Int. J. of Mathematical Sciences and Applications,
Vol. 1, No. 3, September 2011

www.journalshub.com

Intuitionistic fuzzy KU-ideals in KU-algebras

Samy M. Mostafa, Mokhtar A. Abdel Naby and Osama R. Elgendy
Ain Shams University, Roxy, Cairo, Egypt
e-mail: samymostafa@yahoo.com; abdelnaby@hotmail.com

Abstract

We consider the Intuitionistic fuzzification of the concept KU-ideal and the image (preimage) of KU-ideal in KU-algebra, and investigate some of there properties. Moreover, we introduce the notion of product of intuitionistic fuzzy KU-ideal in KU-algebras, and investigate some related properties.

Keywords:KU-algebras, fuzzy KU -ideals in KU -algebras, intuitionistic fuzzy KU-ideal, intuitionistic fuzzy image (preimage) of KU-ideal .

Introduction:

In 1965, L. A. Zadeh [11] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world. As generalization of intuitionistic fuzzy subset was defined by K. T. Atanassov [1, 2] and it was extended to intuitionistic fuzzy ideal by Basnet and Benerjee [3, 4]. Intuitionistic fuzzy sets have also been defined by G. Takeuti and S. Titanti in [9]. G. Takeuti and S. Titanti however considered intuitionistic fuzzy logic in the narrow sense and derived a set theory from logic which they called intuitionistic fuzzy set theory. In [8], C. Prabpayak and U. Leerawat studied ideals and congruences of BCC-algebras. ([5], [6]) and introduce a new algebraic structure which is called KU-algebras and investigated some related properties. Xi [10] applied the concept of fuzzy set to BCK-algebras and gave some of its properties. Samy M. Mostafa, Mokhtar A. Abdel Naby and Moustafa M. Youssef [7] introduced fuzzy KU-ideals in KU-algebras.. In this paper, we introduce the notion of intuitionistic fuzzy KU-ideals in KU-algebras and fuzzy intuitionistic image (preimage) of KU-ideals in KU-algebras. We also introduce the Cartesian product of two intuitionistic fuzzy KU-ideals in KU-algebras and investigate some results.

2000 Mathematics subjects classification: 03B52, 06F35, 03G25, 94D05.

2. Preliminaries:

Definition 2.1 [8]:

An algebraic system $(X,*,0)$ of type (2, 0) is called a KU-algebra if it satisfying the following conditions:

(1) $(x*y)*[(y*z)*(x*z)]=0,$

(2) $0*x=x,$

(3) $x*0=0,$

(4) $x*y=0=y*x$ implies $x=y,$ for all $x,y,z\in X$.

In a KU-algebra *X*, we get $(0*0)*[(0*x)*(0*x)]=0$. It follows that $x*x=0$ for all $x\in X$. And if we put $y=0$ in (1), we obtain $z*(x*z)=0$ for all $x,z\in X$. A subset *S* of a KU-algebra *X* is called subalgebra of *X*, if $x,y\in S$, implies $x*y\in S$.

Definition 2.2:

A non empty subset *I* of a KU-algebra *X* is said to be an KU-ideal of *X* if it satisfies:

(K_1) $0\in I,$

(K_2) $x*(y*z)\in I$ and $y\in I$ imply $x*z\in I$ for all x, y and $z\in X$.

Lemma 2.3[7]:

In KU-algebra $(X,*,0)$, the following is hold

(i) $x\le y$ implies $y*z\le x*z$,

(ii) $z*(y*x)=y*(z*x)$,

(iii) $y*[(y*x)*x]=0$. for all x, y and $z\in X$.

Example 2.4:

Let X = {0,1,2,3,4} be a set with a binary operation $*$ defined by the following table:

$*$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	3
2	0	0	0	1	4
3	0	0	0	0	3
4	0	0	0	0	0

Then $(X,*,0)$ is a KU-algebra.

3. fuzzy KU-ideal:

Definition 3.1[7]:

Let X be a KU-algebra. A fuzzy set μ in X is called a fuzzy KU-ideal of X if it satisfies:

(FK_1) $\mu(0)\ge\mu(x)$,

(FK_2) $\mu(x*z)\ge\min\{\mu(x*(y*z)),\mu(y)\}$, for all x, y and $z\in X$.

