

Viscosity On The Stability Of The Plane Interface Separating Two Incompressible, Superposed Fluids Of Uniform Densities

Lekh Raj

Research Scholar PhD(Mathematics)
Department of Mathematics,
CMJ University, SHILLONG.INDIA

Dr. Mohd. A.A.Ansari

Reader, Department of Mathematics,
Govt. P. G. College, Harakh, Barabanki (U.P.)

Abstract

In this paper treats the Kelvin-Helmholtz instability arising at the interface separating 2 superposed, viscous, electrically conducting fluids of related densities through a porous medium in the presence of a uniform 2D horizontal magnetic field. The stability motion was also assumed to be uniform, 2D, and horizontal. By applying the normal mode technique to the linearized perturbation equations, the dispersion relation was derived. The stability analysis was carried out for fluids of high kinematic viscosities. It was found that both viscosity and porosity suppressed the stability, while streaming motion had a destabilizing influence. The Kelvin-Helmholtz discontinuity arising at the plane interface between 2 superposed streaming fluids is of prime importance in various astrophysical, geophysical, and laboratory situations. The Kelvin-Helmholtz instability arises in such situations as when air is blown over mercury, highly ionized hot plasma is surrounded by a slightly cold gas, or a meteor enters the earth's atmosphere. It is interesting to note that for the special case when perturbations in the direction of streaming are ignored, the system is unstable for potentially unstable configuration and the system is stable for potentially stable configuration and not depending upon kinematic visco-elasticity, medium porosity and medium permeability.

Key words:- 2 incompressible, superposed fluids, Kelvin-Helmholtz(KH), magnetic field etc.

Introduction

A comprehensive account of investigations of these problems in hydrodynamics and hydro-magnetics was given by Chandrasekhar (1961) in his monograph. These problems of instabilities in hydrodynamic and hydro-magnetic configurations continue to attract the attention of researchers due to their importance in actual physical situations. The problems of non-streaming, superposed instability (Rayleigh-Taylor instability) and Kelvin-Helmholtz instability have been investigated by several researchers from different points of view. Jukes (1964) investigated the problem by incorporating finite electrical conductivity effects and concluded that this inclusion introduced new and unexpected

solutions. Gerwin (1968) studied the stability problem of non-conducting, streaming gas flowing over an incompressible conducting liquid. The influence of viscosity on the stability of the plane interface separating 2 incompressible, superposed fluids of uniform densities was studied by Bhatia (1974), who concluded that viscosity has a stabilizing influence. A comprehensive account of various hydrodynamic stability problems was also given by Joseph (1976) and by Drazin and Reid (1981). Several researchers have examined the Kelvin-Helmholtz instability in superposed fluids in hydrodynamic, hydro-magnetic, and plasma regimes from different points of views.

Sengar (1984) analyzed the stability of 2 superposed gravitating streams in a uniform, vertical magnetic field in the presence of effects of magnetic resistivity. He found that magnetic resistivity had a destabilizing effect on the system. Mehta and Bhatia (1988) studied the Kelvin-Helmholtz instability of 2 viscous, superposed plasmas in the presence of finite Larmor radius (FLR) effects and showed that both viscosity and FLR effects suppressed the instability. D'avalos-Orozco and Aguilar-Rosas (1989a, 1989b) and D'avalos-Orozco (1996) examined the effects of a general rotation and a horizontal magnetic field on the stability of superposed inviscid fluids. El-Sayeed (2003) examined the effect of viscosity and finite ion Larmor radius on the hydrodynamic transverse instability problem.

An excellent reappraisal of the Kelvin-Helmholtz problem was given by Benjamin and Bridges (1997), who gave the Hamiltonian formulation of the classic Kelvin-Helmholtz problem in hydrodynamics. Allah (1998) investigated the effects of magnetic field, heat, and mass transfer on the Kelvin-Helmholtz instability of superposed fluids. El-Ansary et al. (2002) studied the effects of rotation on the hydrodynamic stability of 3 layers. Meignin et al. (2001) and Watson et al. (2004) studied the Kelvin-Helmholtz instability in a Hele-Shaw cell and a weakly ionized medium, respectively. In recent years, investigations of the flow of fluids of related densities through porous media have become an important topic due to the recovery of crude oil from the pores of reservoir rocks. McDonnell (1978) pointed out the physical properties of comets; meteorites and interplanetary dust strongly suggest the significance of the effects of porosity in the astrophysical context.

Several applications of the problems of flow through a porous medium in geophysics may be found in the work of Phillips (1991), Ingham and Pop (1998), and Nield and Bejan (1999). Several researchers (e.g. Vaghela and Chhajlani (1988), Samaria et al. (1990), Sharma and Kumar (1997), and Khan and Bhatia (2003)) have studied the effects of the permeability of a porous medium on different problems in hydrodynamic and hydromagnetic stability in view of the importance of such studies in rocks and heavy oil recovery.

