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On Slightly continuous functions via $\tilde{g}_\alpha$-sets

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Abstract

The aim of this paper is to define a new class of functions namely slightly $\tilde{g}_\alpha$-continuous functions and study its properties. We introduce $\tilde{g}_\alpha$-connected, $\tilde{g}_\alpha$-normal and $\tilde{g}_\alpha$-compact spaces and study their properties in terms of slightly $\tilde{g}_\alpha$-continuous functions. We prove that the slightly $\tilde{g}_\alpha$-continuous function is weaker than $\tilde{g}_\alpha$-continuous functions.

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1 Introduction

The generalized closed sets has been extended to study the characterization of continuity, compactness, connectedness etc. Slightly continuity was introduced by Jain[4] and has been applied for semi-open and pre-open sets by Nour[5] and Baker[1] respectively. Jafari et al. [3] have introduced $\tilde{g}_\alpha$-closed sets in topology and proved that it forms a topology. In this paper slightly $\tilde{g}_\alpha$-continuous functions has been defined and proved that it includes the class of slightly continuous and $\tilde{g}_\alpha$-continuous functions. We investigated the properties of slightly $\tilde{g}_\alpha$-continuous functions in terms of composition and restriction. The behavior of $\tilde{g}_\alpha$-compactness and $\tilde{g}_\alpha$-connectedness is studied under slightly $\tilde{g}_\alpha$-continuous functions.

2 Preliminaries

We list some definitions in a topological space (X, τ) which are useful in the following sections. The interior and the closure of a subset A of (X, τ) are denoted by Int(A) and Cl(A), respectively. Throughout the present paper (X, τ) and (Y, σ)(or X and Y) represent topological spaces on which no separation axiom is defined, unless otherwise mentioned.

Definition 2.1 A subset A of a topological space (X, τ) is called

(i) an ω-closed set [7] (= $\hat{g}-$ closed) if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ),
(ii) a *g-closed set[10] if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ω-open in (X, τ) ,
(iii) a #g-semi-closed set(briefly #gs-closed)[11] if sCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is *g-open in (X, τ) and
(iv) a $\tilde{g}_\alpha$-closed set[3] if αCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is #gs-open in (X, τ)

The complement of ω-closed(resp *g-closed, #gs-closed, $\tilde{g}_\alpha$-closed)set is said to be ω-open (resp *g-open,

$^{\#}$gs-open, $\widetilde{g}_{\alpha}$-open)

Definition 2.2 A function f: $(X,\tau) \to (Y,\sigma)$ is called

(i) $\widetilde{g}_{\alpha}$-continuous[8] if the inverse image of every closed set in Y is $\widetilde{g}_{\alpha}$-closed in X.

(ii) $\widetilde{g}_{\alpha}$-irresolute[8] if the inverse image of every $\widetilde{g}_{\alpha}$-closed set in Y is $\widetilde{g}_{\alpha}$-closed in X.

(iii) $\widetilde{g}_{\alpha}$-closed[9] if the image of every closed set in X is $\widetilde{g}_{\alpha}$-closed in Y.

(iv) strongly $\widetilde{g}_{\alpha}$-closed[9] if the image of every $\widetilde{g}_{\alpha}$-closed set in X is $\widetilde{g}_{\alpha}$-closed in Y.

(v) slightly continuous[4] if the inverse image of every clopen set in Y is open in X.

Definition 2.3[3] Let (X, **τ)** be a topological space and E be a subset of X. We define the $\widetilde{g}_{\alpha}$-Interior of E denoted by $\widetilde{g}_{\alpha}$-Int(E) to be the union of all $\widetilde{g}_{\alpha}$-open sets contained in E. Similarly the $\widetilde{g}_{\alpha}$-closure of E denoted as $\widetilde{g}_{\alpha}$-Cl(E) is defined to be the intersection of all $\widetilde{g}_{\alpha}$-closed sets containing E.

Remark 2.4 The collection of all $\widetilde{g}_{\alpha}$-closed ($\widetilde{g}_{\alpha}$-open sets) are denoted by $\widetilde{G}_{\alpha}$C(X) ($\widetilde{G}_{\alpha}$O(X)) respectively. Similarly the $\widetilde{g}_{\alpha}$–clopen subsets of X is denoted by $\widetilde{G}_{\alpha}$CO(X) and the clopen subsets of X is denoted as CO(X).

