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ORDERED FUZZY SCALABLE PRESEMI λ COMPACT SPACES

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Abstract

In this article, the initiation of ordered fuzzy scalable structure spaces is given and FQ_X^{**} presemi λ compactness is explored. Also some interesting properties of ordered fuzzy presemi λ - compact spaces are studied.

Keywords: Ordered fuzzy scalable structure spaces, FQ_X^{**} presemi λ cover and FQ_X^{**} presemi λ compact space

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1 Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper [14] in 1965. Fuzzy sets have applications in many fields such as information [9] and control [12]. The theory of fuzzy topological spaces was introduced and developed by Chang [5] and since then various notions in classical topology have been extended to fuzzy topological spaces. In 1968 Chang [4] was the first who used the term fuzzy compact. The concept of fuzzy paracompactness was introduced by M. K. Luo[7]. S.J. John and T. Baiju [6] introduced the concept of metacompactness in L-topological spaces. The idea of scale Q in a topological space was introduced by M. Burgin[2]. The notions of fuzzy scalable structure space and a FQ_X^{**} presemi λ closed sets were introduced by D.Amsaveni[1]. Nachbin[8-10] gave some results on the relationship between topological and order structures and several other authors have continued his work. In this paper, the concepts of an order and a fuzzy scalable structure space are combined and ordered FQ_X^{**} presemi λ compact spaces,ordered FQ_X^{**} presemi λ paracompact spaces, ordered FQ_X^{**} presemi λ metacompact spaces are introduced. Also some of their properties are studied.

2 Preliminaries

Definition 2.1. [3]
Let X be a non empty set. An order on X is a relation $\leq$ on X which is reflexive, antisymmetric and transitive.

Definition 2.2. [13]
Let X be a non-empty set and I be the unit interval [0, 1]. A fuzzy set in X is an element of the set I^X of all functions from X to I.

Definition 2.3. [3]
A fuzzy set μ, in a ordered set X, is called:

(i) Increasing if X $\leq$ Y implies $\mu(x) \leq \mu(y)$
(ii)Decreasing if X $\leq$ Y implies $\mu(x) \geq \mu(y)$

Definition 2.4. [5]
A fuzzy topology T on X is a collection of subsets of I^X such that
(i) 0, 1 $\in$ T
(ii) If $\lambda, \mu \in T$, then $\lambda \wedge \mu \in T$,
(iii) If $\lambda_i \in T$ for each $i \in \Lambda$ then $\vee \lambda_i \in T$.
The ordered pair (X, T) is called a fuzzy topological space. A fuzzy set λ in a fuzzy topological space is called a fuzzy closed set, if its complement λ^c or $(1 - \lambda)$ is fuzzy open.

Definition 2.5. [4]
Given fuzzy topological space (X, τ) and (Y, δ), a function f : X $\leftarrow$ Y is fuzzy continuous if the inverse image under f of any open fuzzy set in Y is an open fuzzy set in X, that is if $f^{-1}(\mu) \in \tau$ whenever $\mu \in \delta$.

Definition 2.6. [4]
Let (X$_i$,δ_i) be a fuzzy topological space for each index i $\in$ I. The product fuzzy topology $\delta = \Pi_i \delta_i$ on the set X $= \Pi_i X_i$ is the coarsest fuzzy topology on X making all the projection mappings $\pi_i : X \to X_i$ fuzzy continuous.

Definition 2.7. [5]
A family A of fuzzy sets is a cover of a fuzzy set B iff $B \subset \cup \{A \mid A \in A\}$. It is an open cover iff each member of A is an open fuzzy set. A subcover of A is a subfamily of A which is also a cover.

Definition 2.8. [5]
A fuzzy topological space (X, τ) is compact if every cover of X by members of τ contains a finite subcover.

Definition 2.9. [13]
A family of interval valued fuzzy sets on X is said to be inadequate iff it fails to cover X and finitely inadequate iff no finite subfamily of it covers X.

Definition 2.10. [3]
An ordered set on which there is given a fuzzy topology is called a ordered fuzzy topological space.

Definition 2.11. [7]
Let A be a family of sets and B be a set in fuzzy topological spaces (X, τ). We say that A is * locally finite in B if for each point e in B, there exists a U in neighbourhood of e is intersects with at most a finite number of sets of A.

