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Particle in a Box : Semi Macro and Micro system of Energy

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Abstract

If a gaseous particle having mass 'm' and velocity 'v' traces a path as circular, inside a box, then supposing the path circular, centripetal force comes into play. The energy required to rotate the particle on circular path, fuel, as energy, obtained by the moving particle, is equal to half to the product of centripetal force 'F' and radial distance 'r'.

Key Words: Atomic and Nuclear physics – Matter wave

Let us consider a particle of mass m in a box (Fig. 01). The particle is restrained to move only in one dimension, say along the x-axis. At the position x=0 and x=a, the particle undergoes perfect elastic collision with the wall of the box. The particle does not experience any other force. The momentum of the particle = p_x = p. It remains constant but but only changes in sign when it collides against the wall

The phase integral for the periodic motion of the particle is given by-

$$J = \oint p_x \, dx$$

$$J = \oint p_x \, dx \qquad \text{……..….. (1)}$$

$$J = p \; x \; 2pa \qquad \text{………….. (2)}$$

According to Wilson-Somerfield quantization rule-

$$\oint p \, dx = nh \qquad \text{………… (3)}$$

Or $$2pa = nh$$

Or $$P = \frac{nh}{2a}$$

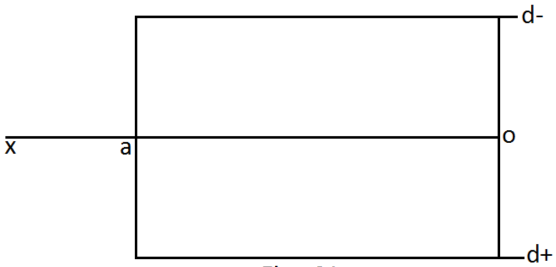

Fig.: 01

The total energy of the system is only due to its kinetic energy-

$$E_n = \frac{1}{2}mu^2 = \frac{p^2}{2m} \qquad \text{............ (4)}$$

Substituting the value of *p*, we get

$$E_n = \frac{n^2h^2}{8ma^2} \qquad \text{............ (5)}$$

Here n = 0, 1, 2, 3… etc.

For n = 1

$$E_1 = \frac{h^2}{8ma^2} \qquad \text{............ (6)}$$

For n = 2

$$E_2 = 4\left(\frac{h^2}{8ma^2}\right) \qquad \text{............ (7)}$$

For n = 3

$$E_3 = 9\left(\frac{h^2}{8ma^2}\right)$$

Therefore energy values of the partial are discrete and non-continuous. Moreover, they are not equally spaced as shown in fig.: 02

From equation (3), the energy E_n depends on m and a^2

Here $$E \propto \frac{1}{m}$$

And $$E \propto \frac{1}{a2}$$

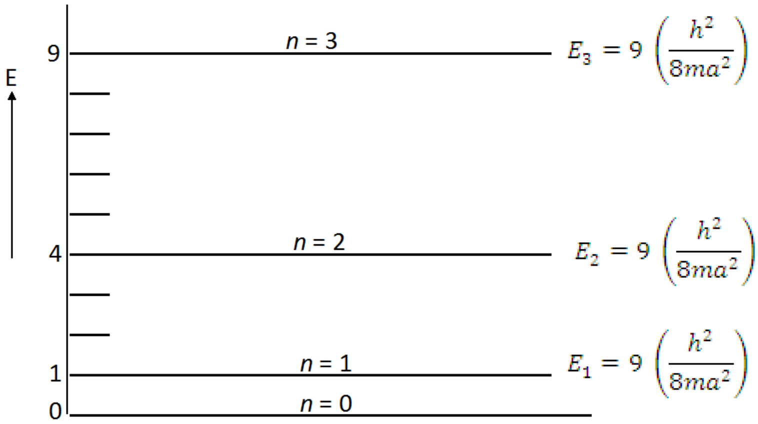

Fig.: 02

Macroscopic System:

In the case of macroscopic system say large *m* and large *a,* we suppose

And $$m = 1\ kg$$

$$a = 1\ \text{m},$$

Then $$E_n = \frac{n^2h^2}{8}$$

Here *En* is of the order of 2

i.e En is the order of 10-68 J.

For n= 1,2,3….etc.

The difference in any two energy states is extremely small and not detectable. Hence for microscopic systems discrete values are not detectable and appear to be continuous.

Microscopic System:

In the case of microscopic systems for an electron moving in a box, the values of *a* and *m* are very small. For and electron *a* is the order of 10^{-10m} and *m* is of the order of 10^{-30} kg.

Hence the value *E1* and *E2* are given by-

$$E_n = \frac{n^2h^2}{8ma^2}$$

$$E_1 = \frac{(6.624 \; x \; 10^{-34})^2}{8 \; x \; (9.1) x \; 10^{-31} x (10^{-10})^2}$$

$$E_1 = 0.6 \; x \; 10^{-17} J$$

Similarly $$E_2 = 2.4 \; x \; 10^{-17} J$$

$$E_3 = 5.4 \; x \; 10^{-17}$$

Hence the different between *E2* and *E1*

$$= 2.4 \times 10^{-17} - 0.6 \times 10^{-17} = 18 \times 10^{-17} J$$

= 112.5 eV

Similarly $E_3 - E_2$ = 3 x 10-17J = 187.5 eV

These values are quite appreciable.

Hence in microscopic systems the energy values are significant, discrete and non-continuous.

Semi micro and semi macro system of energy:-

It is not always true that a gaseous particle enclosed in a box just bears micro and macro system of energy.

If due to the collision of gaseous particle, we may imagine such a particle which traces a circular path, then the centripetal force comes into play, and the centripetal force is given by-

$$F = \frac{mv^2}{r}$$

There fore $$F \times r = mv^2$$

Where *m* is the mass of gaseous particle, *v* is the velocity of the gaseous particle and *r* is the radial distance of the particle form the center of the circular path. Further the kinetic energy possessed by the same particle is given by-

$$K.E. = \frac{1}{2} mv^2$$

There fore $$2K.E. = mv^2$$

2K.E. = F x r

$$K.E. = \frac{1}{2}F \times r$$

i.e. The kinetic energy bearded by the gaseous particle (Electron) is equal to half of the product of centripetal force (F) and radial distance (r).

It has been a brain storming problem for the research scholar that, if an electron is moving on a circular path, what is the supplier source of energy to the electron to move on the circular path? Above result makes it clear that centripetal force and radial distance are the energy supplier.

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