Lemma 3.2 [7]:

Let μ be a fuzzy ideal of KU-algebra X. if the inequality $x*y\le z$ hold in X, then $\mu(y)\ge\min\{\mu(x),\mu(z)\}$.

Lemma 3.3 [7]:

If μ be a fuzzy ideal of KU-algebra X and if $x\le y$, then $\mu(x)\ge\mu(y)$.

Definition 3.4:

Let μ be a fuzzy set on a KU-algebra X, then μ is called a fuzzy KU-subalgebra of X if $\mu(x*y)\ge\min\{\mu(x),\mu(y)\}$, for all $x, y\in X$.

4. Intuitionistic fuzzy KU-ideal in KU-algebra.

Definition 4.1:

An Intuitionistic fuzzy set (briefly IFS) A in a nonempty set X is an object having the form $A=\{(x,\alpha_A(x),\beta(x))\mid x\in X\}$, where the function $\alpha_A:\ X\to[0,1]$ and $\beta_A:\ X\to[0,1]$ denote the degree of membership and degree of non membership, respectively and $0\le\alpha_A(x)+\beta_A(x)\le1$ for all $x\in X$. An Intuitionistic fuzzy set $A=\{(x,\alpha_A(x),\beta(x))\mid x\in X\}$ in X can be identified to an order pair (α_A,β_B) in $I^X\times I^X$.

We shall use the symbol $A=(\alpha_A,\beta_A)$ for IFS $A=\{(x,\alpha_A(x),\beta(x))\mid x\in X\}$.

Definition 4.2:

An IFS $A=(\alpha_A,\beta_A)$ in a KU-algebra X is called an intuitionistic fuzzy KU-subalgebra of X if it satisfies the following

($I\ S_1$) $\alpha_A(x*y)\ge\min\{\alpha_A(x),\alpha_A(y)\}$

($I\ S_2$) $\beta_A(x*y)\le\max\{\beta_A(x),\beta_A(y)\}$, for all $x, y\in X$.

Example 4.3:

Let $X=\{0,1,2,3,4\}$ as in example 2.4, and $A=(\alpha_A,\beta_A)$ be an IFS in X defined by $\alpha_A(0)=\alpha_A(2)=\alpha_A(3)=\alpha_A(4)=0.7<\ 0.3=\alpha_A(1)$, and $\beta_A(0)=\beta_A(2)=\beta_A(3)=\beta_A(4)=0.2<0.5=\beta_A(1)$. Then $A=(\alpha_A,\beta_A)$ is an intuitionistic fuzzy KU-subalgebra of *X*.

Lemma 4.4:

Every, intuitionistic fuzzy subalgebra $A=(\alpha_A,\beta_A)$ of *X* satisfies the inequalities $\alpha_A(0)\geq\alpha_A(x)$, and $\beta_A(0)\leq\beta_A(x)$ for all $x\in X$.

Proof:

$\alpha_A(0)=\alpha_A(x*x)\geq\min\{\alpha_A(x),\alpha_A(x)\}=\alpha_A(x),$ and

$\beta_A(0)=\beta_A(x*x)\leq\max\{\beta_A(x),\beta_A(x)\}=\beta_A(x).$ 1

Definition 4.5:

An IFS $A=(\alpha_A,\beta_A)$ in *X* is called an intuitionistic fuzzy KU-ideal of *X* if it satisfies the following inequalities:

(IFK$_1$) $\alpha_A(0)\geq\alpha_A(x)$ and $\beta_A(0)\leq\beta_A(x),$

(IFK$_2$) $\alpha_A(x*z)\geq\min\{\alpha_A(x*(y*z)),\alpha_A(y)\},$

(IFK$_3$) $\beta_A(x*z)\leq\max\{\beta_A(x*(y*z)),\beta_A(y)\},$ for all $x,y,z\in X$.

