Int Jr. of Mathematics Sciences & Applications
Vol.3, No.1, January-June 2013

ISSN No: 2230-9888
www.journalshub.com

A NEW VIEW ON COMPACTIFICATION IN A CENTRED QUASI UNIFORM STRUCTURE SPACE

G. Vasuki
vasukigj@yahoo.in

E. Roja and M.K. Uma
R. Narmada Devi
Department of Mathematics,
Sri Sarada College for Women,
Salem - 636 016
Tamil Nadu.

Abstract

In this paper the concepts of centred quasi uniform structure space, centred quasi uniform structure point symmetric space, centred quasi uniform structure bi-completion and centred T_0^* - compactification are introduced and some of its properties are discussed.

Keywords

Centred quasi uniform structure space, centred quasi uniform structure point symmetric space, centred quasi uniform structure bi-completion and centred T_0^* - compactification.

2010 AMS subject classification Primary : 54A05, 54A10, 54A20.

INTRODUCTION

Certain concepts like uniform continuity, cauchy sequence, completeness etc. defined in a metric space and they play very significant roles in the general theory of metric spaces and analysis. These concepts cannot be applied in a topological space in general. So attempts were made to define structures. In 1937 Andre Weil [1] formulated the concept of uniform space which is a generalisation of a metric space. After that quasi uniformities has emerged since 1948 when Nachbin [10] began the study of these structures. Fletcher and Lindgren [2] studied the construction of compactifications for Hausdorff quasi-uniform spaces. Further contributions in this direction were given in [3] and [12]. Later on a study and description of the structure of T_0^* - compactification of a quasi uniform was carried out in [13]. In this paper we introduce Centred quasi uniform structure space, centred quasi uniform structure point symmetric space, centred quasi uniform structure bi-completion and centred T_0^* - compactification, and as well as its relation with the compactness of the stability space which was recently introduced and discussed by [9].

2. Preliminaries

Definition 2.1.1 [5]

Let (X, T) be a Hausdorff Space. The system P = $\{A_i\}_{i\in\Lambda}$ of all open sets of (X, T) is said to be centred if any finite collections of sets $\{A_i\}_{i=1}^{n}$ such that $\bigcap_{i=1}^{n} A_i \neq \phi$. The system P is called maximal centred system (or) an end. If it cannot be included in any larger centred system.

Definition 2.1.2 [11]

A filter on a set X is a set $\mathcal{F}$ of subsets of X which has the following properties

(i). Every subset of X which contains a set $\mathcal{F}$ belongs to $\mathcal{F}$

(ii). Every finite intersection of sets of $\mathcal{F}$ belongs to $\mathcal{F}$

(iii). The empty set is not in $\mathcal{F}$

Definition 2.1.3 [11]

A uniformity for a set X is a non-void family $\mathcal{U}$ of subsets of $X \times X$ such that

(i). Each member of $\mathcal{U}$ contains the diagonal Δ

(ii). If $U \in \mathcal{U}$ the $U^{-1} \in U$

(iii). If $U \in \mathcal{U}$, then $V \circ V \subset U$ for some V in $\mathcal{U}$

(iv). If U and V are members of $\mathcal{U}$, then $U \cap V \in \mathcal{U}$

(v). If $U \in \mathcal{U}$ and $U \subset V \subset X \times X$, then $V \in \mathcal{U}$.

The pair (X, $\mathcal{U}$) is a uniform space. The sets of $\mathcal{U}$ called entourages of the uniformity defined on X by $\mathcal{U}$.

Definition 2.1.4 [4] A quasi-uniformity on a set X is a filter $\mathcal{U}$ on $X \times X$ such that each member of $\mathcal{U}$ contains the diagonal of $X \times X$ and if $U \in \mathcal{U}$, then $V \circ V \subseteq U$ for some $V \in \mathcal{U}$. The pair (X, $\mathcal{U}$) is called a quasi-uniform space. $\mathcal{U}$ generates a topology $\tau(\mathcal{U})$ containing all subsets G of X such that for $x \in G$, there exists $U \in \mathcal{U}$ with $U(x) \subseteq G$, where $U(x) = \{ y \in X : (x,y) \in U \}$.

Definition 2.1.5 [6] A topological space is a T_0 - space iff for each pair x and y of distinct points, there is a neighbourhood of one point to which the other does not belong.

Definition 2.1.6 [6] A filter $\mathcal{F}$ on a uniform space X is called a Cauchy filter if for each entourage V of X there is a subset of X which is V-small and belongs to $\mathcal{F}$. In otherwords a Cauchy filter is one containing arbitrarily small sets.

Definition 2.1.7 [7] The minimal elements of the set of Cauchy filters on a uniform space X are called minimal Cauchy filters on X.

Definition 2.1.8 [7] A quasi uniform space (X, $\mathcal{U}$) is called point symmetric if $\mathcal{T}(\mathcal{U}) = \mathcal{T}(\mathcal{U}^{-1})$.