3 Slightly $\widetilde{g}_{\alpha}$-continuous function

Definition 3.1 A function f: $(X,\tau) \to (Y,\sigma)$ is said to be slightly $\widetilde{g}_{\alpha}$-continuous at a point x$\in$X if for each clopen subset V of Y containing f(x), there exists a $\widetilde{g}_{\alpha}$-open subset U in X containing x such that f(U)$\subseteq$V. T he function f is said to be slightly $\widetilde{g}_{\alpha}$ continuous if f is slightly $\widetilde{g}_{\alpha}$-continuous at each of its points.

Definition 3.2 A function f: $(X,\tau) \to (Y,\sigma)$ is said to be slightly $\widetilde{g}_{\alpha}$-continuous if the inverse image of every clopen set in Y is $\widetilde{g}_{\alpha}$-clopen in X.

Example 3.3 Let X={a,b,c}=Y, τ ={ϕ, X,{a},{b},{a,c},{a,b}}, σ ={ϕ,Y,{a},{b,c}}. $\widetilde{G}_{\alpha}$O(X)={ϕ,X,{a},[b},{a,c},{a,b}}. The function f is defined as f(a)=c,f(b)=a,f(c)=b. The function f is slightly $\widetilde{g}_{\alpha}$-continuous .

Proposition 3.4 The definitions 3.1 and 3.2 are equivalent.

Proof. Suppose that definition 3.1 holds. Let V be a clopen set in Y and let x$\in$ f^{-1}(V). Then f(x) $\in$V and thus there exists a $\widetilde{g}_{\alpha}$-open set U_x such that x$\in U_x$ and f(U_x)$\subseteq$V. Now x$\in U_x \subseteq f^{-1}$(V) and f^{-1}(V)=$\bigcup_{x\in f^{-1}(V)} U_x$. Since arbitrary union of $\widetilde{g}_{\alpha}$-open sets is $\widetilde{g}_{\alpha}$-open, f^{-1}(V) is $\widetilde{g}_{\alpha}$-open in X and therefore f is slightly $\widetilde{g}_{\alpha}$-continuous.

Let definition 3.2 holds and f(x) $\in$V where V is a clopen set in Y. Since f is slightly $\widetilde{g}_{\alpha}$-continuous, we x$\in$ f^{-1}(V) where f^{-1}(V) is $\widetilde{g}_{\alpha}$-open . Let U= f^{-1}(V) . Then U is $\widetilde{g}_{\alpha}$-open in X, x$\in$U and f(U) $\subseteq$V.

Theorem 3.5 Let f: $(X,\tau) \to (Y,\sigma)$ be a function then the following are equivalent.

(i) f is slightly $\widetilde{g}_{\alpha}$-continuous.

(ii) The inverse image of every clopen set V of Y is $\widetilde{g}_{\alpha}$-open in X.

(iii) The inverse image of every clopen set V of Y is $\widetilde{g}_{\alpha}$-closed in X.

(iv) The inverse image of every clopen set V of Y is $\widetilde{g}_{\alpha}$-clopen in X.

Proof. (i)$\Leftrightarrow$ (ii) Follows from the Proposition 3.4.

(ii)$\Rightarrow$(iii) Let V be clopen in Y which implies that V^c is clopen in Y. By (ii) f^{-1}(V^c) is $\widetilde{g}_{\alpha}$-open in X. Since (f^{-1}(V))^c= f^{-1}(V^c), f^{-1}(V) is $\widetilde{g}_{\alpha}$-closed.

(iii)$\Rightarrow$(iv) By (ii) and (iii) f^{-1}(V) is $\widetilde{g}_{\alpha}$-clopen in X.

(iv)$\Rightarrow$(i) Let V be a clopen subset of Y containing f(x). By (iv) f^{-1}(V) is $\tilde{g}_\alpha$-clopen in X. Put U= f^{-1}(V) then f(U)$\subseteq$V. Hence f is slightly $\tilde{g}_\alpha$-continuous.

4 Comparison

Proposition 4.1 Every slightly continuous function is slightly $\tilde{g}_\alpha$-continuous.