In the absence of a magnetic field, Allah (2002) investigated the stability of superposed Newtonian fluids of related densities through a porous medium in the presence of the effects of surface tension, while Kumar and Lal (2005) recently studied the stability in 2 superposed Rivlin-Ericksen viscoelastic fluids of related densities through a porous medium. Kumar et al. (2006) studied the instability of a rotating, superposed Walters B' viscoelastic fluid through a porous medium. More recently, Kumar et al. (2007) investigated the effect of viscosity on stratified, superposed, non-Newtonian fluids. In all of the above mentioned studies of flow and stability in Newtonian and non-Newtonian fluids through a porous medium, the effects of streaming motions were not included. It would be of interest to examine the instability in superposed, streaming, viscous, electrically conducting fluids through a porous medium in the presence of a magnetic field. One can study the problem of the stability of the horizontal layer of stratified fluids and the stability of 2 superposed fluids, whether streaming or not, in the presence of a horizontal or a vertical magnetic field. Khan and Bhatia (2001) studied the stability of 2 non-streaming, superposed, visco-elastic fluids in a horizontal magnetic field. In this chapter, we have examined the stability of a planar interface separating 2 streaming, electrically conducting, viscous fluids in a horizontal magnetic field through a porous medium. For a uniform vertical magnetic field, Bhatia and Sharma (2003) studied the problem of Kelvin-Helmholtz instability in superposed viscous fluids of related densities through a porous medium.

Mathematical formulation & Graphically representation : We considered the motion of an incompressible, viscous, infinitely electrically conducting fluid of uniform viscosity μ , moving with a

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uniform horizontal velocity $\vec{U} = (U_x, U_y, 0)$ through a porous medium in the presence of uniform 2D magnetic field $\vec{H} = (H_x, H_y, 0)$. The relevant linearized perturbation equations are:

$$\frac{\rho}{\varepsilon} \frac{\partial \vec{u}}{\partial t} + \frac{\rho}{\varepsilon} (\vec{U} \cdot \nabla) \vec{u} = -\nabla \delta p + \vec{g} \delta \rho + (\nabla \times \vec{h}) \times \vec{H} + \frac{\mu}{\varepsilon} \nabla^2 \vec{u} - \frac{\mu}{Q} \vec{u} \quad (1)$$

$$\varepsilon \frac{\partial}{\partial t} (\delta \rho) + (\vec{U} \cdot \nabla) \delta \rho + (\vec{u} \cdot \nabla) \rho = 0 \quad (2)$$

$$\varepsilon \frac{\partial \vec{h}}{\partial t} + (\vec{U} \cdot \nabla) \vec{h} = (\vec{H} \cdot \nabla) \vec{u} \quad (3)$$

$$\nabla \cdot \vec{h} = 0 \quad (4)$$

$$\nabla \cdot \vec{u} = 0 \quad (5)$$

where $\vec{h}(h_x, h_y, h_z)$, $\delta \rho$ and δp are the perturbations, respectively, in magnetic field \vec{H} , density ρ and pressure p , resulting from the disturbance, the Darcian velocity $\vec{u}(u, v, w)$ to the system. In the above equations, μ is the coefficient of viscosity, $\vec{g} = (0, 0, -g)$ is the acceleration due to gravity, Q is the permeability of the porous medium and ε is the medium's porosity. Analyzing in terms of normal modes, we assumed that the perturbed quantities have the space (x, y, z) and time (t) dependence of the form:

$$f(z) \exp(ik_x x + ik_y y + nt) \quad (6)$$

Where $f(z)$ is some function of z , k_x and k_y are the horizontal wave numbers ($k^2 = k_x^2 + k_y^2$) and n is the rate at which the system departs away from equilibrium. Eliminating some of the variables, we get the following equation in w :

$$\frac{n'}{\varepsilon} [\rho k^2 w - D(\rho D w)] - \frac{g k^2}{n' \varepsilon} (D \rho) w - \frac{(\vec{k} \cdot \vec{H})^2}{n' \varepsilon} (D^2 - k^2) w + \frac{\mu}{\varepsilon} (D^2 - k^2)^2 w - \frac{\mu}{Q} (D^2 - k^2) w = 0 \quad (7)$$

where we have written $n + i(\vec{k} \cdot \vec{U}) = n'$ and $D \equiv \frac{d}{dz}$,

Eq. (7) holds for all $\rho(z)$.