Definition 2.12. [1]
Let X be a non empty set, $\tau_i : i \in J$ be the topologies associated with X and let τ_X^* be the collection of all fuzzy open sets of (X, τ_i) and a function $Q_X : X \rightarrow \tau_X^*$ be the fuzzy scale.
A fuzzy scalable structure is denoted by Q_X^{**} and defined by
$Q_X^{**} = \{0, 1\} \bigcup Q_X^*$ where $Q_X^* = \{Q_X(x) : x \in X\}$.

Definition 2.13. [1]
Let X be a non empty set, τ_X^* be the collection of all fuzzy open sets of X and Q_X^{**} be a fuzzy scalable structure on X. Then, the triad (X, τ_X^*, Q_X^{**}) is called a fuzzy scalable structure space.

Every member of Q_X^{**} is called a fuzzy Q_X^{**} open set (briefly, FQ_X^{**} closed).

Definition 2.14. [1]
Let (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$) and (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$) be any two fuzzy scalable structure spaces. Then the function f : (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$) $\rightarrow$ (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$) is said to be FQ_X^{**} pre semi λ continuous if for every FQ_X^{**} closed set in (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$) there exists a FQ_X^{**} pre semi λ closed set in (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$).

3 Ordered Fuzzy Scalable Presemi λ Compact Space

Definition 3.1. Let (X, τ_X^*, Q_X^{**}) be a fuzzy scalable structure space. Let μ be any fuzzy set. Then μ is said to be an
(i) increasing fuzzy set if $\mu(x) \leq \mu$(y) and
(ii) decreasing fuzzy set if $\mu(x) \geq \mu$(y)
whenever x $\leq$ y and x, y $\in$ X.

Definition 3.2. Let X be an ordered set, τ_X^* be the collection of all fuzzy open sets of X and Q_X^{**} be a fuzzy scalable structure on X. Then (X, τ_X^*, Q_X^{**}, $\leq$) is said to be an ordered fuzzy scalable structure space.

Definition 3.3. Let (X, τ_X^*, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space. Let μ be a fuzzy set in X. Then
(i) $FQ_X^{**}I^{cl}(\mu) = \wedge\{\gamma : \gamma$ is a FQ_X^{**} increasing closed set and $\gamma \geq \mu\}$.
(ii)$FQ_X^{**}D^{cl}(\mu) = \wedge\{\gamma : \gamma$ is a FQ_X^{**} decreasing closed set and $\gamma \geq \mu\}$.
(iii)$FQ_X^{**}I^{o}(\mu) = \vee\{\gamma : \gamma$ is a FQ_X^{**} increasing open set and $\gamma \leq \mu\}$.

(iv)FQ_X^{**} $D^o(\mu) = \vee$ $\{\gamma : \gamma$ is a FQ_X^{**} decreasing open set and $\gamma \leq \mu$ $\}$.

clearly FQ_X^{**} $I^{cl}(\mu)$ is the smallest increasing (resp., decreasing) FQ_X^{**} closed set containing μ and FQ_X^{**} $I^o(\mu)$ (resp., $FQ_X^{**} D^o(\mu)$ is the largest increasing (resp., decreasing) FQ_X^{**} open set contained in μ.

Definition 3.4. Let (X, τ_X^*, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space. A collection $\mathcal{C} = \{\mu_i : i \in J\}$ of increasing (resp. decreasing) fuzzy sets in X is said to be an increasing (resp., decreasing) FQ_X^{**} cover of 1_X if $\vee\mu_i = 1_X$ and it is said to be increasing (resp., decreasing) FQ_X^{**} presemi λ open cover iff each member of $\mathcal{C}$ is an increasing (resp.,decreasing) FQ_X^{**} presemi λ open set.

Definition 3.5. Let (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) and (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) be any two ordered fuzzy scalable structure spaces. A function f : (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) $\rightarrow$ (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) is said to be an upper (resp., lower) FQ_X^{**} presemi λ continuous function if for each increasing (resp., decreasing) $FQ_{X_2}^{**}$ presemi λ open set γ in (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$), the inverse image $f^{-1}(\gamma)$ is an increasing (resp., decreasing) $FQ_{X_1}^{**}$ presemi λ open set in (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$).