Definition 2.1.9 [7] A compactification of a T_1 quasi- uniform space (X, $\mathcal{U}$) is a compact T_1 quasi-uniform space (Y, $\mathcal{V}$) that has a $\mathcal{T}(\mathcal{V})$ dense subspace quasi- isomorphic to (X, $\mathcal{U}$).

Definition 2.1.10 [7] A quasi-uniform space (X, $\mathcal{U}$) is said to be bicomplete if each Cauchy filter on (X, $\mathcal{U}^*$) converges with respect to the topology $\mathcal{T}(\mathcal{U}^*)$ i.e., if the uniform space (X, $\mathcal{U}^*$) is complete.

Lemma 2.1.1 [8] A T_1 quasi-uniform space (X, $\mathcal{U}$) is *-compactifiable iff it is point symmetric and its bicompletion is compact.

Lemma 2.1.2 [8] Let (X, $\mathcal{U}$) be a T_1 quasi - uniform space such that $\mathcal{U}^{-1}$ is hereditarily precompact. Then (X, $\mathcal{U}$) is *- compactifiable iff it is point symmetric and precompact.

3. T_0^* - compactification in a centred quasi-uniform structure space.

Definition 3.1.1 Let $\mathbb{P}_X = \{P_i : i \in \Lambda\}$ be a nonempty set where P_i' s are centred systems in a Hausdorff spaces (X, T) and P_i's are called centred points in $\mathbb{P}_X$. Then $(P_i, P_j) = \{A \times B : A \in P_i, B \in P_j\}$ where $P_i, P_j \in \mathbb{P}_X$. A nonempty set $\mathbb{P}_X \times \mathbb{P}_X$ is the collection of all ordered pair (P_i, P_j) where eachP_i, $P_j \in \mathbb{P}_X$.

Notation 3.1.1

(i) $\mathcal{P}(\mathbb{P}_X)$ denotes the power set of $\mathbb{P}_X$.

(ii) $\mathcal{P}(\mathbb{P}_X \times \mathbb{P}_X)$ denotes the power set of $\mathbb{P}_X \times \mathbb{P}_X$.

Definition 3.1.2 Let $\mathbb{P}_X = \{ P_i : i \in \Lambda \}$ be a nonempty set where P_i' s are centred systems in a Hausdorff space (X, T). For $A \in \mathcal{P}(\mathbb{P}_X)$ and$U \in \mathcal{P}(\mathbb{P}_X \times \mathbb{P}_X)$, the image of A under the relation U is defined by $U(A) = \{ Q \in \mathbb{P}_X : (P, Q) \in U, \text{ for } P \in A \}$.

In particular, for $P \in \mathbb{P}_X$, $U(P) = \{ Q \in \mathbb{P}_X : (P, Q) \in U \}$.

Definition 3.1.3 A family $\mathfrak{F} \subset \mathcal{P}(\mathbb{P}_X)$ is said to be centred filter on $\mathbb{P}_X$ if it satisfies the following conditions

(i). $\Phi \notin \mathfrak{F}$

(ii). If $A, B \in \mathfrak{F}$ then $A \cap B \in \mathfrak{F}$

(iii). If $A \in \mathfrak{F}$, $B \in \mathcal{P}(\mathbb{P}_X)$ such that $A \subseteq B$ then $B \in \mathfrak{F}$

Definition 3.1.4 A centred quasi uniformity on a set $\mathbb{P}_X$ is a structure given by a set $\mathcal{U}$ of subsets of $\mathbb{P}_X \times \mathbb{P}_X$ which satisfies the following conditions:

(i). Every set belonging to $\mathcal{U}$ contains the diagonal $\Delta = \{ (P_1, P_1) \mid P_1 \in \mathbb{P}_X \}$

(ii). If $U, V \in \mathcal{U}$ then $U \cap V \in \mathcal{U}$

(iii). If $U \in \mathcal{U}$ and $U \subset V \subset \mathbb{P}_X \times \mathbb{P}_X$, then $V \in \mathcal{U}$

(iv). If $U \in \mathcal{U}$, V in $\mathcal{U}$ then $V \circ V \subset \mathcal{U}$ where $\circ$ is defined as

$U \circ V = \{ (P_1, P_3) : \exists P_2 \in \mathbb{P}_X;\ (P_1, P_2) \in U, (P_2, P_3) \in V \}$

The sets of $\mathcal{U}$ are called centred entourages of the centred-quasi uniformity $\mathcal{U}$ on $\mathbb{P}_X$. The set $\mathbb{P}_X$ endowed with a centred quasi uniformity $\mathcal{U}$ is called a centred quasi uniform space. The pair $(\mathbb{P}_X, \mathcal{U})$ is called a centred quasi uniform space.

Definition 3.1.5 Let $(\mathbb{P}_X, \mathcal{U})$ be a centred quasi uniform space and let V be any centred entourages of $\mathbb{P}_X$. A subset A of $\mathbb{P}_X$ is said to be a centred V-small set if $A \times A \subset V$.

