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PROFIT FUNCTION OF A TWO-NON IDENTICAL WARM STANDBY SYSTEM SUBJECT TO FOG WITH SWITCH FAILURE AND REPAIR FACILITY AS FIRST COME LAST SERVE.

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ABSTRACT

To transfer a unit from the standby state to the online state, a device known as 'switching device' is required. Generally, we assume that (1) the switching device is perfect in the sense that it does not fail and (2) the repair facility is first come first serve basis.. However, there are practical situations where the **switching device** can also **fail** and the repair facility is **FCLS** i.e. **First Come Last Serve**. We have taken units failure, switch failure distribution as exponential and repair time distribution as General. We have find out MTSF, Availability analysis, the expected busy period of the server for repair the failed unit under fog in (0,t], expected busy period of the server for repair in(0,t], the expected busy period of the server for repair of switch failure in (0,t], the expected number of visits by the repairman for failure of units in (0,t], the expected number of visits by the repairman for switch failure in (0,t] and Profit benefit analysis using regenerative point technique. A special case using failure and repair distributions as exponential is derived and graphs have been drawn.

Keyword Warm Standby , Fog, First Come Last Serve, MTSF, Availability, Busy period, cost-benefit Function

INTRODUCTION

To transfer a unit from the standby state to the online state, a device known as 'switching device' is required. Generally, we assume that the switching device is perfect in the sense that it does not fail. However, there are practical situations where the switching device can also fail. This has been pointed out by Gnedenko et al(1969).Such system in which the switching device can fail are called systems with imperfect switch. In the study of redundant systems it is generally assumed that when the unit operating online fails, the unit in standby is automatically switched online and the switchover from standby state to online state is instantaneous.In this paper, we have **FOG** which are non-instantaneous in nature. We assume that the **FOG** cannot occur simultaneously in both the units and when there occurs **FOG** of the non –instantaneous nature the operation of the unit stop automatically. Here, we investigate a two-unit (identical) cold standby –a system in which offline unit cannot fail with switch failure under the influence of **FOG** . The **FOG** cannot occur simultaneously in both the units and when there are less **FOG** that is within specified limit of a unit, it operates as normal as before but if these are beyond the specified limit the operation of the unit is avoided and as the **FOG** goes on some characteristics of the stopped unit change which we call failure of the unit. After the **FOG** are over the failed unit undergoes repair immediately according to first come last served discipline. For example , when a train came on the junction Station it waits for crossing of other train. When the other train came it stops and departs first according to FCLS.

ASSUMPTIONS

The system consists of two dissimilar warm standby units and the fog and failure time distribution are exponential with rates λ_1, λ_2 and λ_3 whereas the repairing rates for repairing the failed system due to fog and due to switch failure are arbitrary with CDF $G_1(t)$ & $G_2(t)$ respectively.

2. The operation of units stops automatically when FOG occurs so that excessive damage of the unit can be prevented.

3. The fog actually failed the units. The fog is non-instantaneous and it cannot occur simultaneously in both the units.

4. The repair facility works on the first come last serve (FCLS) basis.

5. The switches are imperfect and instantaneous.

6. All random variables are mutually independent.

Symbols for states of the System

Superscripts O, WS, SO, F, SFO

Operative, cold Standby, Stops the operation, Failed, Switch failed but operable respectively
Subscripts nf, uf,ur, wr, uR
No fog, under fog, under repair, waiting for repair, under repair continued respectively

Up states – 0,1,2,6 ; down states – 3, 4,5,7,8
States of the System

0(O_{nf} , WS_{nf})
One unit is operative and the other unit is warm standby and there are no fog in both the units.

1(SO_{nf} , O_{nf})
The operation of the first unit stops automatically due to fog and warm standby units starts operating.

2(SO_{nf} , $SFO_{nf,ur}$)
The operation of the first unit stops automatically due to fog and during switchover to the second unit switch fails and undergoes repair..

3(F_{ur} , O_{uf})
The first one unit fails and undergoes repair after the fog are over and the other unit continues to be operative with no fog.

4(F_{ur} , SO_{uf})
The one unit fails and undergoes repair after the fog are over and the other unit also stops automatically due to fog.

5(F_{uR} , F_{wr})
The repair of the first unit is continued from state 4 and the other unit is failed due to fog in it & is waiting for repair.