Example 4.6:

Let $X=\{0,1,2,3,4\}$ as in example 2.4, Define IFS $A=(\alpha_A,\beta_A)$ in *X* as follows $\alpha_A(0)=\alpha_A(2)=1,\ \alpha_A(1)=\alpha_A(3)=\alpha_A(3)=t$. $\beta_A(0)=\beta_A(2)=0,$ $\beta_A(1)=\beta_A(3)=\beta_A(4)=s$. Where $t,s\in[0,1]$ and $t+s\leq 1$. By routine calculations we know that $A=(\alpha_A,\beta_A)$ is an intuitionistic fuzzy KU-ideal of *X*.

Definition 4.7:

For any $t\in[0,1]$ and a fuzzy set μ in a nonempty set *X*, the set $U(\mu,t):=\{x\in X\mid\mu(x)\geq t\}$ is called an upper t-level cut of μ, and the set $L(\mu,t):=\{x\in X\mid\mu(x)\leq t\}$ is called a lower t-level cut of μ.

Theorem 4.8:

An IFS $A=(\alpha_A,\beta_A)$ is an intuitionistic fuzzy KU-ideal of *X* if and only if for all $s,t\in[0,1]$, the set $U(\alpha_A,t)$ and $L(\beta_A,s)$ are either empty or KU-ideal of *X*.

Proof. Let $A=(\alpha_A,\beta_A)$ be an intuitionistic fuzzy KU-ideal of *X* and $U(\alpha_A,t)\neq\phi\neq L(\beta_A,s)$. Since $\alpha_A(0)\geq t$ and $\beta_A(0)\leq s$, let $x,y,z\in X$ be such that $x*(y*z)\in U(\alpha_A,t)$. $y\in U(\alpha_A,t)$, then $\alpha_A(x*(y*z))\geq t$ and $\alpha_A(y)\geq t$, it follows that $\alpha_A(x*z)\geq\min\{\alpha_A(x*(y*z)),\alpha_A(y)\}\geq t$, so that $x*z\in U(\alpha_A,t)$.
Hence $U(\alpha_A,t)$ is an KU-ideal of *X*. let $x,y,z\in X$ be such that $x*(y*z)\in L(\beta_A,s)$ and $y\in L(\beta_A,s)$, then $\beta_A(x*(y*z))\leq s$ and $\beta_A(y)\leq s$ which imply that $\beta_A(x*z)\leq\max\{\beta_A(x*(y*z)),\beta_A(y)\}\leq s$. Thus $x*z\in L(\beta_A,s)$ and therefore $L(\beta_A,s)$ is a KU-ideal of *X*.
Conversely, assume that for each $s,t\in[0,1]$, the sets $U(\alpha_A,t)$ and $L(\beta_A,s)$ are either empty or KU-ideal of *X*. For any $x\in X$, let $\alpha_A(x)=t$ and $\beta_A(x)=s$. Then $x\in U(\alpha_A,t)\bigcap L(\beta_A,s)$ and so $U(\alpha_A,t)\neq\phi\neq L(\beta_A,s)$. Since $U(\alpha_A,t)$ and $L(\beta_A,s)$ are KU-ideal of *X*, therefore $0\in U(\alpha_A,t)\bigcap L(\beta_A,s)$. Hence $\alpha_A(0)\geq t=\alpha_A(x)$ and $\beta_A(0)\leq s=\beta_A(x)$ for all $x\in X$.
If there exist $x',y',z'\in X$ be such that $\alpha_A(x'*z')\geq\min\{\alpha_A(x'*(y'*z')),\alpha_A(y')\}$. Then by taking $t_0:=\frac{1}{2}\{\alpha_A(x'*z')+\min\{\alpha_A(x'*(y'*z')),\alpha_A(y')\}\}$, we get

$\alpha_A(x'*z') < t_0 < \min\{\alpha_A(x'*(y'*z')), \alpha_A(y')\}$ and hence $(x'*y') \notin U(\alpha_A, t_0)$, $x'*(y'*z') \in U(\alpha_A, t_0)$ and $y' \in U(\alpha_A, t_0)$, i.e. $U(\alpha_A, t_0)$ is not an KU-ideal of *X*, which make a contradiction. Finally assume that there exist $a, b, c \in X$ such that $\beta_A(a*c) > \max\{\beta_A(a*(b*c)), \beta_A(b)\}$. Then by taking $s_0 := \frac{1}{2}\{\beta_A(a*c) + \max\{\alpha_A(a*(b*c)), \beta_A(b)\}\}$, we get $\max\{\beta_A(a*(b*c)), \beta_A(b)\} < s_0 < \beta_A(a*c)$ therefore $(a*(b*c)) \in L(\beta_A, s_0)$ and $b \in L(\beta_A, s_0)$ but $(a*c) \notin L(\beta_A, s_0)$, which make a contradiction. This completes the proof.

