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ONPAIRWISE FUZZY σ-RESOLVABLE SPACES

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ABSTRACT

In this paper the concepts of pairwise fuzzy σ-resolvable spaces are introduced and several characterizations of pairwise fuzzy σ-resolvable spaces are studied. The inter-relations between pairwise fuzzy resolvable spaces, pairwisefuzzy almost resolvable spaces, pairwisefuzzy submaximal spaces, pairwisefuzzy hyperconnected spaces, are also investigated.

KEYWORDS : Pairwise fuzzy dense set, pairwise fuzzy open set ,pairwisefuzzy

almost resolvable, space,pairwisefuzzysubmaximal space, pairwise fuzzy

hyperconnected space, pairwise fuzzy resolvable space.

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1.INTRODUCTION

The fundamentalconcept of afuzzy set introduced by **L.A.ZADEH** [15] in 1965,provides a natural foundation for building new branches of fuzzy mathematics.In 1968**C.L.CHANG** [3] introduced the concept of fuzzy topological spaces as a generalizations of topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

E.HEWIT [6] introduced the concepts of resolvability and irresolvabilityin topological spaces. **A.G.EL'KIN** [5] introduced openhereditarily irresolvable spaces in classical topology. The concept of almost resolvable spaces was introduced by**RICHARD BOLSTEIN**[8] as a generalization of resolvable spaces of E.HEWIT The concept of σ-resolvable spaces was introduced by **BRANISLAV.NOVOTNY** [2] in classical topology. In 1989, **A.KANDIL**[7] introduced the concept of fuzzybitopological spaces as an extension of fuzzy topological spaces. In this paper the concepts of pairwise fuzzy σ- resolvable spaces are introduced and characterizations of pairwise fuzzy σ-resolvable spaces are studied. The inter-relations between pairwisefuzzy σ-resolvable spaces,pairwise fuzzy resolvable spaces,pairwise fuzzy almost resolvable spaces, pairwisefuzzy submaximal spaces and pairwise fuzzyhyperconnectedspaces, are also investigated.

2. PRELIMINARIES

Now we introduce some basic notions and results used in the sequel. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to CHANG(1968). By a fuzzy bitopologicalspace (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X. The complement $\lambda'$of a fuzzy set λ is defined by $\lambda'(x)$ =1 –λ (x) ($x \in$ X).

Definition 21 : Let λ and μ be fuzzy sets in X. Then, for all $x \in X$,

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$
(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$
(iii) $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$
(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \psi(x) = \min\{\lambda(x), \mu(x)\}$
(v) $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$

For a family$\{\lambda_i / i \in I\}$ of fuzzy sets in (X,T), the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$, are defined respectively as

(vi) $\psi(x) = sup_i\{\lambda_i(x) / x \in X\}$
(vii) $\delta(x) = inf_i\{\lambda_i(x) / x \in X\}$

Definition 2.2 : Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define the interior and the closure of λ respectively as follows :

(i) Int (λ) = $\vee\{\mu / \mu \leq \lambda, \mu \in T\}$,
(ii) Cl (λ) = $\wedge\{\mu / \lambda \leq \mu, 1-\mu \in T\}$.

Lemma 2.1 [1] : For a fuzzy set λ of a fuzzy topological space X,

(i). $1 -$ Int (λ) = Cl $(1-\lambda)$,
(ii). $1 -$ Cl (λ) = Int $(1-\lambda)$.

DEFINITION 2.3 [12] : A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ (i = 1,2). The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set.

Definition 2.4 [9] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1$.

Definition 2.5 [10] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1} cl_{T_2}(\lambda) = int_{T_2} cl_{T_1}(\lambda) = 0$.

Lemma 2.2 [1] : For a family $\mathcal{A}$ of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X,T), $\vee$ cl $(\lambda_\alpha) \leq$ cl $(\vee\lambda_\alpha)$. In case$\mathcal{A}$ is a finite set, $\vee$ cl (λ_α)= cl$(\vee(\lambda_\alpha))$. Also $\vee$int$(\lambda_\alpha) \leq$int $(\vee\lambda_\alpha)$ in (X,T).

Definition 2.8[10] : If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X,T_1,T_2), then the fuzzy set (1 – λ) is called a pairwise fuzzy residual set in (X,T_1,T_2).