Definition 3.1.6 Let $\mathbb{P}_X = \{ P_i : i \in \Lambda \}$ be a nonempty set where P_i' s are centred systems in a Hausdorff space (X, T). If $\mathcal{U}$ is a centred quasi uniformity on a set $\mathbb{P}_X$, then $\mathcal{U}^{-1} = \{ U^{-1} : U \in \mathcal{U} \}$ is also a centred quasi uniformity on a set $\mathbb{P}_X$ and it is called the centred conjugate of $\mathcal{U}$. Also, the centred quasi uniformity $\mathcal{U} \cup \mathcal{U}^{-1}$will be denoted by $\mathcal{U}^*$. If $U \in \mathcal{U}$ the entourage $U \cap U^{-1}$ of $\mathcal{U}^*$ will be denoted by U*.

Definition 3.1.7 Let $\mathbb{P}_X = \{ P_i : i \in \Lambda \}$ be a nonempty set where P_i' s are centred systems in a Hausdorff space (X, T). Let $\mathcal{U}$ be a centred quasi-uniformity on $\mathbb{P}_X$. Then a centred quasi uniform structure $\mathbb{T}_{\mathcal{U}}$ induced by a centred quasi-uniformity $\mathcal{U}$ on $\mathbb{P}_X$ is defined by

$\mathbb{T}_{\mathcal{U}} = \{ A \subseteq \mathbb{P}_X : \text{for each } P \in A, \text{ there is } U \in \mathcal{U} \text{ such that } U(P) \subseteq A \}$

The pair $(\mathbb{P}_X, \mathbb{T}_{\mathcal{U}})$ is called a centred quasi uniform structure space. Each element of $\mathbb{T}_{\mathcal{U}}$ is a $\mathbb{T}_{\mathcal{U}}$ open set. The complement of $\mathbb{T}_{\mathcal{U}}$ open set is $\mathbb{T}_{\mathcal{U}}$ closed set.

Definition 3.1.8 Let $(\mathbb{P}_X, \mathbb{T}_{\mathcal{U}})$ be a centred quasi uniform structure space. A centred point $P \in \mathbb{P}_X$ is said to be a centred quasi uniform closed point if {P} is a $\mathbb{T}_{\mathcal{U}}$ closed set.

Definition 3.1.9 Let $(\mathbb{P}_X, \square_{\square})$ be a centred quasi uniform structure space. Let A and B be any two sets in $\square(\mathbb{P}_X)$. Then A is a $\square_{\square}$ neighbourhood of B if there exists a $\square_{\square}$ open set O such that $B \subseteq O \subseteq A$.

Note 3.1.1 Let A be a $\square_{\square}$ neighbourhood of B. If $P \in B$, then A is also $\square_{\square}$ neighbourhood of P, where $P \in \mathbb{P}_X$.

Definition 3.1.10 Let $(\mathbb{P}_X , \square_{\square})$ be a centred quasi uniform structure space and let A be any set in $\square(\mathbb{P}_X)$. Then the centred quasi uniform structure closure of A (inshort, $centQ_{\square}cl(A)$) is defined as cent $Q_{\square}cl(A)= \cap \{ B \in \square(\mathbb{P}_X) :$ $A \subseteq B$ and B is $\square_{\square}$ closed set$\}$.

Definition 3.1.11 Let $(\mathbb{P}_X , \square_{\square})$ be a centred quasi uniform structure space and let A be any set in $\square(\mathbb{P}_X)$. Then the centred quasi uniform structure interior of A (inshort, $centQ_{\square}int(A)$) is defined as $centQ_{\square}int(A)= \cup \{ B \in \square(\mathbb{P}_X) :$ $A \supseteq B$ and B is $\square_{\square}$ open set$\}$.

Definition 3.1.12 Let $(\mathbb{P}_X , \square_{\square})$ be a centred quasi uniform structure space. Let $\square = \{A_i \}$ where each A_i is a $\square_{\square}$ open sets in $\mathbb{P}_X$. Then $\square$ is said to be $\square_{\square}$ open cover of $\mathbb{P}_X$ if $\bigcup_{A_i \in \square} A_i = \mathbb{P}_X$ where $i \in I$.

Definition 3.1.13 Let $(\mathbb{P}_X , \square_{\square})$ be a centred quasi uniform structure space. Then $(\mathbb{P}_X , \square_{\square})$ is said to be a centred quasi uniform structure compact space if every $\square_{\square}$ open cover $\square$ of $\mathbb{P}_X$ contains a finite subcollection that also covers $\mathbb{P}_X$.

Definition 3.1.14 Let $(\mathbb{P}_X , \square_{\square})$ be a centred quasi uniform structure space. A centred quasi uniform structure net in $\mathbb{P}_X$ is a function $f : D \rightarrow \mathbb{P}_X$ defined by $f(m)=F(m) \in \mathbb{P}_X \ \forall\ m \in D$ where D is the direct set.

Definition 3.1.15 A centred quasi uniform net $\{ p_m : m \in D \}$ in the centred quasi uniform structure space $(\mathbb{P}_X , \square_{\square})$ is said to be a centred quasi uniform structure Cauchy net iff for each member U of $\square$, there is N in D such that $(p_n, p_m) \in U$ whenever both n and m follow N in the ordering of the direct set D.