5. Homomorphism of KU-algebra:

Definition 5.1:

Let $(X, *, 0)$ and $(Y, *', 0')$ be KU-algebras. A mapping $f: X \to Y$ is said to be a homomorphism if $f(x*y) = f(x)*' f(y)$ for all $x, y \in X$. Note that if $f: X \to Y$ is a homomorphism of KU-algebras, then $f(0) = 0'$. Let $f: X \to Y$ be a homomorphism of KU-algebras for any I F S $A = (\alpha_A, \beta_A)$ in *Y*, we define new I F S $A^f = (\alpha_A^f, \beta_A^f)$ in *X* by $\alpha_A^f(x) := \alpha_A(f(x))$, and $\beta_A^f(x) := \beta_A(f(x))$ for all $x \in X$.

Theorem 5.2:

Let $f: X \to Y$ be a homomorphism of KU-algebras. If the IFS $A = (\alpha_A, \beta_A)$, is an intuitionistic fuzzy KU-ideal of *Y*, then the IFS $A^f = (\alpha_A^f, \beta_A^f)$ in *X* is an intuitionistic fuzzy KU-ideal of *X*.

Proof. $\alpha_A^f(x) := \alpha_A(f(x)) \leq \alpha_A(0) = \alpha_A(f(0)) = \alpha_A^f(0)$, and $\beta_A^f(x) := \beta_A(f(x)) \geq \beta_A(0) = \beta_A(f(0)) = \beta_A^f(0)$, for all $x, y \in X$. And

$\alpha_A^f(x*z) := \alpha_A(f(x*z)) = \alpha_A(f(x)*' f(z)) \geq \min\{\alpha_A(f(x)*'(f(y)*' f(z))), \alpha_A(f(y))\}$
$= \min\{\alpha_A(f(x)*' f(y*z)), \alpha_A(f(y))\} = \min\{\alpha_A(f(x*(y*z)), \alpha_A(f(y))\}$
$= \min\{\alpha_A^f(x*(y*z)), \alpha_A^f(y)\}$, and $\beta_A^f(x*z) := \beta_A(f(x*z)) = \beta_A(f(x)*' f(z)) \leq$
$\max\{\beta_A((f(x)*' f(y))*' f(z)), \beta_A(f(y))\} = \max\{\beta_A(f(x*(y*z)), \beta_A(f(y))\}$
$= \max\{\beta_A(f(x*(y*z)), \beta_A(f(y))\} = \max\{\beta_A^f(x*(y*z)), \beta_A^f(y)\}$.

Hence $A^f = (\alpha_A^f, \beta_A^f)$ is an intuitionistic fuzzy KU-ideal in *X*.

6. Product of intuitionistic fuzzy KU-ideal.

Definition 6.1:

Let μ and λ be are two fuzzy sets in the set *X*. the Cartesian product $\lambda \times \mu : X \times X \to [0,1]$ is defined by, $(\lambda \times \mu)(x, y) = \min\{\lambda(x), \mu(y)\}$, and $\lambda_A \times \lambda_B : X \times X \to [0,1]$ is defined by $(\lambda_A \times \lambda_B)(x, y) = \max\{\lambda_A(x), \lambda_B(y)\}$ for all $x, y \in X$.

Definition 6.2:

Let $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ are two IFS of *X*, the Cartesian product $A \times B = (X \times X, \mu_A \times \mu_B, \lambda_A \times \lambda_B)$ is defined by $(\mu_A \times \mu_B)(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $(\lambda_A \times \lambda_B)(x, y) = \max\{\lambda_A(x), \lambda_B(y)\}$ where $\mu_A \times \mu_B : X \times X \to [0,1]$, for all $x, y \in X$.

Remark 6.3:

Let *X* and *Y* be a KU-algebras, we define $*$ on $X \times Y$ by: For every $(x, y), (u, v) \in X \times Y$ $(x, y)*(u, v) = (x*u, y*v)$ then clearly $(X*Y; *, (0,0))$ is KU-algebra.