A function f: (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) $\rightarrow$ (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) is said to be an ordered FQ_X^{**} presemi λ continuous function if its both upper and lower FQ_X^{**} presemi λ continuous function.

Definition 3.6. Let (X, τ_X^*, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space. Then (X, ${\tau_X}^*$, ${Q_X}^{**}$, $\leq$) is said to be an upper (resp., lower) FQ_X^{**} presemi λ compact space if for each increasing (resp., decreasing) FQ_X^{**} presemi λ open cover of $\mathcal{C}$ has a finite subcollection $\mathcal{S} = \{\mu_j : j \in J\}$ with $\vee_{j=1}^n \mu_j = 1_X$

Definition 3.7. An ordered fuzzy scalable structure space (X, τ_X^*, Q_X^{**}, $\leq$) is said to be an ordered fuzzy FQ_X^{**} presemi λ compact space if it is both upper FQ_X^{**} presemi λ compact space and lower FQ_X^{**} presemi λ compact space.

Definition 3.8. Let (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) and (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) be any two ordered fuzzy scalable structure spaces with (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) is an upper (resp., lower) $FQ_{X_2}^{**}$ presemi λ compact space. Let $X_1 \times X_2$ be an ordered fuzzy scalable structure space. If $\eta(x)$ is an icreasing (resp., decreasing) fuzzy set that contain a slice in $X_1 \times X_2$, then there exists a tube in the slice and also in $\eta(x)$.

Proposition 3.1. Let (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) and (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) be any two ordered fuzzy scalable structure spaces. If (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) is an upper $FQ_{X_1}^{**}$ presemi λ compact space and (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) is an upper $FQ_{X_2}^{**}$ presemi λ compact space then $X_1 \times X_2$ is an upper FQ_X^{**} presemi λ compact space.
Proof
Let (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) and (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) be an upper $FQ_{X_1}^{**}$ presemi λ compact space and upper $FQ_{X_2}^{**}$ presemi λ compact space respectively. Let $\mathcal{C}$ be an increasing FQ_X^{**} presemi λ open cover of $X_1 \times X_2$. For any x $\in X_1$, $\{x\}\times$ 1_{X_2} is an upper FQ_X^{**} presemi λ compact space. Then there exists an increasing FQ_X^{**} presemi λ open subcover $\{$ μ_{i_j} : i and j $\in$ J $\}$

$\in \mathcal{C}$ and $\eta(x) = \vee_{j=1}^{n}\{\mu_{i_j}\}$ is the union of increasing finite element. By the, Definition 3.8 there exists an increasing FQ_X^{**} presemi λ open set $\gamma(x) \times X_2$, where $\{x\}\times 1_{X_2} \leq \gamma(x)\times 1_{X_2} \leq\eta(x)$.

Since $\gamma(x)$ is an increasing $FQ_{X_1}^{**}$ presemi λ open cover S of $(X_1, \tau_{X_1}^{*}, Q_{X_1}^{**}, \leq)$, $\gamma(x)$ have an increasing $FQ_{X_1}^{**}$ presemi λ subcover $\gamma(x_n) =\vee_{j=1}^{n}\{\omega(\mathrm{x}_j)\}$ in $\eta(x_n)$. Then for each x_n, $\gamma(x_n) \times 1_{X_2}$ is also in $\eta(x_n)$. Since $\eta(x_n)$ is an increasing finite union of all elements of S and the finite subcollection $\vee_{j=1}^{n}\{\omega(\mathrm{x}_j)\times X_2\}$ is the increasing FQ_X^{**} presemi λ open cover for $X_1 \times X_2$, the family $\eta(x_n) = \vee_{j=1}^{n}\{\mu(\mathrm{x}_j)\}$ is the increasing FQ_X^{**} presemi λ open cover for $X_1 \times X_2$.

Proposition 3.2. Let (X, τ_X^{*}, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space. If γ is a ordered FQ_X^{**} presemi λ compact subspace of (X, τ_X^{*}, Q_X^{**}, $\leq$) then for every family $\{\mu_i : i \in I\}$ of FQ_X^{**} presemi λ open sets of X such that $\gamma \leq \vee_i\mu_i$ there exists a finite set $\{i_1, i_2, ..i_n\} \subset I$ such that $\gamma \leq \vee_{j=1}^{n}\mu_{i_j}$.
Proof
It follows from the definition of subspace.