6(F_{uR} , SO_{uf})
The repair of the first unit is continued from state 3 and in the other unit fog occur and stops automatically due to fog.

7(F_{wr} , SFO_{uR})
The repair of failed switch is continued from state 2 and the first unit is failed after fog and waiting for repair.

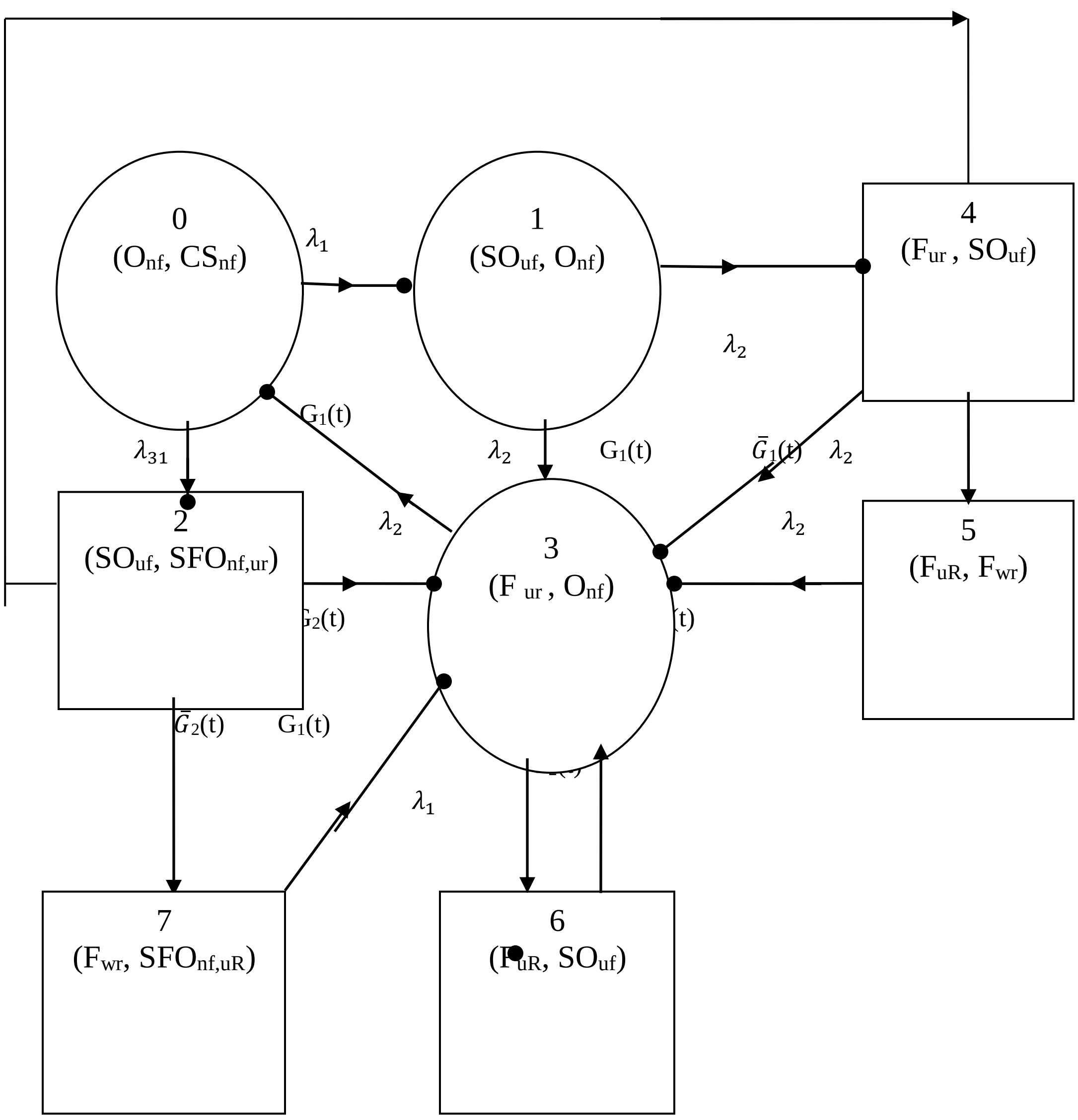

Fig.1 The State Transition Diagram
regeneration point

Up State
Down State

Transition Probabilities
Simple probabilistic considerations yield the following expressions :