Proof. Let U be a clopen set in Y then f^{-1}(U) is open in X. Since every open set is $\tilde{g}_\alpha$-open, f^{-1}(U) is $\tilde{g}_\alpha$-open. Hence f is slightly $\tilde{g}_\alpha$-continuous.

Remark 4.2 The converse of the proposition 4.1 need not be true.

Example 4.3 Let X={a,b,c,d},Y={p,q,r},
τ={ϕ, X,{b,c},{b,c,d},{a,b,c}}, σ={ϕ,Y,{p},{q,r}}.

The function f is defined as f(a)=p,f(b)=q,f(c)=f(d)=r. The function f is slightly $\tilde{g}_\alpha$-continuous but not slightly continuous.

Proposition 4.4 Every $\tilde{g}_\alpha$-continuous function is slightly $\tilde{g}_\alpha$-continuous.

Proof. Let U be a clopen set in Y then f^{-1}(U) is $\tilde{g}_\alpha$-open in X. Hence f is slightly $\tilde{g}_\alpha$-continuous.

Remark 4.5 The converse of the Proposition 4.4 need not be true.

Example 4.6 Let X={a,b,c},Y={p,q},
τ={ϕ, X,{a},{b},{a,b}}, σ={ϕ,Y,{p}}.

The function f is defined as f(a)=q,f(b)=f(c)=p. The function f is slightly $\tilde{g}_\alpha$-continuous but not $\tilde{g}_\alpha$-continuous. Since f^{-1}({p})={b,c} is not $\tilde{g}_\alpha$-open in X.

Definition 4.7 A space (X, τ) is called

(i) a locally indiscrete space if every open subset is closed[2].

(ii) a $^{\#}T_{\tilde{g}_\alpha}$-space if every $\tilde{g}_\alpha$-closed subset in it is closed[3].

Theorem 4.8 If the function f: $(X,\tau)\to(Y,\sigma)$ is slightly $\tilde{g}_\alpha$-continuous and (Y, σ) is a locally indiscrete space then f is $\tilde{g}_\alpha$-continuous.

Proof. Let U be an open subset of Y. Since Y is locally indiscrete U is closed in Y. Since f is slightly $\tilde{g}_\alpha$-continuous f^{-1}(U) is $\tilde{g}_\alpha$-open in X. Hence f is $\tilde{g}_\alpha$-continuous.

Theorem 4.9 If the function f: $(X,\tau)\to(Y,\sigma)$ is slightly $\tilde{g}_\alpha$-continuous and (X, τ) is a $^{\#}T_{\tilde{g}_\alpha}$-space then f is slightly continuous.

Proof. Let U be a clopen subset of Y. Since f is slightly $\tilde{g}_\alpha$-continuous f^{-1}(U) is $\tilde{g}_\alpha$-open in X. Since X is a $^{\#}T_{\tilde{g}_\alpha}$, f^{-1}(U) is open in X. Hence f is slightly continuous.

Hence f is $\tilde{g}_\alpha$-continuous.

Theorem 4.10 Let f: $(X,\tau)\to(Y,\sigma)$ and g: (Y, σ)$\to(Z,\eta)$ be functions.

(i) If f is $\tilde{g}_\alpha$-irresolute and g is slightly $\tilde{g}_\alpha$-continuous thengof: (X, τ)$\to(Z,\eta)$ is slightly $\tilde{g}_\alpha$-continuous.

(ii) If f is $\tilde{g}_\alpha$-irresolute and g is $\tilde{g}_\alpha$-continuous then gof: (X, τ)$\to(Z,\eta)$ is slightly $\tilde{g}_\alpha$-continuous.

(iii) If f is $\tilde{g}_\alpha$-irresolute and g is slightly continuous then gof: (X, τ)$\to(Z,\eta)$ is slightly $\tilde{g}_\alpha$-continuous.

Proof. (i) Let U be a clopen set in (Z,η). Then $g^{-1}(U)$ is $\tilde{g}_\alpha$-open in Y. Since f is $\tilde{g}_\alpha$-irresolute $f^{-1}(g^{-1}(U))$ is $\tilde{g}_\alpha$-open in X. Since $(gof)^{-1}(U)=$ $f^{-1}(g^{-1}(U))$, gof is slightly $\tilde{g}_\alpha$-continuous.