Definition 3.1.16 Let $(\mathbb{P}_X , \square_{\square})$ be a centred quasi uniform structure space. Then $(\mathbb{P}_X , \square_{\square})$ is said to be a centred quasi uniform structure complete space iff every centred quasi uniform structure Cauchy net in the centred quasi uniform structure space converges to a centredpoint of the centred quasi uniform structure space.

Definition 3.1.17 A centred quasi-uniform structure space $(\mathbb{P}_X , \square_{\square})$ is said to be a centred quasi-uniform structure totally bounded space if $(\mathbb{P}_X , \square_{\square})$ is both centred quasi-uniform structure compact space and centred quasi-uniform structure complete space.

Definition 3.1.18 Let A be any set in $\square(\mathbb{P}_X)$. Then A is called a centred quasi uniform structure dense in a centred quasi uniform structure space $(\mathbb{P}_X , \square_{\square})$ iff the centred quasi uniform structure closure of A is $\mathbb{P}_X$.

Definition 3.1.19 A centred quasi-uniform structure space $(\mathbb{P}_X , \square_{\square})$ is said to be centred quasi uniform structure point symmetric if $\square_{\square} \subseteq \square_{\square^{-1}}$.

Definition 3.1.20 A centred quasi-uniform structure space $(\mathbb{P}_X , \square_{\square})$ is said to be centred quasi uniform structure precompact space if for each $U \in \square$ there is a finite subset A of $\mathbb{P}_X$ such that $U(A)= \mathbb{P}_X$.

Definition 3.1.21 A centred quasi-uniform structure space $(\mathbb{P}_X , \square_{\square})$ is said to be centred quasi-uniform structure hereditarily precompact space if any subspace of $(\mathbb{P}_X , \square_{\square})$ is centred quasi-uniform structure precompact space and it is centred quasi uniform structure totally bounded space provided that $\square^{*}$ is a centred totally bounded uniformity on $\mathbb{P}_X$.

Definition 3.1.22 Let $(\mathbb{P}_X , \square_{\square})$ be a centred quasi uniform structure space. For any $P \in \mathbb{P}_X$, $\mathcal{F}_P = \{ B : B \in \square(\mathbb{P}_X)$ and B is a neighbourhood of P $\}$. If $\mathcal{F}_P$ satisfies the axioms of centred filter, $\mathcal{F}_P$ is called a centred neighbourhood filter of P.

Definition 3.1.23 Let A be any set in $\square(\mathbb{P}_X)$. If $P \in \mathbb{P}_X$. Then P is said to be centred cluster point of A if every centred neighbourhood filter of P which intersects A in some centred point other than P itself.

Definition 3.1.24 A centred filter $\mathcal{F}$ on a centred quasi-uniform structure space $(\mathbb{P}_X , \square_{\square})$ is called centred quasi uniform structure stable filter if for each $U \in \square$, $\bigcap_{F \in \mathcal{F}} U(F) \in \mathcal{F}$ and $\mathcal{F}$ is called centred quasi uniform structure doubly stable filter if it is centred quasi uniform structure stable filter both for $(\mathbb{P}_X , \square_{\square})$ and $(\mathbb{P}_X , \square_{\square^{-1}})$.

Definition 3.1.25 A centred filter $\mathcal{F}$ on a centred quasi-uniform structure space $(\mathbb{P}_X , \square_{\square})$ is said to be

(i). centred quasi uniform structure Cauchy filter if it contains arbitrarily centred V-small sets, for each $V \in \square$

(ii). centred quasi uniform structure left K-Cauchy filter if for each $U \in \square$ there exists $F \in \mathcal{F}$ such that $U(P) \in \mathcal{F}$ for all $P \in \mathcal{F}$.

Definition 3.1.26 Let $(\mathbb{P}_X , \square_\square)$ be a centred quasi-uniform structure space. Then $(\mathbb{P}_X , \square_\square)$ said to be a centred quasi uniform structure left K-complete space iff every centred quasi uniform structure left K-Cauchy filter converges with respect to $\square_\square$.

Definition 3.1.27 A centred quasi uniform structure space $(\mathbb{P}_X , \square_\square)$ is called a centred quasi uniform structure bicomplete space if each centred quasi uniform structure Cauchy filter on $(\mathbb{P}_X , \square_{\square^*})$ converges with respect to $\square_{\square^*}$. That is, if the centred quasi uniform structure space $(\mathbb{P}_X , \square_{\square^*})$ is centred quasi uniform structure complete space.

Definition 3.1.28 A centred quasi uniform structure bicompletion of a centred quasi uniform structure space $(\mathbb{P}_X , \square_\square)$ is a centred quasi uniform structure bicomplete space $(\mathbb{P}_Y , \square_\square)$ that has $\square(\square*)$ centred quasi uniform structure dense subspace quasi isomorphic to $(\mathbb{P}_X , \square_\square)$. More formally, there is a centred quasi-uniform structure embedding $i : (\mathbb{P}_X , \square_\square) \to (\mathbb{P}_Y , \square_\square)$.