$p_{01} = \frac{\lambda1}{\lambda1+\ \lambda3}$, $p_{02} = \frac{\lambda3}{\lambda1+\ \lambda3}$

$p_{13} = \frac{\lambda2}{\lambda1+\ \lambda2}$, $p_{14} = \frac{\lambda1}{\lambda1+\ \lambda2}$

$p_{23}= \lambda_1 G_2^*(\lambda_2)$, $p_{23}^{(7)} = \lambda_2 G_2^*(\lambda_2)$, $p_{24}= \overline{G}_2^*(\lambda_2)$,

$p_{30}= G_1^*(\lambda_1)$, $p_{33}^{(6)}= \overline{G}_1^*(\lambda_1)$

$p_{43} = G_1^*(\lambda_2)$, $p_{43}^{(5)} = G_1^*(\lambda_2)$ (1)

We can easily verify that

$p_{01} + p_{02} = 1$, $p_{13} + p_{14} = 1$, $p_{23} + p_{23}^{(1)} + p_{24}= 1$, $p_{30} + p_{33}^{(6)} = 1$,

$p_{43} + P_{43}^{(5)} = 1$ (2)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt = -1/\lambda1$

Similarly

$\mu_1 = 1/\lambda2$, $\mu_2 = \int_0^\infty e^{-}\lambda1\overline{G}1(t)dt$,

$\mu_4 = \int_0^\infty e^{-}\lambda2\overline{G}1(t)dt$ (3)

Mean Time to System Failure

We can regard the failed state as absorbing

$$\theta_0(t) = Q_{01}(t)[s]\theta_1(t) + Q_{02}(t)$$

$$\theta_1(t) = Q_{13}(t)[s]\theta_3(t) + Q_{14}(t) , \theta_3(t) = Q_{30}(t)[s]\theta_0(t) + Q_{33}^{(6)}(t)$$

(4-6)

Taking Laplace-Stieltjes transforms of eq. (4-6) and solving for

$$Q_0^*(s) = N_1(s) / D_1(s) \quad (7)$$

Where

$N_1(s) = Q_{01}^*(s) \{ Q_{13}^*(s) Q_{33}^{(6)*}(s) + Q_{14}^*(s) \} + Q_{02}^*(s)$

$D_1(s) = 1 - Q_{01}^*(s) \quad Q_{13}^*(s) Q_{30}^*(s)$

Making use of relations (1) & (2) it can be shown that $Q_0^*(0) = 1$, which implies

that $\theta_1(t)$ is a proper distribution.

$$MTSF = E[T] = \left. \frac{d}{ds} Q_{01}^*(s) \right|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$= \quad (\mu_0 + p_{01}\ \mu_1 + p_{01}\ p_{13}\ \mu_3) / (1 - p_{01}\ p_{13}\ p_{30})$ (8)

where
$\mu_0 = \mu_{01} + \mu_{02}$, $\mu_1 = \mu_{13} + \mu_{14}$, $\mu_2 = \mu_{23} + \mu_{23}^{(1)} + \mu_{24}$, , $\mu_3 = \mu_{30} + \mu_{33}^{(6)}$

$\mu_3 = \mu_{43} + \mu_{43}^{(5)}$

Availability analysis
Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state belonging to E. By probabilistic arguments, we have
The value of $M_0(t) = e^{-\lambda_1 t} e^{-\lambda_3 t}$ $M_1(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$
$M_3(t) = e^{-\lambda_1} \overline{G}_1(t)$. (9)
The point wise availability $A_i(t)$ have the following recursive relations
$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$
$A_1(t) = M_1(t) + q_{13}(t)[c]A_3(t) + q_{14}(t)[c]A_4(t)$,
$A_2(t) = \{q_{23}(t) + q_{23}^{(7)}(t)\}[c]A_3(t) + q_{33}^{(6)}(t)\,[c]A_3(t)$
$A_4(t) = \{q_{43}(t) + q_{43}^{(5)}(t)[c]A_3(t)$
(10 - 14)
Taking Laplace Transform of eq. (10-14) and solving for $\hat{A}_0(s)$
$\hat{A}_0(s) = N_2(s) / D_2(s)$ (15)