(ii) Let U be a clopen set in (Z, η). Then $g^{-1}(U)$ is $\widetilde{g}_{\alpha}$-open in Y. Since f is $\widetilde{g}_{\alpha}$-irresolute $f^{-1}(g^{-1}(U))$ is $\widetilde{g}_{\alpha}$-open in X. Since $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$, gof is slightly $\widetilde{g}_{\alpha}$-continuous.

(iii) Let U be a clopen set in (Z, η). Then $g^{-1}(U)$ is open in Y and any open set is $\widetilde{g}_{\alpha}$-open, $f^{-1}(g^{-1}(U))$ is $\widetilde{g}_{\alpha}$-open in X. Since $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$, gof is slightly $\widetilde{g}_{\alpha}$-continuous.

Theorem 4.11 Let f: $(X,\tau) \to (Y,\sigma)$ and g: (Y, σ) $\to (Z,\eta)$ be functions. If f is surjective and strongly $\widetilde{g}_{\alpha}$-open and gof: (X, τ) $\to (Z,\eta)$ is slightly $\widetilde{g}_{\alpha}$-continuous then g is slightly $\widetilde{g}_{\alpha}$-continuous.

Proof. Let U be a clopen set in (Z, η) then $f^{-1}(g^{-1}(U))$ is $\widetilde{g}_{\alpha}$-open in X. Since f is surjective f($f^{-1}(g^{-1}(U)) = g^{-1}(U)$ is $\widetilde{g}_{\alpha}$-open in Y. Hence g is slightly $\widetilde{g}_{\alpha}$-continuous.

Theorem 4.12 Let f: $(X,\tau) \to (Y,\sigma)$ and g: (Y, σ) $\to (Z,\eta)$ be functions. If f is surjective and strongly $\widetilde{g}_{\alpha}$-open and $\widetilde{g}_{\alpha}$-irresolute then gof: (X, τ) $\to (Z,\eta)$ is slightly $\widetilde{g}_{\alpha}$-continuous if and only if g is slightly $\widetilde{g}_{\alpha}$-continuous.

Proof. Let U be a clopen set in (Z, η) then $f^{-1}(g^{-1}(U))$ is $\widetilde{g}_{\alpha}$-open in X. Since f is surjective f($f^{-1}(g^{-1}(U)) = g^{-1}(U)$ is $\widetilde{g}_{\alpha}$-open in Y. Hence g is slightly $\widetilde{g}_{\alpha}$-continuous.

Conversely let g be slightly $\widetilde{g}_{\alpha}$-continuous and U be a clopen set in (Z, η) then $g^{-1}(U)$ is $\widetilde{g}_{\alpha}$-open in Y. Since f is $\widetilde{g}_{\alpha}$-irresolute $f^{-1}(g^{-1}(U))$ is $\widetilde{g}_{\alpha}$-open in X. Hence gof is slightly $\widetilde{g}_{\alpha}$-continuous.

5 Applications

Lemma 5.1[3] Let A $\subseteq$ B $\subseteq$ $\widetilde{G}_{\alpha}$ O(B) and B is open in X then A is $\widetilde{g}_{\alpha}$-open in X.

Theorem 5.2 If f: $(X,\tau) \to (Y,\sigma)$ is slightly $\widetilde{g}_{\alpha}$-continuous and A is an open subset of X then the restriction $f_{/A} : (A, \tau_A) \to (Y,\sigma)$ is slightly $\widetilde{g}_{\alpha}$-continuous.

Proof. Let V be a clopen subset of Y. Then $(f_{/A})^{-1}(V) = f^{-1}(V) \cap A$. Since f^{-1}(V) is $\widetilde{g}_{\alpha}$-open and A is open $(f_{/A})^{-1}(V)$ is $\widetilde{g}_{\alpha}$-open in the relative topology of A. Hence $f_{/A}$ is slightly $\widetilde{g}_{\alpha}$-continuous.

Theorem 5.3 Let f: $(X,\tau) \to (Y,\sigma)$ be a function and $\Lambda = \{U_i : i \in I\}$ be a $\widetilde{g}_{\alpha}$-open cover of X. If $f_{/U_i}$ is slightly $\widetilde{g}_{\alpha}$-continuous for each i $\in$ I then f is a slightly $\widetilde{g}_{\alpha}$-continuous function.