Proposition 3.3. Let (X, τ_X^{*}, Q_X^{**}, $\leq$) be an ordered FQ_X^{**} presemi λ compact space . If μ is FQ_X^{**} presemi λ closed set of X then it is FQ_X^{**} presemi λ compact set.
proof
Let μ be a FQ_X^{**} presemi λ closed set of the ordered FQ_X^{**} presemi λ compact space (X, τ_X^{*}, Q_X^{**}, $\leq$) and let C be both increasing and decreasing FQ_X^{**} presemi λ open cover of μ by FQ_X^{**} presemi λ open sets in (X, τ_X^{*}, Q_X^{**}, $\leq$). Since μ is FQ_X^{**} presemi λ closed, then 1_X - μ is FQ_X^{**} presemi λ open and C* = C $\vee$ (1_X - μ) is both increasing and decreasing FQ_X^{**} presemi λ open cover of (X, τ_X^{*}, Q_X^{**}, $\leq$). Since (X, τ_X^{*}, Q_X^{**}, $\leq$) is ordered FQ_X^{**} presemi λ compact space, it has a finite increasing and decreasing subcover, containing only finitely many members $\mathcal{C}_1, \mathcal{C}_2 ... \mathcal{C}_n$ of $\mathcal{C}$ and may contain 1_X - μ . Since 1_X = (1_X - μ)$\vee\vee_{i=1}^{n}\mathrm{C}_i$ it follows that $\mu \leq \vee_{i=1}^{n}\mathrm{C}_i$ and has finite increasing and decreasing FQ_X^{**} presemi λ subcover.

Corollary 3.4. Let (X, τ_X^{*}, Q_X^{**}, $\leq$) be an ordered FQ_X^{**} presemi λ compact space. Let μ be an FQ_X^{**} presemi λ open set of X. If a family $\{\gamma_i : i \in I\}$ of FQ_X^{**} presemi λ closed sets of X contains atleast one FQ_X^{**} presemi λ compact set and $\wedge_{i\in I}\gamma_i \leq \mu$, then there exists a finite set $\{i_1, i_2, ..i_n\} \subset I$ such that $\wedge_{j=1}^{n}\gamma_{i_j} \leq \mu$.
Proof
Let (X, τ_X^{*}, Q_X^{**}, $\leq$) be an ordered FQ_X^{**} presemi λ compact space, μ be an FQ_X^{**} presemi λ open set and a family $\{\gamma_i : i \in I\}$ of FQ_X^{**} presemi λ closed sets of X. Then $1_X \setminus \mu = \gamma$ is presemi λ closed set and $1_X \setminus \gamma_i = \eta_i$ is presemi λ open set. By Proposition 3.3 γ is FQ_X^{**} presemi λ compact set. Then by Proposition 3.2, there exists a finite set $\{i_1, i_2, ..i_n\} \subset I$ such that $\gamma \leq\vee_{j=1}^{n}\eta_{i_j}$. This implies that $\wedge_{j=1}^{n}\gamma_{i_j} \leq \mu$.

Proposition 3.5. Let (X_1, $\tau_{X_1}^{*}$, $Q_{X_1}^{**}$, $\leq$) be an ordered $FQ_{X_1}^{**}$ presemi λ compact space and (X_2, $\tau_{X_2}^{*}$, $Q_{X_2}^{**}$, $\leq$) be a ordered fuzzy scalabel structure space. If a function f : (X_1, $\tau_{X_1}^{*}$, $Q_{X_1}^{**}$, $\leq$) $\to$ (X_2, $\tau_{X_2}^{*}$, $Q_{X_2}^{**}$, $\leq$) is an ordered FQ_X^{**} presemi λ continuous function, then the