Proposition 6.4
Let $A=(X,\lambda_A,\mu_A)$, $B=(X,\lambda_B,\mu_B)$ are intuitionistic fuzzy KU-ideal of *X*, then $A\times B$ is intuitionistic fuzzy KU-ideal of $X\times X$.

Proof. $\mu_A\times\mu_B(0,0)=\min\{\mu_A(0),\mu_B(0)\}\geq\min\{\mu_A(x),\mu_B(y)\}=\mu_A\times\mu_B(x,y)$, for all $x,y\in X$.
And $\lambda_A\times\lambda_B(0,0)=\max\{\lambda_A(0),\lambda_B(0)\}\leq\max\{\lambda_A(x),\lambda_B(y)\}=\lambda_A\times\lambda_B(x,y)$, for all $x,y\in X$.
Now let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$, then
$\min\{(\mu_A\times\mu_B)((x_1,x_2)*((y_1,y_2)*(z_1,z_2))),(\mu_A\times\mu_B)(y_1,y_2)\}$
$=\min\{(\mu_A\times\mu_B)((x_1,x_2)*((y_1,z_1)*(y_2,z_2)),(\mu_A\times\mu_B)(y_1,y_2)\}$
$=\min\{(\mu_A\times\mu_B)(x_1*(y_1*z_1),x_2*(y_2*z_2)),(\mu_A\times\mu_B)(y_1,y_2)\}$
$=\min\{\{\mu_A(x_1*(y_1*z_1)),\mu_B(x_2*(y_2*z_2))\},\{\mu_A(y_1),\mu_B(y_2)\}\}$
$=\min\{\min\{\mu_A(x_1*(y_1*z_1)),\mu_A(y_1)\},\min\{\mu_B(x_2*(y_2*z_2)),\mu_B(y_2)\}$
$=\min\{\min\{\mu_A(x_1*(y_1*z_1)),\mu_A(y_1)\},\min\{\mu_B(x_2*(y_2*z_2)),\mu_B(y_2)\}\leq\min\{\mu_A(x_1*z_1),\mu_B(x_2*z_2)\}$
$=(\mu_A\times\mu_B)(x_1*z_1,x_2*z_2)$. And $\max\{(\lambda_A\times\lambda_B)((x_1,x_2)*((y_1,y_2)*(z_1,z_2))),(\lambda_A\times\lambda_B)(y_1,y_2)\}$
$=\max\{(\lambda_A\times\lambda_B)((x_1,x_2)*((y_1*z_1),(y_2*z_2))),(\lambda_A\times\lambda_B)(y_1,y_2)\}$
$=\max\{(\lambda_A\times\lambda_B)(x_1*(y_1*z_1),x_2*(y_2*z_2)),(\lambda_A\times\lambda_B)(y_1,y_2)\}$
$=\max\{\{\lambda_A(x_1*(y_1*z_1)),\lambda_B(x_2*(y_2*z_2))\},\{\lambda_A(y_1),\lambda_B(y_2)\}\}$
$=\max\{\max\{\lambda_A(x_1*(y_1*z_1)),\lambda_A(y_1)\},\max\{\lambda_B(x_2*(y_2*z_2)),\lambda_B(y_2)\}$
$=\max\{\max\{\lambda_A(x_1*(y_1*z_1)),\lambda_A(y_1)\},\max\{\lambda_B(x_2*(y_2*z_2)),\lambda_B(y_2)\}\geq\max\{\lambda_A(x_1*z_1),\lambda_B(x_2*z_2)\}=$
$(\lambda_A\times\lambda_B)(x_1*z_1,x_2*z_2)$. This completes the proof.

Definition 6.5:
Let $A=(X,\lambda_A,\mu_A)$ and $B=(X,\lambda_B,\mu_B)$ are intuitionistic fuzzy subset of KU-algebras *X*. for $s,t\in[0,1]$ the set $U(\mu_A\times\mu_B,s):=\{(x,y)\in X\times X\mid(\mu_A\times\mu_B)(x,y)\geq s\}$ is called upper level of $(\mu_A\times\mu_B)(x,y)$ and $L(\lambda_A\times\lambda_B,t):=\{(x,y)\in X\times X\mid(\lambda_A\times\lambda_B)(x,y)\leq t\}$ is called lower level of $(\lambda_A\times\lambda_B)(x,y)$.