Definition 3.1.29 A centred quasi uniform structure space $(\mathbb{P}_X , \square_\square)$ is centred T_0 quasi uniform structure space iff for every pair $P,Q \in \mathbb{P}_X$ with $P\neq Q$ there exist $\square_\square$ open sets A and B such that $P \in A$ and $Q \notin A$ (or) $Q \in B$ and $P \notin B$.

Definition 3.1.30 Let $(\mathbb{P}_X , \square_\square)$ be centred quasi uniform structure space. Let $\mathcal{F}_\square$ be a centred quasi uniform structure Cauchy filters on $\mathbb{P}_X$. The centred quasi uniform structure minimalCauchy filter on a centred quasi uniform space $(\mathbb{P}_X , \square_\square)$ is defined by $\mathcal{F}_\square = \bigcap_{i\in\Lambda} \mathcal{F}_\square$

Notation 3.1.2

(i). Let $\mathbb{P}_{\tilde{X}}$ denote the collection of all centred quasi uniform structure minimalCauchy filters in centred quasi uniform structure space $(\mathbb{P}_X , \square_{\square^*})$ and that the family $\{ \widetilde{U} : U\in \square \}$ is a base for $\tilde{\square}$ where for each $U\in \square$, $\widetilde{U} = \{ (\mathcal{F},\square) \in \mathbb{P}_{\tilde{X}} \times \mathbb{P}_{\tilde{X}} :$ there are $F\in \mathcal{F}$ and $G\in \square$ with $F \times G \subseteq U \}$ and the centred quasi uniform structure embedding $i : (\mathbb{P}_X , \square_\square)\to (\mathbb{P}_{\tilde{X}} , \square_{\tilde{\square}})$ is given as follows: for $P \in \mathbb{P}_X$, we have that i(P) is the $\square_{\square^*}$-centred neighbourhood filter at P.

(ii). Each centred quasi uniform structure space $(\mathbb{P}_X , \square_\square)$ has an unique T_0 centred T_0 quasi uniform structure bicompletion which will be denoted by $(\mathbb{P}_{\tilde{X}} , \square_{\tilde{\square}})$.

Definition 3.1.31 A centred T_0^*-compactification of a centred T_0 quasi-uniform structure space $(\mathbb{P}_X , \square_\square)$ is a centred compact T_0 quasi uniform space $(\mathbb{P}_Y, \square_\square)$ that has a centred dense subspace $(\mathbb{P}_Y , \square_{\square^*})$ isomorphic to $(\mathbb{P}_X , \square_\square)$.

Definition 3.1.32 A centred T_0 quasi uniform structure space is centred T_0^*-compactifiable iff its centred quasi uniform structure bicompletion is centred quasi uniform structure compact.

Definition 3.1.33 Let $(\mathbb{P}_X , \square_\square)$ be a centred quasi-uniform structure space. Let $P_0(\mathbb{P}_X)$ be the collection of all nonempty centred subsets of $\mathbb{P}_X$. Then, the Hausdorff - Bourbaki centred quasi structure uniformity of $(\mathbb{P}_X , \square_\square)$ is the centred quasi uniformity $\square_H$ on $P_0(\mathbb{P}_X)$ which has a base, the family of sets of the form

$U_H = \{ (A,B) \in P_0(\mathbb{P}_X)\times P_0(\mathbb{P}_X) : B \subseteq U(A), A\subseteq U^{-1}(B) \}$ where $U \in \square$.

If $(\mathbb{P}_X , \square_\square)$ is a centred T_0 quasi uniform structure space, then $(P_0(\mathbb{P}_X), \square_H)$ is not necessarily centred T_0 quasi uniform structure space. Hence it is possible to construct the set

$C_n(\mathbb{P}_X)= \{ A' : A \in P_0(\mathbb{P}_X) \}$ where $A'=$ cent $Q_\square cl(A) \cap$ cent $Q_{\square^{-1}} cl(A)$.

Definition 3.1.34 Let $(\mathbb{P}_X , \square_\square)$ be a centred quasi-uniform structure space. Let $A, B \in P_0(\mathbb{P}_X)$ and $U\in \square$. Then

(i). If $B \in U_H(A)$, then $B, B' \in (U^2)_H(A')$ and $B' \in (U^2)_H(A)$

(ii). If $B' \in U_H(A')$, then $B', B \in (U^2)_H(A)$ and $\square \in (U^2)_H(A')$

Proposition 3.1.1 Let $(\mathbb{P}_X , \square_\square)$ be a centred quasi uniform structure space. Then $(C_n(\mathbb{P}_X), \square_{\square_H})$ is centred quasi uniform structure compact iff $(P_0(\mathbb{P}_X), \square_{\square_H})$ is centred quasi-uniform structure compact.