Where
$N_2(s) = (1 - \hat{q}_{33}^{(6)}(s))\, \widehat{M}_0(s) + [\ \hat{q}_{01}(s)\{\widehat{M}_1(s) + (\ \hat{q}_{13}(s) + \ \hat{q}_{14}(s)\ (\ \hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s)\ ))\} + \hat{q}_{02}(s)\{\ \hat{q}_{23}(s)) + \hat{q}_{23}^{(1)}(s)) + \hat{q}_{24}(s)(\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\}]\, \widehat{M}_3(s)$
$D_2(s) = (1 - \hat{q}_{33}^{(6)}(s)) - \hat{q}_{30}(s)\ [\hat{q}_{01}(s)\{\ \hat{q}_{13}(s) + \hat{q}_{14}(s)\ (\ \hat{q}_{43}(s) + \ \hat{q}_{43}^{(5)}(s))\ \}$
$+ \ \hat{q}_{20}(s)\{\ \hat{q}_{23}(s) + \hat{q}_{23}^{(7)}(s) + \ \hat{q}_{24}(s)(\hat{q}_{43}(s)) \ + \hat{q}_{43}^{(5)}(s)\ )\}]$
The steady state availability
$A_0 = \lim_{t\to\infty}[A_0(t)] \quad = \lim_{s\to 0}[s\,\hat{A}_0(s)] \quad = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$
Using L' Hospitals rule, we get
$A_0 = \lim_{s\to 0}\frac{N_2(s) + s\ N_2{'}(s)}{D_2{'}(s)} = \frac{N_2(0)}{D_2{'}(0)}$ (16)
Where
$N_2(0) = p_{30}\,\widehat{M}_0(0) + p_{01}\widehat{M}_1(0)\ \ \widehat{M}_3(0)\ )$
$D_2{'}(0) = \mu_3 + [\ \mu_0 + p_{01}\ (\mu_1 + p_{14}\ \mu_4 + p_{02}(\ \mu_2 + p_{24}\,\mu_4\ )]\ p_{30}$
The expected up time of the system in (0,t] is
$\lambda_u(t) = \int_0^{\propto} A_0(z)dz$ So that $\widehat{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (17) The expected down time of the system in (0,t] is
$\lambda_d(t) = t - \lambda_u(t)$ So that $\widehat{\lambda_d}(s) = \frac{1}{s^2} - \widehat{\lambda_u}(s)$ (18)

The expected busy period of the server for repairing the failed unit under fog in (0, t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$
$R_1(t) = S_1(t) + q_{13}(t)[c]R_3(t) + q_{14}(t)[c]R_4(t)$,
$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{23}^{(7)}(t)[c]R_3(t) + q_{24}(t)[c]R_4(t)$
$R_3(t) = q_{30}(t)[c]R_0(t) + q_{33}^{(6)}(t)[c]R_3(t)$,
$R_4(t) = S_4(t) + (q_{43}(t) + q_{43}^{(5)}(t))\ [c]R_3(t)$ (19-23)
Where
$S_1(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$, $S_2(t) = e^{-\lambda_1 t}\,\overline{G}_2(t)$, $S_4(t) = e^{-\lambda_1 t}\,\overline{G}_1(t)$ (24)
Taking Laplace Transform of eq. (19-23) and solving for $\widehat{R_0}(s)$
$\widehat{R_0}(s) = N_3(s) / D_2(s)$ (25)

Where
$N_3(s) = (1 - \hat{q}_{33}^{(6)}(s))\ [\ \hat{q}_{01}(s)(\ \hat{S}_1(s) + \hat{q}_{14}(s)\,\hat{S}_4(s) + \hat{q}_{02}(s)(\ \hat{S}_2(s) + \hat{q}_{24}(s)\,\hat{S}_4(s))]$ and $D_2(s)$ is already defined.
In the long run, $R_0 = \frac{N_3(0)}{D_2{'}(0)}$ (26)
where $N_3(0) = p_{30}\,[\ p_{01}\ (\hat{S}_1(0) + p_{14}\,\hat{S}_4(0)\ ) + p_{02}\ (\hat{S}_2(0) + p_{24}\,\hat{S}_4(0)\ )$ and $D_2{'}(0)$ is already defined.
The expected period of the system under fog in (0,t] is
$\lambda_{rv}(t) = \int_0^{\propto} R_0(z)dz$ So that $\widehat{\lambda_{rv}}(s) = \frac{\hat{R}_0(s)}{s}$ (27)
The expected busy period of the server for repair of dissimilar units by the repairman in (0, t]
$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$