Theorem 6.6:
An intuitionistic fuzzy set $A=(X,\lambda_A,\mu_A)$ and $B=(X,\lambda_B,\mu_B)$ are intuitionistic fuzzy KU-ideal of *X* if and only if the non empty set upper *s*-level cut $U(\mu_A\times\mu_B,s)$ and the non empty *t*-level cut $L(\lambda_A\times\lambda_B,t)$ are KU-ideal of $X\times X$ for any $s,t\in[0,1]$.

Proof. Let $A=(X,\lambda_A,\mu_A)$ and $B=(X,\lambda_B,\mu_B)$ are intuitionistic fuzzy KU-ideal of *X*, therefore for any $(x,y)\in X\times X$, $\mu_A\times\mu_B(0,0)=\min\{\mu_A(0),\mu_B(0)\}\geq\min\{\mu_A(x),\mu_B(y)\}=\mu_A\times\mu_B(x,y)$ and for $s\in[0,1]$, if $(\mu_A\times\mu_B)(x_1*x_2,z_1*z_2)=(\mu_A\times\mu_B)(x_1*z_1,x_2*z_2)\geq s$, therefore $(x_1*z_1,x_2*z_2)\in U(\mu_A\times\mu_B,s)$ .Let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$ be such that $((x_1,x_2)*((y_1,y_2)*(z_1,z_2)))\in U(\mu_A\times\mu_B,s)$, and $(y_1,y_2)\in U(\mu_A\times\mu_B,s)$.
Now
$(\mu_A\times\mu_B)((x_1,x_2)*(z_1,z_2))=(\mu_A\times\mu_B)(x_1*z_1,x_2*z_2)\geq$
$\min\{(\mu_A\times\mu_B)((x_1,x_2)*((y_1,y_2)*(z_1,z_2))),(\mu_A\times\mu_B)(y_1,y_2)\}=$

$\min\{(\mu_A \times \mu_B)(x_1 * (y_1 * z_1), x_2 * (y_2 * z_2)), (\mu_A \times \mu_B)(y_1, y_2)\} \geq \min\{s, s\} = s$, therefore $(x_1, x_2) * (z_1, z_2) \in U((\mu_A \times \mu_B)(x, y), s)$ is KU-ideal of $X \times X$. In a similar way, we can prove that $L((\lambda_A \times \lambda_B)(x, y), t)$ is a KU-ideal of $X \times X$. This completes the proof.

References

[1] K. Atanassov, intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1) (1986), 87-96

[2] K. Atanassov, New operations defined over the intuitionistic fuzzy sets, fuzzy sets and systems, 61 (2) 1994, 137-142.

[3] D. Basnet, intuitionistic fuzzy ideals, J. of fuzzy mathematics, 15 (4) 2007, 811-819.

[4] B. Benerjee and D. Basnet, intuitionistic fuzzy subrings and ideals, J. of fuzzy mathematics, 11 (1) 2003, 139-155.

[5] W. A. Dudek and X. Zhang, On ideal and congruences in BCC-algebras, Czechoslovak Math. Journal,48(1998), No. 123, 21-29.

[6] W. A. Dudek, On proper BCC-algebras, Bull. Ins. Math. Academic Science, 20(1992),137-150.

[7] Samy M. Mostafa, Mokhtar A. Abdel Naby and Moustafa M. Youssef Fuzzy KU-ideals in KU-algebras. Submitted.

[8] C. Prabpayak and U. Leerawat, On ideals and congruences in KU-algebras, Scientia Magna Journal, 5(2009), No. 1, 54-57.

[9] G. TaKeuti and S. Titants, Intuitionistic fuzzy Logic and Intuitionistic fuzzy set theory, Journal of Symbolic Logic, 49 (1984), 851-866.

[10] O. G. Xi, Fuzzy BCK-algebra, Math. Japon. 36 (1991) 935-942.

[11] L. A. Zadeh, Fuzzy sets, Inform. And Control 8 (1965) 338-353.