Definition 2.10[12] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy G_δ -set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy open sets in (X,T_1,T_2).

Definition 2.11 [12] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy F_σ -set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy closed sets in (X,T_1,T_2).

Definition 2.10 [14] : A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy almost resolvable space if $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets (λ_k) in (X,T_1,T_2) are such that $\operatorname{int}_{T_1}\operatorname{int}_{T_2}(\lambda) = \operatorname{int}_{T_2}\operatorname{int}_{T_1}(\lambda) = 0$. Otherwise (X,T_1,T_2) is called a pairwise fuzzy almost irresolvable space.

3. PAIRWISE FUZZY σ-RESOLVABLE SPACES

Definition 3.1: A fuzzy bitopological space (X,T_1,T_2) is called a pairwisefuzzy σ-resolvablespace if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where (λ_i)'s are pairwise fuzzy dense setsin (X,T_1,T_2) and $\lambda_i \leq 1-\lambda_j$ for $i \neq j$. That is, (X,T_1,T_2) is a pairwise fuzzyσ-resolvable space if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where $\operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(\lambda_i) = 1 = \operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(\lambda_i)$ and $\lambda_i \leq 1-\lambda_j$ for $i \neq j$.

Afuzzybitopological space(X,T_1,T_2)which is not pairwise fuzzy σ-resolvable, is calleda pairwise fuzzyσ-irresolvablespace.

Proposition 3.1: If a fuzzybitopologicalspace (X, T_1, T_2) is a pairwise fuzzyσ-resolvable space ,then $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where $\operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(\lambda_i)=1$ and$\operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(\lambda_i)=1$

and$\operatorname{int}_{T_1}\operatorname{int}_{T_2}(\lambda_i) = 0$ and$\operatorname{int}_{T_2}\operatorname{int}_{T_1}(\lambda_i) = 0$.

Proof: Let (X, T_1, T_2) be a pairwisefuzzy σ-resolvable space.Then,$\vee_{i=1}^{\infty}(\lambda_i) = 1$and $\operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(\lambda_i) = 1 = \operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(\lambda_i)$ and $\lambda_i \leq 1-\lambda_j$, for $i \neq j$.Now $\lambda_i \leq 1-\lambda_j$ implies that$\operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(\lambda_i) \leq \operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(1-\lambda_j)$ and$\operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(\lambda_i) \leq \operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(1-\lambda_j)$.Then wehave $1 \leq \operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(1-\lambda_j)$ and $1 \leq \operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(1-\lambda_j)$.That is,$\operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(1-\lambda_j) = 1$ and $\operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(1-\lambda_j) = 1$and hence$\operatorname{int}_{T_1}\operatorname{int}_{T_2}(\lambda_j) = 0$ and $\operatorname{int}_{T_2}\operatorname{int}_{T_1}(\lambda_j) = 0$.Hence we have, $\vee_{i=1}^{\infty}(\lambda_i) = 1$ and $\operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(\lambda_i) = 1 = \operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(\lambda_i)$and $\operatorname{int}_{T_1}\operatorname{int}_{T_2}(\lambda_i) = 0$ and $\operatorname{int}_{T_2}\operatorname{int}_{T_1}(\lambda_i) = 0$.

Remark 3.1 : In the above proposition, the fuzzy sets (λ_i)'s are not non-zero pairwise fuzzy open sets in (X,T_1,T_2). For, if (λ_i)'s are pairwise fuzzy open sets in (X,T_1,T_2), then $(1-\lambda_i)$'sare pairwise fuzzy closed sets in (X,T_1,T_2). Then $\operatorname{cl}_{T_i}(1-\lambda_i) = 1-\lambda_i$.Now$\operatorname{int}_{T_1}\operatorname{int}_{T_2}(\lambda_i) = 0$ and$\operatorname{int}_{T_2}\operatorname{int}_{T_1}(\lambda_i) = 0$ implies that$\operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(1-\lambda_i) = 1$ and $\operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(1-\lambda_i) = 1$andhence $\operatorname{cl}_{T_1}(1-\lambda_i) = 1$ and $\operatorname{cl}_{T_2}(1-\lambda_i) = 1$.Therefore we have $\operatorname{cl}_{T_i}(1-\lambda_i) = 1$and hence $1 - \operatorname{int}(\lambda_i) = 1$.This implies that$1-(\lambda_i) = 1$ and hence $\lambda_i = 0$. Therefore thefuzzy sets (λ_i)'s are not non-zero pairwise fuzzy open sets in (X,T_1,T_2).