$B_1(t) = q_{13}(t)[c]B_3(t) + q_{14}(t)[c]B_4(t)$,
$B_2(t) = q_{23}(t)[c] B_3(t) + q_{23}^{(7)}(t)[c]B_3(t) + q_{24}(t)[c] B_4(t)$
$B_3(t) = T_3(t)+ q_{30}(t)[c] B_0(t) + q_{33}^{(6)}(t)[c]B_3(t)$

$B_4(t) = T_4(t)+ \{ q_{43}(t) + q_{43}^{(5)}(t)\} [c]B_3(t)$ (28-32)

Where

$T_3(t) = e^{-\lambda_2 t}\, \overline{G}_1(t)$ $T_4(t) = e^{-\lambda_1 t}\, \overline{G}_1(t)$ (33)

Taking Laplace Transform of eq. (28-32) and solving for $\widehat{B_0}(s)$

$$\widehat{B_0}(s) = N_4(s) / D_2(s) \quad (34)$$

Where

$N_4(s) = \widehat{T}_3(s) [\hat{q}_{01}(s) \{ \hat{q}_{13}(s)+ \hat{q}_{14}(s) (\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\}+ \hat{q}_{02}(s) \{ \hat{q}_{23}(s)+ \hat{q}_{23}^{(7)}(s)+ \hat{q}_{24}(s) (\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\}]+ \widehat{T}_4(s) [\hat{q}_{01}(s) \hat{q}_{44}(s)(1 - \hat{q}_{33}^{(6)}(s)) + (\hat{q}_{02}(s) \hat{q}_{24}(s)(1- \hat{q}_{33}^{(6)}(s))]$

And $D_2(s)$ is already defined.

In steady state, $B_0 = \frac{N_4(0)}{D_2'(0)}$ (35)

where $N_4(0)= \widehat{T}_3(0)+ \widehat{T}_4(0) \{ p_{30} (p_{01}p_{14} + p_{02} p_{24}) \}$ and $D_2'(0)$ is already defined.

The expected busy period of the server for repair in (0,t] is

$\lambda_{ru}(t) = \int_0^{\infty} B_0(z)dz$ So that $\widehat{\lambda_{ru}}(s) = \frac{\widehat{B}_0(s)}{s}$ (36)

The expected Busy period of the server for repair of switch in (o, t]

$P_0(t) = q_{01}(t)[c]P_1(t) + q_{02}(t)[c]P_2(t)$
$P_1(t) = q_{13}(t)[c]P_3(t) + q_{14}(t)[c]P_4(t)$,
$P_2(t) = L_2(t)+ q_{23}(t)[c]P_3(t) + q_{23}^{(7)}(t)[c]P_3(t)+ q_{24}(t)[c]P_4(t)$
$P_3(t) = q_{30}(t)[c]P_0(t) + q_{33}^{(6)}(t)[c]P_3(t)$,
$P_4(t) = (q_{43}(t)+ q_{43}^{(5)}(t)) [c]P_3(t)$ (37-41)

Where $L_2(t) = e^{-\lambda_1 t}\, \overline{G}_2(t)$

Taking Laplace Transform of eq. (37-41) and solving for $\widehat{P_0}(s)$

$$\widehat{P_0}(s) = N_5(s) / D_2(s) \quad (43)$$

where $N_5(s) = \hat{q}_{02}(s)\, \hat{L}_2(s) (1 - \hat{q}_{33}^{(6)}(s))$ and $D_2(s)$ is defined earlier.

In the long run , $P_0 = \frac{N_5(0)}{D_2'(0)}$ (44)

where $N_5(0)= p_{30}\, p_{02}\, \hat{L}_2(0)$ and $D_2'(0)$ is already defined.