Proof. Let V be any clopen subset of Y. Since $f_{/U_i}$ is slightly $\widetilde{g}_{\alpha}$-continuous for each i $\in$ I, it follows that $(f_{/U_I})^{-1}(V)$ is $\widetilde{g}_{\alpha}$-open in U_i. We have $f^{-1}(V) = \bigcup_{i\in I}(f^{-1}(V) \cap U_i) = \bigcup_{i\in I}(f_{/U_i})^{-1}(V)$. Then by lemma 5.1 f^{-1}(V) is $\widetilde{g}_{\alpha}$-open in X. Hence f is slightly $\widetilde{g}_{\alpha}$-continuous.

Definition 5.4 A space (X, τ) is called $\widetilde{g}_{\alpha}$-connected if X is not the union of two disjoint non empty $\widetilde{g}_{\alpha}$-open sets.

Theorem 5.5 If f: $(X,\tau) \to (Y,\sigma)$ is a slightly $\widetilde{g}_{\alpha}$-continuous surjective function and X is $\widetilde{g}_{\alpha}$-connected then Y is connected.

Proof. Suppose Y is not connected. Then there exists non-empty disjoint open sets U and V such that Y=U $\cup$ V. Therefore U and V are clopen sets in Y. Since f is slightly $\widetilde{g}_{\alpha}$-continuous, f^{-1}(U), f^{-1}(V) are non empty disjoint $\widetilde{g}_{\alpha}$-open sets in X. Also f^{-1}(Y)=X= f^{-1}(U) $\cup$ f^{-1}(V). This shows that X is not $\widetilde{g}_{\alpha}$-connected, a contradiction and hence Y is connected.

Theorem 5.6 If f is a slightly $\widetilde{g}_{\alpha}$-continuous function from a $\widetilde{g}_{\alpha}$-connected space (X, τ) onto a space (Y, σ) then Y is not a discrete space.

Proof. Suppose that Y is a discrete space. Let A be a proper nonempty open subset of Y. Then f^{-1}(A) is a proper nonempty $\widetilde{g}_\alpha$-clopen subset of X, which is a contradiction to the assumption that X is $\widetilde{g}_\alpha$-connected.

Theorem 5.7 A space X is $\widetilde{g}_\alpha$-connected if every slightly $\widetilde{g}_\alpha$-continuous function from X into any T_0 space Y is constant.

Proof. Let every slightly $\widetilde{g}_\alpha$-continuous function from a space X into Y be constant. If X is not $\widetilde{g}_\alpha$-connected there exists a proper nonempty $\widetilde{g}_\alpha$-clopen subset A of X. Let (Y, σ) be such that Y={a,b}, $\tau = \{\phi$, Y,{a},{b}} be a topology. Let f:X→Y be any function such that f(A)={a} and f(X-A)={b}. Then f is a non-constant and slightly $\widetilde{g}_\alpha$-continuous function such that Y is T_0 which is a contradiction. Hence X is $\widetilde{g}_\alpha$-connected.

Theorem 5.8 If a function f:X→$\prod Y_\alpha$ is slightly $\widetilde{g}_\alpha$-continuous, then P_α o f:X→Y_α is slightly $\widetilde{g}_\alpha$-continuous for each $\alpha \in \Lambda$, where each P_α is the projection of $\prod Y_\alpha$ onto Y_α.

Proof. Let V_α be any clopen subset of Y_α. Then $P_\alpha^{-1}(V_\alpha)$ is clopen in $\prod Y_\alpha$ and hence (P_α o f)$^{-1}$(V_α)= $f^{-1}(P_\alpha^{-1}(V_\alpha))$ is $\widetilde{g}_\alpha$-open in X. Therefore P_α o f is slightly $\widetilde{g}_\alpha$-continuous.

Theorem 5.9 If a function f:$\prod X_\alpha \to \prod Y_\alpha$ is slightly $\widetilde{g}_\alpha$-continuous then $f_\alpha : X_\alpha \to Y_\alpha$ is slightly $\widetilde{g}_\alpha$-continuous for each $\alpha \in \Lambda$.