Remark 3.2: Also, the (λ_i)'s$(\neq 1)$ in the above proposition, are notpairwise fuzzy closed sets in (X,T_1,T_2). For, if (λ_i)'s$(\neq 1)$ are pairwise fuzzy closed sets in (X,T_1,T_2), then$\operatorname{cl}_{T_i}(\lambda_i) = \lambda_i (i = 1,2)$. Then, $\operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(\lambda_i) = 1$ and$\operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(\lambda_i) = 1$. That is,$\operatorname{cl}_{T_1}(\lambda_i)) = 1$ and$\operatorname{cl}_{T_2}(\lambda_i)=1$. That is, $\operatorname{cl}_{T_i}(\lambda_i) = 1$ and hence $\operatorname{cl}_{T_i}(\lambda_i) = \lambda_i$implies that$\lambda_i = 1$. Therefore the fuzzysets (λ_i)'s$(\neq 1)$ are not pairwise fuzzy closed sets in (X,T_1,T_2).

Proposition 3.2 : If a fuzzybitopological space (X, T_1, T_2) is a pairwise fuzzyσ-resolvable space, then (X, T_1, T_2) is a pairwise fuzzy almost resolvable space.

Proof :Suppose that thefuzzybitopological space (X, T_1, T_2) is a pairwise fuzzy σ-resolvable space. Then by proposition 3.1,$\vee_{i=1}^{\infty}(\lambda_i) = 1$, where $\operatorname{cl}_{T_1}\operatorname{cl}_{T_2}(\lambda_i) = 1$ and$\operatorname{cl}_{T_2}\operatorname{cl}_{T_1}(\lambda_i) = 1$ in (X,T_1,T_2) and$\operatorname{int}_{T_1}\operatorname{int}_{T_2}(\lambda_i) = 0$ and $\operatorname{int}_{T_1}\operatorname{int}_{T_2}(\lambda_i) = 0 = \operatorname{int}_{T_2}\operatorname{int}_{T_1}(\lambda_i)$. Hencewe have$\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets$(\lambda_i)'s$ in

(X, T_1, T_2) are such thatint$\operatorname{int}_{T_1}\operatorname{int}_{T_2}(\lambda_i) = 0 = \operatorname{int}_{T_2}\operatorname{int}_{T_1}(\lambda_i)$ .Therefore (X,T_1, T_2) is a pairwise fuzzy almost resolvable space .

Theorem 3.1 [14]: If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy almost resolvable space if and only if $\wedge_{k=1}^{\infty}(\mu_k) = 0$, where (μ_k)'s are pairwise fuzzy dense sets in (X, T_1, T_2) .

Proposition 3.3 : If a fuzzybitopological space (X, T_1, T_2) is a pairwise fuzzyσ-resolvable space, then$\wedge_{k=1}^{\infty}(\mu_k) = 0$, where (μ_k)'s are pairwise fuzzy dense sets in (X, T_1, T_2) .

Proof : Supposethat thefuzzybitopological space (X, T_1, T_2) is a pairwise fuzzyσ-resolvable space. Then by proposition 3.2, (X,T_1, T_2) is a pairwise fuzzy almost resolvable space .By theorem 3.1,then$\wedge_{k=1}^{\infty}(\mu_k) = 0$, where$(\mu_k)$'s are pairwise fuzzy dense sets in (X, T_1, T_2) .

Definition 3.2 [9] : A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy resolvable space if there exists a pairwise fuzzy dense set λ in (X,T_1,T_2) such that $1-\lambda$ is also a pairwise fuzzy dense set in (X,T_1,T_2). That is, $cl_{T_1}cl_{T_2}(1-\lambda) = 1 = cl_{T_2}cl_{T_1}(1-\lambda)$, for a pairwise fuzzy dense set λ in (X,T_1,T_2).

Theorem 3.2 [9] : A fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy resolvable space if $\vee_{k=1}^{N}(\lambda_k) = 1$, wherethe fuzzy sets (λ_k)'s are such that $int_{T_1}int_{T_2}(\lambda_k) = 0 = int_{T_2}int_{T_1}(\lambda_k)$, in ((X, T_1, T_2) .

