \dots, s; j = 1, \dots, T \quad (5.14)$$

and express the factors occuring in R.H.S. Of (5.8) in terms of following Mellin-Barnes contour integral , we obtain :

$$\prod_{j=1}^W (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} = \prod_{j=1}^W \left[ \frac{(U_i^{(j)} u_i + V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})} \right] \frac{1}{(2\pi\omega)^W} \int_{L'_1} \cdots \int_{L'_W} \prod_{j=1}^W \left[ \Gamma(-\zeta'_j) \Gamma(-K_i^{(j)} + \zeta'_j) \right. \\ \left. \prod_{j=1}^W \left[ \frac{(U_i^{(j)}(x_i - u_i))}{(u_i U_i^{(j)} + V_i^{(j)})} \right]^{\zeta'_j} \right] d\zeta'_1 \cdots d\zeta'_W \quad (5.15)$$

and

$$\prod_{j=W+1}^T (U_i^{(j)} x_i + V_i^{(j)})^{K_i^{(j)}} = \prod_{j=W+1}^T \left[ \frac{(U_i^{(j)} v_i + V_i^{(j)})^{K_i^{(j)}}}{\Gamma(-K_i^{(j)})} \right] \frac{1}{(2\pi\omega)^{T-W}} \int_{L'_{W+1}} \cdots \int_{L'_T} \prod_{j=W+1}^T \left[ \Gamma(-\zeta'_j) \Gamma(-K_i^{(j)} + \zeta'_j) \right. \\ \left. \prod_{j=W+1}^T \left[ \frac{(U_i^{(j)}(x_i - v_i))}{(v_i U_i^{(j)} + V_i^{(j)})} \right]^{\zeta'_j} \right] d\zeta'_{W+1} \cdots d\zeta'_T \quad (5.16)$$

We apply the Fubini's theorem for multiple integral. Finally evaluating the innermost  $\mathbf{x}$ -integral with the help of (3.2) and reinterpreting the multiple Mellin-Barnes contour integral in terms of multivariable A-function of  $(r+T)$ -variables, we obtain the formula (5.8).

## 6. Multivariable H-function

If  $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$  and  $m = 0$ , the multivariable A-function reduces to multivariable H-function defined by Srivastava et al [10], we obtain the two following formulas.

### Formula 1

$$\int_{u_1}^{v_1} \cdots \int_{u_s}^{v_s} \prod_{i=1}^s \left[ (x_i - u_i)^{\alpha_i - 1} (v_i - x_i)^{\beta_i - 1} \prod_{j=1}^T (u_i^{(j)} x_i + v_i^{(j)})^{\sigma_i^{(j)}} \right] \\ A_{p,q}^{m,n} \left[ \begin{matrix} \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{a_i} (v_i - x_i)^{b_i}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{c_i^{(j)}}} \right] \end{matrix} \right] S_{N_1, \dots, N_u}^{M_1, \dots, M_u} \left( \begin{matrix} y_1 \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\alpha_i'} (v_i - x_i)^{\beta_i'}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\xi_i^{(j,1)}}} \right] \\ \vdots \\ y_u \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\alpha_i^{(u)}} (v_i - x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\xi_i^{(j,u)}}} \right] \end{matrix} \right)$$