Proof. Let V_α be any clopen subset of Y_α. Then $P_\alpha^{-1}(V_\alpha)$ is clopen in $\prod Y_\alpha$ and hence (P_α o f)$^{-1}$(V_α)= $f^{-1}(P_\alpha^{-1}(V_\alpha)) = f_\alpha^{-1}(V_\alpha) \times \prod\{X_\alpha : \alpha \in \Lambda - \{\alpha\}\}$. Since f is slightly $\widetilde{g}_\alpha$-continuous $f^{-1}(P_\alpha^{-1}(V_\alpha))$ is $\widetilde{g}_\alpha$-open in $\prod X_\alpha$ and the projection P_α of $\prod X_\alpha$ onto X_α is open and continuous, $f_\alpha^{-1}(V_\alpha)$ is $\widetilde{g}_\alpha$-open in X_α and hence f_α is slightly $\widetilde{g}_\alpha$-continuous.

Definition 5.10 A space (X, τ) is said to be

(i) mildly compact[6] if every clopen cover of X has a finite subcover.

(ii) $\widetilde{g}_\alpha$-compact[8] if every $\widetilde{g}_\alpha$-open cover of X has a finite subcover.

(iii) A subset A of a space X is said to be mildly compcat[6] relative to X if every cover of A by clopen sets of X has a finite subcover.

(iv) A subset A of a space X is said to be $\widetilde{g}_\alpha$-compact[8] relative to X if every cover of A by $\widetilde{g}_\alpha$-open sets of X has a finite subcover.

(v) A subset A of a space X is said to be mildly compact[6] if the subspace A is mildly compact.

(vi) A subset A of a space X is said to be $\widetilde{g}_\alpha$-compact[8] if the subspace A is $\widetilde{g}_\alpha$-compact.

Theorem 5.11 If a function f: $(X,\tau) \to (Y,\sigma)$ is a slightly $\widetilde{g}_\alpha$-continuous and A is $\widetilde{g}_\alpha$-compact relative to X then f(A) is mildly compact in Y.

Proof. Let $\{H_\alpha : \alpha \in I\}$ be any cover of f(A) by clopen sets of the subspace f(A).. For each $\alpha \in$ I there exists a clopen set A_α of Y such that $H_\alpha = A_\alpha \cap f(A)$. For each x∈A, there exists $\alpha_x \in I$ such that f(x) $\in A_{\alpha_x}$ and there exists $U_x \in \widetilde{G}_\alpha$O(X) containing x such that $f(U_x) \subseteq A_{\alpha_x}$. Since the family $\{U_x$:x∈A} is a cover of A by $\widetilde{g}_\alpha$-open sets of X, there exists a finite subset A_0 of A such that $A \subseteq \{U_x : x \in A_0\}$. Therefore we get $f(A) \subseteq \bigcup\{f(U_x : x \in A_0\}$ which is a subset of $\bigcup\{A_{\alpha_x} : x \in A_0\}$. Thus f(A)= $\bigcup\{A_{\alpha_x} : x \in A_0\}$ and f(A) is mildly compact.

Corollary 5.12 If f: $(X,\tau) \to (Y,\sigma)$ is a slightly $\widetilde{g}_\alpha$-continuous, surjective and X is $\widetilde{g}_\alpha$-compact then Y is mildly compact.

Proof. Let {V_α: V_α ∈CO(Y), α ∈I} be a cover of Y. Since f is slightly $\widetilde{g}_\alpha$-continuous

{ $f^{-1}(V_\alpha)$: $\alpha \in$ I} be a $\tilde{g}_\alpha$-open cover of X. So there is a finite subset I_0 of I such that X=$\bigcup_{\alpha\in I_0} f^{-1}(V_\alpha)$. Then Y=$\bigcup_{\alpha\in I_0} V_\alpha$ (since f is surjective). Thus Y is mildly compact.

Definition 5.13 A space X is said to be
(i) mildly countably compact[6] if every countable clopen cover of X has a finite subcover.
(ii) Mildly Lindelof[6] if every cover of X by clopen sets has a countable sub cover.
(iii) Countably $\tilde{g}_\alpha$-compact if every $\tilde{g}_\alpha$-open cover of X has a finite subcover.
(iv) $\tilde{g}_\alpha$-closed compact if every $\tilde{g}_\alpha$-closed cover of X has a finite subcover.
(v) $\tilde{g}_\alpha$-Lidelof if every $\tilde{g}_\alpha$-open cover of X has a countable subcover.
(vi) Countably $\tilde{g}_\alpha$-closed compact if every countable cover of X by $\tilde{g}_\alpha$-closed sets has a finite subcover.
(vii) $\tilde{g}_\alpha$-closed Lindelof if every $\tilde{g}_\alpha$-closed cover of X has a countable subcover.