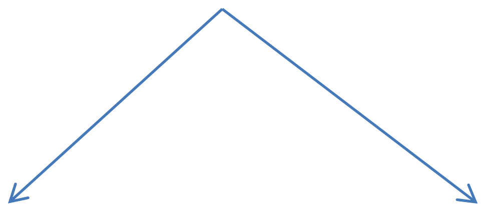
Theorem 3.3 [14]: If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy resolvable space, then(X,T_1,T_2) is apairwisefuzzy almost resolvable space.

Proposition 3.4 : If a fuzzybitopological space (X, T_1, T_2) is a pairwise fuzzyσ-resolvable space, then(X,T_1,T_2) is apairwisefuzzy resolvable space.

Proof : Supposethat thefuzzybitopological space (X, T_1, T_2) is a pairwise fuzzyσ-resolvable space. Then, by proposition 3.1, $\vee_{i=1}^{\infty}(\lambda_i) = 1$ and $cl_{T_1}cl_{T_2}(\lambda_i) =1 = cl_{T_2}cl_{T_1}(\lambda_i)$and $int_{T_1}int_{T_2}(\lambda_i)= 0$ and $int_{T_2}int_{T_1}(\lambda_i) = 0$.Now for thepairwise fuzzy dense set λ_i , we have $cl_{T_1}cl_{T_2}(1-\lambda i) = 1 - int_{T_1}int_{T_2}(\lambda_i)= 1$ and $cl_{T_2}cl_{T_1}(1-\lambda_i)= 1 - int_{T_2}int_{T_1}(\lambda_i) = 1$.Therefore $(X,T_1, T_2)$is a pairwise fuzzy resolvable space .

Remark 3.2: The relation between pairwise fuzzy □-resolvable space, pairwise fuzzy resolvable space and pairwise fuzzy almost resolvable space, can be summarized as follows :

pairwise fuzzy□-resolvable space



pairwise fuzzy resolvable spacepairwise fuzzy almost→resolvable space

Proposition 3.5:If$\wedge_{\square=I}^{\infty}(\mu_i) = 0$,wherethe fuzzy sets$(\mu_i)$'s in (X,T_1,T_2) are such thatint$_{T_I}$int$_{T_2}(\mu_\square) =0 =$ int$_{T_2}$int$_{T_1}(\mu_\square)$and$\mu_i \geq (1- \mu_j)$, for $i \neq j$,then(X,T_1,T_2) is a pairwise fuzzy σ-resolvable space.

Proof: Suppose that$\wedge_{\square=I}^{\infty}(\mu_i) =0$, where $int_{T_I}int_{T_2}(\mu_\square) = int_{T_2}int_{T_1}(\mu_\square) = 0$ and

$\mu_i \geq (1- \mu_j)$, for $i \neq j$.Then $1- \wedge_{\square=I}^{\infty}(\mu_i) = 1$, implies that$\vee_{\square=I}^{\infty}(1 - \mu_\square) = 1$. Also$1 - int_{T_I}int_{T_2}(\mu_\square) =1and1-int_{T_2}int_{T_1}(\mu_\square) = 1$, implies that $cl_{T_1}cl_{T_2}(1 - \mu_\square) = 1$

and $cl_{T_2}cl_{T_1}(1-\mu_{\square})=1$. Now $\mu_i \geq (1-\mu_j)$ implies that $(1-\mu_i) \leq 1-(1-\mu_j)$. Let $(1-\mu_i)=\square_{\square}$ .Hencewe have $\vee_{\square=1}^{\infty}(\square_{\square})=1$, where $cl_{T_1}cl_{T_2}(\square_{\square})=1$ and $cl_{T_2}cl_{T_1}(\square_{\square})=1$ and $\square_{\square} \leq 1-\square_{\square}(i \neq j)$.Therefore (X,T_1,T_2) is a pairwise fuzzy σ-resolvable space.

Proposition3.6: If $\wedge_{\square=1}^{\infty}(\mu_i)=0$, where (μ_i)'s are pairwise fuzzy nowhere dense sets in(X,T_1,T_2) such that $\mu_i \geq (1-\mu_j)$, for $i \neq j$, then (X,T_1,T_2) is a pairwise fuzzy σ-resolvable space.