$$\begin{aligned}
& H_{p,q;Y}^{0,n;X} \left( \begin{array}{c} z_1 \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\delta'_i} (v_i - x_i)^{\eta'_i}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\rho_i^{(j,1)}}} \right] \\ \vdots \\ z_r \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\delta_i^{(r)}} (v_i - x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\rho_i^{(j,r)}}} \right] \end{array} \right) \\
& {}^p F_Q \left[ (A_P); (B_Q); - \sum_{k=1}^l g_k \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\mu_i^{(k)}} (v_i - x_i)^{\theta_i^{(r)}}}{\prod_{j=1}^T (U_i^{(j)} x_i + V_i^{(j)})^{\tau_i^{(j,k)}}} \right] \right] dx_1 \cdots dx_s \\
& = \frac{\prod_{j=1}^Q \Gamma(B_j)}{\prod_{j=1}^P \Gamma(A_j)} \prod_{i=1}^s \left[ (v_i - u_i)^{\alpha_i + \beta_i - 1} \prod_{j=1}^W (u_i U_i^{(j)} + V_i^{(j)})^{\sigma_i^{(j)}} \prod_{j=W+1}^T (u_i U_i^{(j)} + V_i^{(j)})^{\sigma_i^{(j)}} \right] \\
& \sum_{G=1}^m \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} a_u \frac{(-)^g \Omega_{p,q}^{m,n}(\eta_{G,g})}{B_G g!} y_1^{K_1} \cdots y_u^{K_u} \prod_{i=1}^s \left[ (v_i - u_i)^{\eta_{G,g}(a_i + b_i) + \sum_{j=1}^u K_j (\alpha_i^{(j)} + \beta_i^{(j)})} \right] \\
& z^{\eta_{G,g}} \prod_{i=1}^s \left[ \prod_{j=1}^W (u_i U_i^{(j)} + V_i^{(j)})^{-\eta_{G,g} c_i^{(j)} - \sum_{l=1}^u K_l \xi_i^{(l,k)}} \prod_{j=W+1}^T (u_i U_i^{(j)} + V_i^{(j)})^{-\eta_{G,g} c_i^{(j)} - \sum_{l=1}^u K_l \xi_i^{(j,l)}} \right] \\
& H_{p+sT+P+2s;X}^{0,n+sT+P+2s;Y} \left( \begin{array}{c|c} \begin{array}{c} z_1 w_1 \\ \vdots \\ z_r w_r \\ g_1 W_1 \\ \vdots \\ g_l W_l \\ G_1 \\ \vdots \\ G_T \end{array} & \begin{array}{c} A ; A^* : C \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ B ; B^* : D \end{array} \end{array} \right) \tag{6.1}
\end{aligned}$$

under the same notations and conditions thar (4.8) with  $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$  and  $m = 0$

## Formula 2

$$\int_{u_1}^{v_1} \cdots \int_{u_s}^{v_s} \prod_{i=1}^s \left[ (x_i - u_i)^{\alpha_i - 1} (v_i - x_i)^{\beta_i - 1} \prod_{j=1}^T (u_i^{(j)} x_i + v_i^{(j)})^{\sigma_i^{(j)}} \right]$$

$$A_{p,q}^{m,n} \left[ z \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{a_i} (v_i - x_i)^{b_i}}{\prod_{j=1}^T \left( U_i^{(j)} x_i + V_i^{(j)} \right)^{c_i^{(j)}}} \right] \right] R_n^{\alpha,\beta} \left[ Z \prod_{j=1}^s \left[ \frac{(x_i - u_i)^{\zeta_i} (v_i - x_i)^{\lambda_i}}{\prod_{j=1}^T \left( U_i^{(j)} x_i + V_i^{(j)} \right)^{\theta_i^{(j)}}} \right] \right]$$

$$S_{N_1, \dots, N_u}^{M_1, \dots, M_u} \left( \begin{array}{c} y_1 \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\alpha'_i} (v_i - x_i)^{\beta'_i}}{\prod_{j=1}^T \left( U_i^{(j)} x_i + V_i^{(j)} \right)^{\xi_i^{(j,1)}}} \right] \\ \vdots \\ y_u \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\alpha_i^{(u)}} (v_i - x_i)^{\beta_i^{(u)}}}{\prod_{j=1}^T \left( U_i^{(j)} x_i + V_i^{(j)} \right)^{\xi_i^{(j,u)}}} \right] \end{array} \right)$$