Theorem 5.14 Let f: $(X,\tau)\to(Y,\sigma)$ be a slightly $\tilde{g}_\alpha$-continuous surjective function. Then the following hold.
(i) If X is $\tilde{g}_\alpha$-Lindelof then Y is mildly Lindelof.
(ii) If X is countably $\tilde{g}_\alpha$-compact then Y is mildly countably compact.

Proof. (i) Let $\{V_\alpha : \alpha \in$ I} be a clopen cover of Y. Since f is slightly $\tilde{g}_\alpha$-continuous surjection then { $f^{-1}(V_\alpha)$: $\alpha \in$ I} is a $\tilde{g}_\alpha$-open cover of X. Since X is $\tilde{g}_\alpha$-Lindelof there exists a countable subset I_0 of I such that X=$\bigcup$ { $f^{-1}(V_\alpha)$: $\alpha \in I_0$ }. Thus Y=$\bigcup$ $\{V_\alpha : \alpha \in I_0$ } and hence Y is mildly Lidelof.
(ii)Let $\{V_\alpha : \alpha \in$ I} be a clopen cover of Y. Since f is slightly $\tilde{g}_\alpha$-continuous surjection then { $f^{-1}(V_\alpha)$: $\alpha \in$ I} is a $\tilde{g}_\alpha$-open countable cover of X. Since X is countably $\tilde{g}_\alpha$-Lindelof there exists a finite subcover of { $f^{-1}(V_\alpha)$: $\alpha \in$ I} such that X=$\bigcup_{i=1}^{n} f^{-1}(V_i)$. Thus Y= $\bigcup_{i=1}^{n} V_i$ and hence Y is mildly countably compact.

Theorem 5.15 Let f: $(X,\tau)\to(Y,\sigma)$ be a slightly $\tilde{g}_\alpha$-continuous surjective function. Then the following hold.
(i) If X is $\tilde{g}_\alpha$-closed compact, then Y is mildly compact.
(ii) If X is $\tilde{g}_\alpha$-closed Lindelof then Y is mildly Lindelof.
(iii) If X is countably $\tilde{g}_\alpha$-closed compact, then Y is mildly countably compact.

Proof. (i)Let $\{V_\alpha : \alpha \in$ I} be a clopen cover of Y. Since f is slightly $\tilde{g}_\alpha$-continuous surjection then { $f^{-1}(V_\alpha)$: $\alpha \in$ I} is a $\tilde{g}_\alpha$-closed cover of X. Since X is $\tilde{g}_\alpha$-closed compact there exists a finite sub cover of { $f^{-1}(V_\alpha)$: $\alpha \in$ I} such that X=$\bigcup_{i=1}^{n} f^{-1}(V_i)$. Thus Y= $\bigcup_{i=1}^{n} V_i$ and hence Y is mildly compact.
(ii)Let $\{V_\alpha : \alpha \in$ I} be a clopen cover of Y. Since f is slightly $\tilde{g}_\alpha$-continuous surjection then { $f^{-1}(V_\alpha)$: $\alpha \in$ I} is a $\tilde{g}_\alpha$-closed cover of X. Since X is $\tilde{g}_\alpha$-closed Lindelof there exists a countable subset I_0 of I such that X=$\bigcup$ { $f^{-1}(V_\alpha)$: $\alpha \in I_0$ }. Thus Y=$\bigcup$ $\{V_\alpha : \alpha \in I_0$ } and hence Y is mildly Lidelof.
(iii)Let $\{V_\alpha : \alpha \in$ I} be a countable clopen cover of Y. Since f is slightly $\tilde{g}_\alpha$-continuous surjection then { $f^{-1}(V_\alpha)$: $\alpha \in$ I} is a countable $\tilde{g}_\alpha$-closed cover of X. Since X is countably $\tilde{g}_\alpha$-closed compact, there

exists a finite sub cover of { $f^{-1}(V_{\alpha}): \alpha \in$ I} such that X=$\bigcup_{i=1}^{n} f^{-1}(V_i)$. Thus Y= $\bigcup_{i=1}^{n} V_i$ and hence Y is mildly countably compact.

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