image of $(X_1,\, \tau^*_{X_1},\, Q^{**}_{X_1},\, \leq)$ is a $FQ^{**}_{X_2}$ presemi λ compact space.
Proof
Let $(X_1,\, \tau^*_{X_1},\, Q^{**}_{X_1},\, \leq)$ be an ordered $FQ^{**}_{X_1}$ presemi λ compact space and $(X_2,\, \tau^*_{X_2},\, Q^{**}_{X_2},$ $\leq)$ be a ordered fuzzy scalabel structure space. Let μ_i be a $FQ^{**}_{X_2}$ presemi λ open cover of the space $(X_2,\, \tau^*_{X_2},\, Q^{**}_{X_2},\, \leq)$. The family $\{f^{-1}(\mu_i)\}_{i\in I}$ is a $FQ^{**}_{X_1}$ presemi λ open cover of $(X_1,\, \tau^*_{X_1},\, Q^{**}_{X_1},\, \leq)$. Since $(X_1,\, \tau^*_{X_1},\, Q^{**}_{X_1},\, \leq)$ is $FQ^{**}_{X_1}$ presemi λ compact space, $f^{-1}(\mu_{i_1}) \vee f^{-1}(\mu_{i_2}) \vee ... \vee f^{-1}(\mu_{i_n}) = 1_{X_1}$, this implies that $\mu_{i_1} \vee \mu_{i_2} \vee ... \vee \mu_{i_n} = 1_{X_2}$. Hence $(X_2,\, \tau^*_{X_2},$ $Q^{**}_{X_2},\, \leq)$ is $FQ^{**}_{X_2}$ presemi λ compact space.

Definition 3.9. Let (X, τ^*_X, Q^{**}_X, $\leq$) be an ordered fuzzy scalable structure space. A collection $\mathcal{S} = \{\omega_i : i \in I\}$ of increasing (resp. decreasing) $FQ^{**}_{X_1}$ presemi λ open set in X is said to be an increasing (resp., decreasing) FQ^{**}_X presemi λ short if $\mathcal{S}$ is not a increasing (resp., decreasing) FQ^{**}_X presemi λ open cover of X.

$\mathcal{S}$ is said to be an oredred FQ^{**}_X presemi λ short of an ordered fuzzy scalable structure space (X, τ^*_X, Q^{**}_X, $\leq$) if it is both increasing FQ^{**}_X presemi λ short and decreasing FQ^{**}_X presemi λ short.

Definition 3.10. Let (X, τ^*_X, Q^{**}_X, $\leq$) be an ordered fuzzy scalable structure space. A collection $\mathcal{S} = \{\omega_i : i \in I\}$ of increasing (resp. decreasing) $FQ^{**}_{X_1}$ presemi λ open set in X is said to be an increasing (resp. decreasing) FQ^{**}_X presemi λ finite short if no finite subfamily of $\mathcal{S}$ is not a increasing (resp. decreasing) FQ^{**}_X presemi λ open cover of X.

An ordered fuzzy scalable structure space (X, τ^*_X, Q^{**}_X, $\leq$) is said to be an oredred FQ^{**}_X presemi λ finite short if it is both increasing FQ^{**}_X presemi λ finite short and decreasing FQ^{**}_X presemi λ finite short.

Proposition 3.6. An ordered fuzzy scalable structure space (X, τ^*_X, Q^{**}_X, $\leq$) is FQ^{**}_X presemi λ compact space if and only if an ordered FQ^{**}_X presemi λ finite short of (X, τ^*_X, Q^{**}_X, $\leq$) is FQ^{**}_X presemi λ short.
Proof
Let (X, τ^*_X, Q^{**}_X, $\leq$) be an ordered FQ^{**}_X presemi λ compact space and $\mathcal{S}$ be any ordered FQ^{**}_X presemi λ finite short family of X. Assume that $\mathcal{S}$ is not an ordered FQ^{**}_X presemi λ short. Then $\mathcal{S}$ is both increasing and decreasing FQ^{**}_X presemi λ open cover for X which has no increasing and decreasing FQ^{**}_X presemi λ finite sub cover. This implies that (X, τ^*_X, Q^{**}_X, $\leq$) is not an ordered FQ^{**}_X presemi λ compact space. This is a contradiction. Hence, $\mathcal{S}$ is an ordered FQ^{**}_X presemi λ short.

Conversely, assume that every ordered FQ^{**}_X presemi λ finite short of (X, τ^*_X, Q^{**}_X, $\leq$) is FQ^{**}_X presemi λ short. Suppose (X, τ^*_X, Q^{**}_X, $\leq$) is not an ordered FQ^{**}_X presemi λ compact space. This reveals that there exists both increasing and decreasing FQ^{**}_X presemi λ open cover $\mathcal{S}$ for X which has no increasing and decreasing FQ^{**}_X presemi λ finite sub cover. Therefore, $\mathcal{S}$ is an ordered FQ^{**}_X presemi λ finite short but not an ordered FQ^{**}_X presemi λ short. This contradicts to the fact that $\mathcal{S}$ is an ordered FQ^{**}_X presemi λ short. Hence, (X, τ^*_X, Q^{**}_X, $\leq$) is an ordered FQ^{**}_X presemi λ compact space.