Superposed fluids: Consider the case in which 2 superposed fluids, occupying the regions $z > 0$ and $z < 0$, are separated by a horizontal boundary at $z = 0$. In the 2 regions of constant ρ , equation (7) becomes:

$$(D^2 - k^2)(D^2 - M^2) w = 0 \quad (8)$$

where

$$M^2 = k^2 + \frac{n'}{v} \left(1 + \frac{(\vec{k} \cdot \vec{H})^2}{n'^2 \rho} + \frac{v\varepsilon}{Qn'} \right) \quad (9)$$

Where $\nu = \frac{\mu}{\rho}$ is the coefficient of kinematic viscosity. We assumed that the fluid density ρ_1 , kinematic viscosity ν_1 and streaming velocities $\vec{U}_1 = (U_{x1}, U_{y1}, 0)$ occupy the lower region $z < 0$, while the fluid of density ρ_2 , kinematic viscosity ν_2 and streaming velocities $\vec{U}_2 = (U_{x2}, U_{y2}, 0)$ occupy the lower region $z > 0$, we sought the solution of equ. (8) for the 2 fluids moving in the presence of magnetic field \vec{H} and flowing through a porous medium of porosity ε . Since w must be bounded both when $z \rightarrow +\infty$ in the upper fluid, and $z \rightarrow -\infty$ in the lower fluid, the solution of equ. (8), which remained bounded in the 2 regions, are:

$$w_1 = A_1 n_1' e^{kz} + B_1 n_1' e^{M_1 z}, \quad z < 0 \quad (10) \quad w_2 = A_2 n_2' e^{-kz} + B_2 n_2' e^{-M_2 z}, \quad z > 0 \quad (11)$$

where $A_1, B_1, A_2,$ and B_2 are constants of integration, $n_1' = n + i\vec{k} \cdot \vec{U}_1, n_2' = n + i\vec{k} \cdot \vec{U}_2$ and M_1 and M_2 are respectively the square roots of M_2 for the 2 regions. The expressions determining M_1 and M_2 are

$$M_1^2 = k^2 + \frac{n_1'^2}{v_1} \left(1 + \frac{(\vec{k} \cdot \vec{H})^2}{n_1'^2 \rho} + \frac{v_1 \varepsilon}{Qn_1'} \right) \quad (12)$$

$$M_2^2 = k^2 + \frac{n_2'^2}{v_2} \left(1 + \frac{(\vec{k} \cdot \vec{H})^2}{n_2'^2 \rho} + \frac{v_2 \varepsilon}{Qn_2'} \right) \quad (13)$$

In writing the solution of Eqs. (10) and (11), it was assumed that M_1 and M_2 were so defined that their real parts were positive.

Boundary conditions: The solutions for Eqs. (10) and (11) must satisfy certain boundary conditions. These conditions require that at the interface $z = 0$,

$$w \quad (14 \text{ (a)})$$

$$Dw \quad (14 \text{ (b)})$$

$$\mu(D^2 + k^2)w \quad (14 \text{ (c)})$$

must be continuous. These conditions ensure the requirement of the continuity of the normal component of velocity, its derivative, and the tangential stress at the interface. By integrating Eq. (7) across the interface, $z = 0$, we obtain another condition:

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$$\begin{aligned}
 & \left[\left\{ \rho_2 - \frac{\mu_2}{n_2'} (D^2 - k^2) + \frac{\mu_2 \varepsilon}{Q n_2'} + \frac{(k_x H_x + k_y H_y)^2}{n_2'^2} \right\} D w_2 \right]_{z=0} \\
 & - \left[\left\{ \rho_1 - \frac{\mu_1}{n_1'} (D^2 - k^2) + \frac{\mu_1 \varepsilon}{n_1' Q} + \frac{(k_x H_x + k_y H_y)^2}{n_1'^2} \right\} D w_1 \right]_{z=0} \\
 & + g k^2 \left(\frac{\rho_2}{n_2'^2} - \frac{\rho_1}{n_1'^2} \right) w_0 + 2k^2 \left(\frac{\mu_2}{n_2'} - \frac{\mu_1}{n_1'} \right) (D w)_0 = 0 \quad (15)
 \end{aligned}$$

where w_0 and $(Dw)_0$ are the unique values of these quantities at $z = 0$. Applying boundary conditions (14 (a)), (14 (b)), (14 (c)) and (15) to the solutions of Eqs. (10) and (11), we obtain the relations:

$$A_1 + B_1 = A_2 + B_2 \quad (16)$$

$$k A_1 + M_1 B_1 = -k A_2 - M_2 B_2 \quad (17) \quad \mu_1 [2A_1 k^2 + (M_1^2 + k^2) B_1] = \mu_2 [2A_2 k^2 + (M_2^2 + k^2) B_2] \quad (18)$$

$$\begin{aligned}
 & -k \rho_2 A_2 - k \rho_1 A_1 + \left\{ \frac{\mu_2 \varepsilon}{Q n_2'} (-k A_2 - M_2 B_2) - \frac{\mu_1 \varepsilon}{Q n_1'} (k A_1 + M_1 B_1) \right\} - k (k_x H_x + k_y H_y)^2 \left(\frac{A_1}{n_1'^2} + \frac{A_2}{n_2'^2} \right) \\
 & = -\frac{g k^2}{2} \left(\frac{\rho_2}{n_2'^2} - \frac{\rho_1}{n_1'^2} \right) (A_1 + B_1 + A_2 + B_2) - k^2 \left(\frac{\mu_2}{n_2'} - \frac{\mu_1}{n_1'} \right) (k A_1 + M_1 B_1 - k A_2 - M_2 B_2) \quad (19)
 \end{aligned}$$