$$H_{p,q;Y}^{0,n:X} \left( \begin{array}{c} z_1 \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\delta'_i} (v_i - x_i)^{\eta'_i}}{\prod_{j=1}^T \left( U_i^{(j)} x_i + V_i^{(j)} \right)^{\rho_i^{(j,1)}}} \right] \\ \vdots \\ z_r \prod_{i=1}^s \left[ \frac{(x_i - u_i)^{\delta_i^{(r)}} (v_i - x_i)^{\eta_i^{(r)}}}{\prod_{j=1}^T \left( U_i^{(j)} x_i + V_i^{(j)} \right)^{\rho_i^{(j,r)}}} \right] \end{array} \right) dx_1 \cdots dx_s$$

$$= \prod_{i=1}^s \left[ (v_i - u_i)^{\alpha_i + \beta_i - 1} \prod_{j=1}^W \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{\sigma_i^{(j)}} \prod_{j=W+1}^T \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{\sigma_i^{(j)}} \right]$$

$$\sum_{G=1}^m \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} a_u \frac{(-)^g \Omega_{p,q}^{m,n}(\eta_{G,g})}{B_G g!} y_1^{K_1} \cdots y_u^{K_u}$$

$$\sum_{w,v,u,t,e,k_1,k_2} \psi'(w,v,u,t,e,k_1,k_2) Z^R \prod_{i=1}^s \left[ (v_i - u_i)^{\eta_{G,g}(a_i+b_i) + \sum_{j=1}^u K_j(\alpha_i^{(j)} + \beta_i^{(j)})} \right]$$

$$\prod_{i=1}^s \left[ \prod_{j=1}^W \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\eta_{G,g} c_i^{(j)} - \sum_{l=1}^u K_l \xi_i^{(j,l)}} \prod_{j=W+1}^T \left( u_i U_i^{(j)} + V_i^{(j)} \right)^{-\eta_{G,g} c_i^{(j)} - \sum_{l=1}^u K_l \xi_i^{(j,l)}} \right]$$

$$H_{p+sT+2s;q+sT+s;Y}^{0,n+sT+2s;X} \left( \begin{array}{c|c} z_1 w_1 & A ; A^* : C \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r w_r & \cdot \\ & \cdot \\ G_1 & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ G_T & B ; B^* : D \end{array} \right) \quad (6.2)$$

under the same notations and conditions thar (5.8) with  $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$  and  $m = 0$

## 7. Conclusion

In this paper we have evaluated two generalized multiple Eulerian integrals involving the multivariable A-functions defined by Gautam et al [4] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

## REFERENCES

- [1] Agrawal B.D. And Chaubey J.P. Certain derivation of generating relations for generalized polynomials. Indian J. Pure and Appl. Math 10 (1980), page 1155-1157, ibid 11 (1981), page 357-359
- [2] Fujiwara I. A unified presentation of classical orthogonal polynomials. Math. Japon. 11 (1966), page133-148.
- [3] Gautam B.P., Asgar A.S. and Goyal A.N. The A-function. Revista Mathematica. Tucuman (1980)
- [4] Gautam B.P., Asgar A.S. and Goyal A.N. On the multivariable A-function. Vijnana Parishas Anusandhan Patrika Vol 29(4) 1986, page 67-81.
- [5] Raizada S.K. A study of unified representation of special functions of Mathematics Physics and their use in statistical and boundary value problem. Ph.D. Thesis, Bundelkhand University, Jhansi, India, 1991
- [6] Salim T.O. A serie formula of generalized class of polynomials associated with Laplace transform and fractional integral operators. J. Rajasthan. Acad. Phy. Sci. 1(3) (2002), page 167-176.
- [7] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page183-191.
- [8] Srivastava H.M. Gupta K.C. And Goyal S.P. The H-functions of one and two variables with applications. South Asian Publishers Pvt Ltd 1982
- [9] Srivastava H.M. and Karlsson P.W. Multiple Gaussian Hypergeometric series. Ellis.Horwood. Limited. New-York, Chichester. Brisbane. Toronto , 1985.
- [10] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