Definition 3.11. Let (X, τ_X^*, Q_X^{**}, $\leq$) be a ordered fuzzy scalable structure space and α_1,$\alpha_2$$\in$ (0, 1) such that $\alpha_1 \leq \alpha_2$. Let μ be a fuzzy set on (X, τ_X^*, Q_X^{**}, $\leq$) and y $\in$ X. If

$$\mu_L(x) = \begin{cases} \alpha_1 & \text{if y = x} \\ 0 & \text{if y} \neq \text{x} \end{cases}$$

$$\mu_U(x) = \begin{cases} \alpha_2 & \text{if y = x} \\ 0 & \text{if y} \neq \text{x} \end{cases}$$

then μ is called a fuzzy point on (X, τ_X^*, Q_X^{**}, $\leq$) and it is denoted by $y_\alpha = (y_{\alpha_1}, y_{\alpha_2})$ where y_{α_1} denotes the degree of lower approximation of fuzzy point y_α and y_{α_1} denotes the degree of upper approximation of fuzzy point y_α. The point y $\in$ X is called support of fuzzy point y_α and is denoted by supp(y).

Definition 3.12. Let (X, τ_X^*, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space. A fuzzy set μ in (X, τ_X^*, Q_X^{**}, $\leq$) is called a FQ_X^{**} neighborhood of fuzzy point P_x^α if and only if there exists a FQ_X^{**} presemi λ open set γ such that $P_x^\alpha \in \gamma \leq \mu$.

Definition 3.13. Let (X, τ_X^*, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space. A collection $\mathcal{B}$ is said to be an increasing (resp. decreasing) FQ_X^{**} locally finite if each fuzzy point in an ordered fuzzy scalable structure space has an increasing FQ_X^{**} presemi λ neighbourhood that intersects only finitely many of the FQ_X^{**} presemi λ open sets in an increasing FQ_X^{**} presemi λ open cover.

Definition 3.14. Let (X, τ_X^*, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space. Then (X, τ_X^*, Q_X^{**}, $\leq$) is said to be an upper (resp. lower) FQ_X^{**} presemi λ paracompact space if for each increasing FQ_X^{**} presemi λ open cover has an increasing (resp. decreasing) FQ_X^{**} locally finite FQ_X^{**} presemi λ subcover.

An ordered fuzzy scalable structure space (X, τ_X^*, Q_X^{**}, $\leq$) is said to be an FQ_X^{**} presemi λ paracompact space it is both upper and lower FQ_X^{**} presemi λ paracompact space.

Proposition 3.7. Let (X, τ_X^*, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure spaces and γ $\in$ I^X. If (X, τ_X^*, Q_X^{**}, $\leq$) is FQ_X^{**} presemi λ paracompact space then (γ, $\tau_{X|\gamma}^*$, $Q_{X|\gamma}^{**}$, $\leq$) is an upper FQ_X^{**} presemi λ paracompact space

Proof

Let (X, τ_X^*, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space and $\gamma \in I^X$ be an ordered fuzzy scalable structure subspaces of (X, τ_X^*, Q_X^{**}, $\leq$). Let $\{ \eta_j\}_j$$\in$ J be an increasing FQ_X^{**} presemi λ open cover of (γ, $\tau_{X|\gamma}^*$, $Q_{X|\gamma}^{**}$, $\leq$) and defined by $\{\eta_j = \mu_j \wedge \gamma : \mu_j$'s are FQ_X^{**} presemi λ open $\}$. Since $\{\mu_j\}$ is an increasing FQ_X^{**} presemi λ open cover in (X, τ_X^*, Q_X^{**}, $\leq$) that have increasing FQ_X^{**} locally finite FQ_X^{**} presemi λ subcover. Then, $\{ \eta_j\}_j$$\in$ J is an increasing FQ_X^{**} locally finite FQ_X^{**} presemi λ subcover.