Proof: Suppose that $\wedge_{\square=1}^{\infty}(\mu_i)=0$, where (μ_i)'s are pairwise fuzzy nowheredense sets in(X,T_1,T_2) such that $\mu_i \geq (1-\mu_j)$, for $i \neq j$.Since (μ_i)'s are pairwise fuzzy nowhere dense sets in(X,T_1,T_2), $int_{T_1}cl_{T_2}(\mu_i)=0$ and $int_{T_2}cl_{T_1}(\mu_i)=0$. Since $int_{T_1}int_{T_2}(\mu_{\square}) \leq int_{T_1}cl_{T_2}(\mu_i)$ and $int_{T_2}int_{T_1}(\mu_{\square}) \leq int_{T_2}cl_{T_1}(\mu_i)$, we have $int_{T_1}int_{T_2}(\mu_{\square})=0$ and $int_{T_2}int_{T_1}(\mu_{\square})=0$.Then we have $\wedge_{\square=1}^{\infty}(\mu_i)=0$, wherethe fuzzy sets$(\mu_i)$'s in (X,T_1,T_2) are such that $int_{T_1}int_{T_2}(\mu_{\square})=0=int_{T_2}int_{T_1}(\mu_{\square})$ and $\mu_i \geq (1-\mu_j)$, for $i \neq j$ and hencebyproposition 3.3, (X,T_1,T_2) is a pairwise fuzzy □-resolvable space**.**

Definition3.3[13]: A fuzzy set λ in a fuzzybitopologicalspace(X,T_1T_2) is called apairwise fuzzy σ-nowhere dense set if λ is a pairwise fuzzyF_σ-set setin(X,T_1T_2) such that $int_{T_1}int_{T_2}(\square)=0$ and $int_{T_2}int_{T_1}(\square)=0$.

Proposition 3.7 : If $\wedge_{\square=1}^{\infty}(\mu_i)=0$, where (μ_i)'s are pairwise fuzzy σ-nowhere dense sets in(X, T_1, T_2) such that $\mu_i \geq (1-\mu_j)$, for $i \neq j$, then (X, T_1, T_2) is a pairwise fuzzy σ-resolvable space.

Proof: Suppose that $\wedge_{\square=1}^{\infty}(\mu_i)=0$, where (μ_i)'s are pairwise fuzzyσ-nowhere dense sets in(X,T_1,T_2) such that $\mu_i \geq (1-\mu_j)$, for $i \neq j$. Since(μ_i)'s are pairwise fuzzyσ-nowhere dense sets in (X,T_1,T_2), (μ_i)'s are pairwise fuzzy F_σ-sets in (X,T_1T_2) such that $int_{T_1}int_{T_2}(\mu_{\square})=0$ and $int_{T_2}int_{T_1}(\mu_{\square})=0$.Then, we have $\wedge_{\square=1}^{\infty}(\mu_i)=0$, wherethe fuzzysets$(\mu_i)$'s in (X,T_1,T_2) are such that $int_{T_1}int_{T_2}(\mu_{\square})=0=int_{T_2}int_{T_1}(\mu_{\square})$ and $\mu_i \geq (1-\mu_j)$, for $i \neq j$ and hence by proposition 3.3, (X,T_1,T_2) is a pairwise fuzzy σ-resolvable space.

Definition 3.4 [13] : A fuzzy bitopological space (X, T_1,T_2) is called a pairwise fuzzy hyper connected space if □ is apairwise fuzzy set in (X,T_1,T_2) ,then $cl_{T_{\square}}(\lambda)=1$ (i= 1,2).

Remark 3.3 [13] : If (X, T_1,T_2) is a pairwise fuzzy hyper connected space and if □ is apairwise fuzzy open set in (X,T_1,T_2) ,then $cl_{T_1}cl_{T_2}(\lambda)=cl_{T_2}cl_{T_1}(\lambda)=1$.

Proposition: 3.8: If $\vee_{i=1}^{\infty}(\lambda_i)=1$,where $(\lambda_i)'$□ are pairwise fuzzy open sets such that $(\lambda_i) \leq (1-\lambda_j)(i \neq j)$ in a pairwise fuzzy hyperconnected space (X,T_1,T_2), then (X,T_1,T_2)is a pairwisefuzzy σ-resolvable space.