Proof: Suppose that $(C_n(\mathbb{P}_X),\ \square_{\square_H})$ is centred quasi uniform structure compact and let (F_λ) be a centred quasi uniform structure net in $P_0(\mathbb{P}_X)$. Then (F'_λ) is a centred quasi uniform structure net in $C_n(\mathbb{P}_X)$. So it has a centred cluster point $C' \in C_n(\mathbb{P}_X)$. By Definition 4.1.34 it follows that C (and C') is a centred cluster point of (F_λ). Hence $(P_0(\mathbb{P}_X), \square_{\square_H})$ is centred quasi-uniform structure compact.

Conversely, suppose that $(P_0(\mathbb{P}_X), \square_{\square_H})$ is centred quasi uniform structure compact and let (F'_λ) be a centred quasi uniform structure net in $C_n(\mathbb{P}_X)$. Then (F_λ) is a centred quasi uniform structure net in $P_0(\mathbb{P}_X)$. So it has a centred cluster point $C \in P_0(\mathbb{P}_X)$. Hence by Definition 4.1.34, C' is a centred cluster point of (F'_λ). Hence $(C_n(\mathbb{P}_X),\ \square_{\square_H})$ is centred quasi uniform structure compact.

Remark 3.1.1 Let $(\mathbb{P}_X, \square_{\square})$ be a centred quasi uniform structure space. Then $(P_0(\mathbb{P}_X),\ \square_{\square_H})$ is centred quasi uniform structure compact iff $(\mathbb{P}_X, \square_{\square})$ is centred quasi uniform structure compact space and $(\mathbb{P}_{X_m},\ \square_{\square^{-1}})$ is centred quasi uniform structure hereditarily precompact space where $\mathbb{P}_{X_m}=\{ P \in \mathbb{P}_X : P$ is a minimal element in the pre-order of the centred quasi uniform structure space $(\mathbb{P}_X, \square_{\square})\ \}$.

Note 3.1.2

(i). If $\mathbb{P}_{X_m}$ is the set of centred quasi uniform closed points in $(\mathbb{P}_X, \square_{\square})$ if $(\mathbb{P}_X, \square_{\square})$ is centred T_0 quasi uniform structure space.

(ii). $G(\mathbb{P}_X)$ denotes the set of centred quasi uniform closed points in a centred quasi uniform structure space $(\mathbb{P}_{\tilde{X}}, \square_{\tilde{\square}_H})$.

Proposition 3.1.2 Let $(\mathbb{P}_X, \square_{\square})$ be a centred quasi uniform structure space and let the map $\square$: $(C_n(\mathbb{P}_X), \square_{\square_H}) \to (C_n(\mathbb{P}_{\tilde{X}}), \square_{\tilde{\square}_H})$ be defined by $\square(A')=$ cent $Q_{\square}cl(A) \cap$ cent $Q_{\square^{-1}}cl(A)$. Then $\square$ is a quasi isomorphism from $(C_n(\mathbb{P}_X), \square_{\square_H})$ onto $\square((C_n(\mathbb{P}_{\tilde{X}}), \square_{\tilde{\square}_H}))$. Furthermore $\square(C_n(\mathbb{P}_x))$ is centred quasi uniform structure dense in $(C_n(\mathbb{P}_{\tilde{\square}}), \square_{\tilde{\square}^*_H})$.

Proof: Since $\square(A') \cap \mathbb{P}_X \times \mathbb{P}_X = A'$, it follows that $\square$ is injective. We deduce from the Definition 4.1.34 that $\square$ and $\square^{-1}$ are centred quasi uniformly continuous. In order to prove that $\square(C_n(\mathbb{P}_X))$ is centred dense in $(C_n(\square_{\tilde{\square}}), \square_{\tilde{\square}^*_H})$, let $A \subseteq \mathbb{P}_{\tilde{X}}$ and $U \in \square$. Let $A^Q=$ cent $Q_{\tilde{\square}}cl(A) \cap$ cent $Q_{\tilde{\square}^{-1}}cl(A)$. For each $P \;\square\; A$, take $Q_p \in \mathbb{P}_X$ such that $P \;\square\; \tilde{U}^*(Q_p)$ and let $B=\{\ Q_p : P \;\square\; A\ \}$, then $A \subseteq \tilde{U}^*(B)$ and $B \subseteq \tilde{U}^*(A)$ and hence $A \square (\tilde{U}_H)^*(B)$. Which shows that $A^Q \square\ (\tilde{U}_H)^*(B')$.

Proposition 3.1.3 Let $(\square_X, \square_{\square})$ be a centred T_0 quasi uniform structure space. Then $(C_n(\square_{\tilde{X}}), \square_{\tilde{\square}_H})$ is a centred T_0^*-compactification of $(C_n(\square_X), \square_{\square_H})$ iff $(\square_X, \square_{\square})$ is a centred T_0^*-compactifiable and $(G(\mathbb{P}_X), \square_{\tilde{\square}^{-1}})$ is centred quasi uniform structure hereditarily precompact space.