The expected busy period of the server for repair of the switch in (0,t] is

$\lambda_{rs}(t) = \int_0^{\infty} P_0(z)dz$ So that $\widehat{\lambda_{rs}}(s) = \frac{\widehat{P}_0(s)}{s}$ (45)

The expected number of visits by the repairman for repairing the different units in (0, t]

$H_0(t) = Q_{01}(t)[s]H_1(t) + Q_{02}(t)[s]H_2(t)$
$H_1(t) = Q_{13}(t)[s][1+H_3(t)] + Q_{14}(t)[s][1+H_4(t)]$,
$H_2(t) = [Q_{23}(t) + Q_{23}^{(7)}(t)] [s][1+H_3(t)] + Q_{24}(t)[s][1+ H_4(t)]$
$H_3(t) = Q_{30}(t)[s]H_0(t) + Q_{33}^{(6)}(t)[s]H_3(t)$,
$H_4(t) = (Q_{43}(t)+ Q_{43}^{(5)}(t)) [s]H_3(t)$ (46-50)

Taking Laplace Transform of eq. (46-50) and solving for $H_0^*(s)$

$$H_0^*(s) = N_6(s) / D_3(s) \quad (51)$$

Where

$N_6(s) = (1 - Q_{33}^{(6)*}(s))\{Q^*_{01}(s)(Q^*_{13}(s) + Q^*_{14}(s)) + Q^*_{02}(s)(Q^*_{24}(s) + Q^*_{23}(s)+Q^*_{23}^{(7)}(s))\}$

$D_3(s) = (1 - Q_{33}^{(6)*}(s)) - Q^*_{30}(s)[Q^*_{01}(s) \{ Q^*_{13}(s) +Q^*_{14}(s) (Q_{43}^*(s) +Q^*_{43}^{(5)}(s))\} + Q^*_{02}(s)\{ Q^*_{23}(s)+ Q^*_{23}^{(7)}(s)) + Q^*_{24}(s)(Q^*_{43}(s)) + Q^*_{43}^{(5)}(s)) \}]$

In the long run , $H_0 = \frac{N_6(0)}{D_3'(0)}$ (52)

where $N_6(0)= p_{30}$ and $D'_3(0)$ is already defined.

The expected number of visits by the repairman for repairing the switch in (0, t]

$V_0(t) = Q_{01}(t)[s]V_1(t) + Q_{02}(t)[s][1+V_2(t)]$
$V_1(t) = Q_{13}(t)[s]V_3(t) + Q_{14}(t)[s]V_4(t)$,
$V_2(t) = Q_{24}(t)[s][1+V_4(t)] +[Q_{23}(t) + Q_{23}^{(7)}(t)[s][1+V_3(t)]$
$V_3(t) = Q_{30}(t)[s]V_0(t) + Q_{33}^{(6)}(t)[s]V_3(t)$ (53-57)

Taking Laplace-Stieltjes transform of eq. (53-57) and solving for $V_0^*(s)$

$$V_0^*(s) = N_7(s) / D_4(s) \quad (58)$$

where $N_7(s) = (1 - Q_{33}^{(6)*}(s))\{Q^*_{01}(s)(Q^*_{14}(s) + Q^*_{43}(s)) + Q^*_{02}(s)(Q^*_{24}(s) + Q^*_{02}(s)(Q^*_{23}(s) + Q^*_{23}{}^{(7)}(s))\}$

and $D_4(s)$ is the same as $D_3(s)$

In the long run , $V_0 = \frac{N_7(0)}{D_4'(0)}$ (59)

where $N_7(0)= p_{30}\,[p_{01}\ p_{14}\ p_{43} + p_{02}]$ and $D'_3(0)$ is already defined.

Cost Benefit Analysis

The cost-benefit function of the system considering mean up-time, expected busy period of the system under fog when the units stops automatically, expected busy period of the server for repair of unit & switch, expected number of visits by the repairman for unit failure, expected number of visits by the repairman for switch failure.

The expected total cost-benefit incurred in (0,t] is

C(t) = Expected total revenue in (0,t] - expected total repair cost for switch in (0,t]
- expected total repair cost for repairing the units in (0,t]
- expected busy period of the system under fog when the units automatically stop in (0,t]
- expected number of visits by the repairman for repairing the switch in (0,t] - expected number of visits by the repairman for repairing of the units in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$

$= K_1A_0 - K_2P_0 - K_3B_0 - K_4R_0 - K_5V_0 - K_6H_0$

Where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is under switch repair

K_3 - cost per unit time for which the system is under unit repair

K_4 - cost per unit time for which the system is under fog when units automatically stop.

K_5 - cost per visit by the repairman for which switch repair,

K_6 - cost per visit by the repairman for units repair.

Conclusion

After studying the system , we have analyzed graphically that when the failure rate, fog rate increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

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