Proof : Suppose that $\vee_{i=1}^{\infty}(\lambda_i)=1$, where$(\lambda_i)'$□ arepairwise fuzzyopensets such that $\lambda_i \leq (1-\lambda_j)$ ($i \neq j$) in (X,T_1,T_2) . Since (X,T_1,T_2) is a pairwise fuzzy hyper connected space,forthe pairwise fuzzy open sets $(\lambda_i)'$□ in (X,T_1,T_2),we have $cl_{T_1}cl_{T_2}(\lambda_i)=(1)=1$ and $cl_{T_2}cl_{T_1}(\square_{\square})=cl_{T_2}(1)=1$.Hence wehave$\vee_{i=1}^{\infty}(\lambda_i)=1$, where $cl_{T_1}cl_{T_2}(\square_{\square})=1$ and $cl_{T_2}cl_{T_1}(\lambda_i)=1$ and $\lambda_i \leq (1-\lambda_j)$for $i \neq j$.Therefore (X,T_1,T_2) is a pairwise fuzzy σ-resolvable space.

Definition3.5 .[11]: A fuzzybitopological space (X,T_1,T_2) is called apairwise fuzzysubmaximal space iffor each fuzzy set λ in(X,T_1,T_2) such that $cl_{T_1}cl_{T_2}(\lambda)=1$ and $cl_{T_2}cl_{T_1}(\lambda)=1$,then $\lambda \in T_i$, $i=1,2$ in (X,T_1,T_2).

Proposition3.9 : If the fuzzy bitopological space (X, T_1, T_2) is apairwise fuzzy σ-resolvable space, then(X,T_1,T_2) is not a pairwise fuzzy submaximal space.

Proof: Let(X,T_1,T_2) be a pairwise fuzzy σ-resolvable space. Then, by proposition3.1, $\vee_{\square=1}^{\infty}(\square_{\square})=1$, where $cl_{T_1}cl_{T_2}(\square_{\square})=1$ and $cl_{T_2}cl_{T_1}(\square_{\square})=1$ and $int_{T_1}int_{T_2}(\lambda_i)=0$ and $int_{T_2}int_{T_1}(\lambda_i)=0$. Also from the remarks 3.1, thepairwise fuzzy dense sets in (X,T_1,T_2) are not pairwise fuzzy open sets. Therefore (X,T_1,T_2) is not apairwise fuzzy submaximal space.

Definition 3.6[4]: Afuzzybitopological space (X,T_1,T_2) is said to bepairwise fuzzystrongly connectedif it has no proper fuzzy sets□$_1$and□$_2$ $\in T_1^c \cup T_2^c$ such that □$_1$ + □$_2$ ≤ 1, whereT_1^c and T_2^c stand for the family of T_1 fuzzy closed sets and T_2 fuzzy closed sets in X .

Proposition 3.10 : If a fuzzybitopological space (X, T_1, T_2) is a pairwise fuzzy□-resolvable space, then(X,T_1,T_2) is apairwisefuzzy strongly connected space.

Proof : Supposethat thefuzzybitopological space (X, T_1, T_2) is a pairwise fuzzy□-resolvable space. Then, we have $\vee_{i=1}^{\infty} (\lambda_i) = 1$,where$(\lambda_i)$'s are pairwise fuzzy dense setsin (X,T_1,T_2) and$\lambda_i \leq 1 - \lambda_j$for $i \neq j$.By theremark 3. 2, (λ_i)'s ($\neq$ 1), are notpairwise fuzzy closed setsin(X, T_1, T_2) . Therefore (X, T_1, T_2) has no proper pairwise fuzzy closedsetsλ_i, λ_j (for $i \neq j$) such that $\lambda_i + \lambda_j \leq 1$, for $i \neq j$. Hence, (X,T_1,T_2) is apairwisefuzzy strongly connected space.

Remark3.4[4]: Pairwise fuzzy strong connectedness implies pairwise fuzzy connectedness. However the converse is not true.

Proposition 3.11 : If a fuzzybitopological space (X, T_1, T_2) is a pairwise fuzzy□-resolvable space, then(X,T_1,T_2) is apairwisefuzzy connected space.

Proof :The proof follows from proposition 3.10 andremark 3.4.

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