Proof: Suppose that $(C_n(\square_{\tilde{X}}), \square_{\tilde{\square}_H})$ is a centred T_0^*-compactification of $(C_n(\square_X), \square_{\square_H})$. Then $(C_n(\square_{\tilde{X}}), \square_{\tilde{\square}_H})$ is centred quasi uniform structure compact space and by Remark 4.1.1 and Proposition 4.1.2, $(\square_{\tilde{X}}, \square_{\tilde{\square}_H})$ is centred quasi uniform structure compact space and $(G(\mathbb{P}_X), \square_{\tilde{\square}^{-1}})$ is centred quasi uniform structure hereditarily precompact space.

Conversely if $(\square_{\tilde{X}}, \square_{\tilde{\square}})$ is centred quasi uniform structure compact space and $(G(\mathbb{P}_X), \square_{\tilde{\square}^{-1}})$ is centred quasi uniform structure hereditarily precompact space, then $\left(C_n(\square_{\tilde{X}}), \square_{\tilde{\square}_H}\right)$ is centred quasi uniform structure compact space and by Proposition 4.1.2, it is a centred T_0*-compactification of $\left(C_n(\square_X), \square_{\square_H}\right)$.

Proposition 3.1.4 Let (□$_X$, □$_□$) be a centred T_0 quasi uniform structure space such that ($\mathbb{P}_X$, □$_{□^{-1}}$) is centred quasi uniform structure hereditarily precompact space. Then (□$_X$, □$_□$) is a centred T_0^* compactifiable iff it is centred quasi uniform structure precompact space.

Proof: Suppose that (□$_X$, □$_□$) be centred T_0^* compactifiable. Then (□$_{\tilde{X}}$, □$_{\tilde{□}}$) is a centred quasi uniform structure compact space , so it is a (□$_{\tilde{X}}$, □$_{\tilde{□}}$) centred quasi uniform structure precompact space. Thus (□$_X$, □$_□$) is a centred quasi uniform structure precompact space. Conversely, let (□$_X$, □$_□$) is a centred quasi uniform structure precompact space. Then (□$_{\tilde{X}}$, □$_{\tilde{□}}$) is a centred quasi uniform structure precompact space and ($\mathbb{P}_{\tilde{□}}$, □$_{\tilde{□}^{-1}}$) is a centred quasi uniform structure hereditarily precompact space. Now, let $\mathcal{F}$ be a centred quasi uniform structure left K-cauchy ultrafilter on(□$_{\tilde{X}}$, □$_{\tilde{□}}$). Since ($\mathbb{P}_{\tilde{□}}$, □$_{\tilde{□}^{-1}}$) is centred quasi uniform structure hereditarily precompact space, it follows that $\mathcal{F}$ is also centred quasi uniform structure left K-cauchy ultrafilter on (□$_{\tilde{□}}$,□$_{\tilde{□}^{-1}}$), and hence it is a centred quasi uniform structure quasi ultra filter on (□$_{\tilde{□}}$,□$_{\tilde{□}^*}$), so it converges with respect to □$_{\tilde{□}^*}$. Therefore (□$_{\tilde{□}}$,□$_{\tilde{□}^*}$) is a centred quasi uniform structure left K-complete space. And the conclusion of the theorem follows from that every centred quasi uniform structure precompact left K-complete space is centred quasi uniform structure compact space.

Corollary 3.1.1 Let (□$_X$, □$_□$) be a centred T_0 quasi uniform structure precompact space such that ($\mathbb{P}_X$, □$_{□^{-1}}$) is centred quasi uniform structure hereditarily precompact space. Then $\left(C_n(□_X), □_{□_H}\right)$ is centred T_0^* compactifiable and $\left(C_n(□_{\tilde{X}}), □_{\tilde{□}_H}\right)$ is a centred T_0^* compactification of $\left(C_n(□_X), □_{□_H}\right)$.

Notation 3.1.3 The stability space (□$_D$(□), □$_□$) on a quasi uniform space (X, U) was introduced in [24]. In this paper, centred stability space is introduced and is denoted by cent(□$_D$(□$_X$), □$_{□_D}$).

Remark 3.1.2

(i). Given a centred T_0 quasi uniform structure space (□$_X$, □$_□$), then $\left(C_n(□_X), □_{□_H}\right)$ is a centred T_0^*- compactifible iff the centred stability space cent(□$_D$(□$_X$), □$_{□_D}$)is a centred quasi uniform structure compact space.

(ii). Given a centred T_0 quasi uniform structure space (□$_X$, □$_□$) then cent(□$_D$(□$_X$), □$_{□_D}$) is a centred quasi uniform structure precompact space iff (□$_X$, □$_□$) is a centred quasi uniform strucutre precompact space and cent(□$_D$(□$_X$), □$_{□_D}$) is a centred quasi uniform structure totally bounded space iff (□$_X$, □$_□$) is centred quasi uniform structure totally bounded, It follows that if □ is a centred uniformity, then cent(□$_D$(□$_X$), □$_{□_D}$) is a centred quasi uniform structure compact space iff (□$_X$, □$_□$) is a centred quasi uniform strucutre totally bounded space.