Definition 3.15. Let (X, τ_X^*,Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space. A collection $\mathcal{B}$ is said to be an increasing(resp. decreasing) FQ_X^{**} finite if each point of (X, τ_X^*,Q_X^{**}, $\leq$) lies in only finitely many members in an increasing (resp. decreasing) FQ_X^{**} presemi λ open cover.

Definition 3.16. Let (X, τ_X^*, Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space. Then (X, τ_X^*, Q_X^{**}, $\leq$) is said to be an upper (resp. lower) FQ_X^{**} presemi λ metacompact space if for each increasing (resp. decreasing) FQ_X^{**} presemi λ open cover has an increasing (resp. decreasing) FQ_X^{**} finite fuzzy point FQ_X^{**} presemi λ subcover.

An ordered fuzzy scalable structure space (X, τ_X^*, Q_X^{**}, $\leq$) is said to be an FQ_X^{**} presemi λ metacompact space it is both upper and lower FQ_X^{**} presemi λ metacompact space.

Definition 3.17. Let (X, τ_X^*,Q_X^{**}, $\leq$) be an ordered fuzzy scalable structure space and γ be any FQ_X^{**} open set in (X, τ_X^*,Q_X^{**}, $\leq$). Then $Q_{X|}\gamma^{*}* =\{ \mu \wedge \gamma : \mu \in Q_X^{**}\}$ is a fuzzy scalable structure on γ and (γ, $\tau_{X|}\gamma^*$,$Q_{X|}\gamma^{**}$, $\leq$) denote the FQ_X^{**} subspace of (X, τ_X^*,Q_X^{**}, $\leq$).

Proposition 3.8. Let (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) and (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) be any two ordered fuzzy scalable structure spaces. If (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) is an upper $FQ_{X_1}^{**}$ presemi λ metacompact space and (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) is an upper $FQ_{X_2}^{**}$ presemi λ metacompact space then $X_1 \times X_2$ is an upper FQ_X^{**} presemi λ metacompact space.
Proof
Let (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) and (X_2, $\tau_{X_2}^*$, $Q_{X_2}^{**}$, $\leq$) be an upper $FQ_{X_1}^{**}$ presemi λ metacompact space and upper $FQ_{X_2}^{**}$ presemi λ metacompact space respectively. Let $\mathcal{C}$ be an increasing FQ_X^{**} finite fuzzy point of $X_1 \times X_2$. For any $x \in X_1$, { x } $\times$ 1_{X_2} is an upper FQ_X^{**} presemi λ metacompact space. Then there exists an increasing FQ_X^{**} finite FQ_X^{**} presemi λ subcover { μ_{i_j} : i and j $\in$ J } $\in$ $\mathcal{C}$and $\eta(x) = \vee_{j=1}^{n}\{\mu_{i_j}\}$ is the union of increasing finite elements. By the definition 3.8 there exists an increasing FQ_X^{**} presemi λ open set $\gamma(x) \times 1_{X_2}$, where $\{x\}\times$ $1_{X_2} \subset \gamma(x)\times$ 1_{X_2} $\subset\eta(x)$.

Since $\gamma(x)$ is an increasing $FQ_{X_1}^{**}$ presemi λ open cover S of (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$), $\gamma(x)$ have an increasing $FQ_{X_1}^{**}$ finite fuzzy point $FQ_{X_1}^{**}$ presemi λ subcover $\gamma(x_n)$ =$\vee_{j=1}^{n}\{\omega(\mathrm{x}_j)\}$ in $\eta(x_n)$. Consider $\mathcal{C}_{x_n}$ as an increasing FQ_X^{**} presemi λ subcover for S. Let (x, y) $\in X_1 \times X_2$ be any point. Since (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) is an upper $FQ_{X_1}^{**}$ presemi λ metacompact space, there exist an increasing $FQ_{X_1}^{**}$ finite fuzzy point $x_1, x_2, ...x_n \in X_1$ lies in only finitely many member in $\gamma(x_n)$. Let δ be $FQ_{X_1}^{**}$ open set in (X_1, $\tau_{X_1}^*$, $Q_{X_1}^{**}$, $\leq$) having increasing $FQ_{X_1}^{**}$ finite fuzzy point. Then $\delta \times 1_{X_1}$ having increasing finite fuzzy point that lies in only finitely many members in $\mathcal{C}_{x_n}$.

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