Proposition 3.1.5 Let (□$_X$, □$_□$) be a centred T_0 quasi uniform structure space. Let A ⊆ $\mathbb{P}_X$ be such that for each P □ A and U□ □ there exists V□ □ with $V^{-1}(P) \subseteq U(P)$. If cent(□$_D$(□$_X$), □$_{□_D}$)is a centred quasi uniform structure compact space, then (A, $\tilde{□}^{-1}$) is centred quasi uniform structure hereditarily precompact space.

Proof: Suppose that (A, $\tilde{□}^{-1}$) is not centred quasi uniform structure hereditarily precompact space, then there exists B ⊆ A, U_0 □ □ and sequence $(b_n)_{n\in□}$in B such that $b_{n+1} \notin \bigcup_{i=1}^{n} U_0^{-1}(b_i)$ for all n □ N. For n □ N, put $B_n=\{b_i : i \le n\}$. Since cent(□$_D$($\mathbb{P}_X$), □$_D$) is a centred quasi uniform structure compact space, there exists a centred quasi uniform structure doubly stable filter □ □ cent(□$_D$($\mathbb{P}_X$)) such that $(B_n)_{n\in□}$ centred clusters to $\mathcal{F}$. Let U □ □ with $U^2 \subseteq U_0$. Let k□ □ be such that B_k □ $U_D(\mathcal{F})$. Then $B_k \subseteq U(F)$ for each F □ $\mathcal{F}$ and U^{-1} (B_k)□ □. Now consider the centred point Q_{k+1}. Let V□ □ with $V^{-1}(Q_{k+1}) \subseteq U(Q_{k+1})$. And let n > k+1 with B_n □$V_D(\mathcal{F})$. Then $B_n \subseteq V(F)$ for each F □ $\mathcal{F}$ and $V^{-1}(B_k)$ □ $\mathcal{F}$. From this it follows that $B_n \subseteq VU^{-1}(B_k)$. In particular, $Q_{k+1} \subseteq VU^{-1}(B_k)$, and hence $V^{-1}(b_{k+1}) \cap U^{-1}(B_k) \neq$ □. Then $b_{k+1}) \cap U^{-1}(B_k) \neq$ □ and Q_{k+1}□$U^{-2}(B_k) \subseteq U_0^{-1}(B_k)$ which is a contradiction. Therefore (A, $\tilde{□}^{-1}$) is centred quasi uniform structure hereditarily precompact space.

Corollary 3.1.2 Let ((□$_X$, □$_□$) be a centred point symmetric centred T_0 quasi uniform structure space with cent(□$_D$(□$_X$), □$_{□_D}$) is a centred quasi uniform structure compact space. Then ($\mathbb{P}_X$, □$_{□^{-1}}$) is a centred quasi uniform structure hereditarily precompact space.

Corollary 3.1.3 Let (□$_X$, □$_□$) be a centred T_0^* compactifiable centred quasi uniform structure space with cent(□$_D$(□$_X$), □$_{□_D}$)is centred uniform structure compact space. Then (G($\mathbb{P}_X$), □$_{\tilde{□}}$) is a centred quasi uniform structure point symmetric space and (G($\mathbb{P}_X$), □$_{\tilde{□}^{-1}}$) is a centred quasi uniform structure hereditarily precompact space.

Corollary 3.1.4 Let (□$_X$, □$_□$) be a centred T_0^*-compactifible quasi uniform space. Then $\left(C_n(□_X), □_{□_H}\right)$ is centred T_0^*-compactifiable iff (G($\mathbb{P}_X$), □$_{\tilde{□}^{-1}}$) is centred quasi uniform structure hereditarily precompact. Furthermore, in this case $\left(C_n(□_{\tilde{X}}), □_{\tilde{□}_H}\right)$ is centred T_0*-compactification of $\left(C_n(□_X), □_{□_H}\right)$.

Note 3.1.3 Every centred quasi uniform structure T_0 point symmetric space is centred T_1 quasi uniform structure space.

Corollary 3.1.5 Let (□$_X$, □$_□$) be a centred point symmetric centred T_0 quasi-uniform space. Then the following are equivalent:

(i). cent(□$_D$(□$_X$), □$_{□_D}$) is centred quasi uniform structure compact.

(ii). (□$_X$, □$_□$) is centred T_0^*-compactifiable and ($\mathbb{P}_X$, □$_{□^{-1}}$) is centred quasi uniform structure hereditarily precompact.

(iii). (□$_X$, □$_□$) is centred T_0^*-compactifiable and (G(□$_X$), □$_{\tilde{□}^{-1}}$) is centred quasi uniform structure hereditarily precompact.

(iv). $\left(C_n(□_{\tilde{X}}), □_{\tilde{□}_H}\right)$ is centred T_0 quasi uniform structure compact.

(v). $\left(C_n(□_X), □_{□_H}\right)$ is centred T_0^*-compactifiable.

(vi). $\left(C_n(□_{\tilde{X}}), □_{\tilde{□}_H}\right)$ is centred T_0^*-compactification of $\left(C_n(□_X), □_{□_H}\right)$.

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