On eliminating the constants A_1, B_1, A_2 and B_2 and evaluating the determinant of the given matrix of the coefficients in Eqs. (16) through (19), we obtained the following characteristic equation:

$$\begin{aligned}
 & (M_1 - k) \left[\begin{aligned} & 2k^2 (v_1 \alpha_1 - v_2 \alpha_2) \left\{ -c \left(\frac{M_1}{k} - 1 \right) + \frac{(\vec{k} \cdot \vec{V}_A)^2}{n_2'^2} - \frac{v_2 \alpha_2 \varepsilon}{Q n_2'} \left(\frac{M_2}{k} - 1 \right) + \alpha_2 \right\} \\ & + v_2 \alpha_2 (M_2^2 - k^2) \left\{ R - 1 + \frac{\varepsilon}{Q} \left(\frac{v_1 \alpha_1}{n_1'} - \frac{v_2 \alpha_2}{n_2'} \right) + \left(\frac{(\vec{k} \cdot \vec{V}_A)^2}{n_1'^2} - \frac{(\vec{k} \cdot \vec{V}_A)^2}{n_2'^2} \right) \right\} \end{aligned} \right] \\
 & - 2k \left[\begin{aligned} & v_1 \alpha_1 (M_1^2 - k^2) \left\{ -c \left(\frac{M_1}{k} - 1 \right) + \frac{(\vec{k} \cdot \vec{V}_A)^2}{n_2'^2} - \frac{v_2 \alpha_2 \varepsilon}{Q n_2'} \left(\frac{M_2}{k} - 1 \right) + \alpha_2 \right\} \\ & + v_2 \alpha_2 (M_2^2 - k^2) \left\{ c \left(\frac{M_1}{k} - 1 \right) - \frac{(\vec{k} \cdot \vec{V}_A)^2}{n_1'^2} + \frac{v_1 \alpha_1 \varepsilon}{Q n_1'} \left(\frac{M_1}{k} - 1 \right) + \alpha_1 \right\} \end{aligned} \right] \\
 & + (M_2 - k) \left[v_1 \alpha_1 (M_1^2 - k^2) \left\{ R - 1 + \frac{\varepsilon}{Q} \left(\frac{v_1 \alpha_1}{n_1'} - \frac{v_2 \alpha_2}{n_2'} \right) + \left(\frac{(\vec{k} \cdot \vec{V}_A)^2}{n_1'^2} - \frac{(\vec{k} \cdot \vec{V}_A)^2}{n_2'^2} \right) \right\} \right] \\
 & - \left[2k^2 (v_1 \alpha_1 - v_2 \alpha_2) \left\{ c \left(\frac{M_1}{k} - 1 \right) - \frac{(\vec{k} \cdot \vec{V}_A)^2}{n_1'^2} + \frac{v_1 \alpha_1 \varepsilon}{Q n_1'} \left(\frac{M_1}{k} - 1 \right) + \alpha_1 \right\} \right] = 0 \quad (20)
 \end{aligned}$$

$$R = g k \left(\frac{\alpha_2}{n_2'^2} - \frac{\alpha_1}{n_1'^2} \right), \quad c = k^2 \left(\frac{v_2 \alpha_2}{n_2'} - \frac{v_1 \alpha_1}{n_1'} \right), \quad (\vec{k} \cdot \vec{V}_A)^2 = \frac{(k_x H_x + k_y H_y)^2}{(\rho_1 + \rho_2)}$$

where

and \vec{V}_A is the Alfvén velocity. On

substituting the expression for M_1 and M_2 in Eq. (20), we obtained the characteristic equation. Now the values of M_1 and M_2 are given by the square roots of Eqs. (12) and (13). To obtain the values of M_1 and

M_2 , we used the binomial theorem and retained terms up to $\frac{1}{v_{1,2}}$, as in the case of non-streaming fluids

(Bhatia 1974). We could then write M_1 and M_2 as:

$$M_1 = k \left[1 + \frac{n'_1}{2k^2 v_1} + \frac{1}{2k^2 n'_1 v_1 \alpha_1} \left(\vec{k} \cdot \vec{V}_A \right)^2 + \frac{1}{2k^2 Q} \varepsilon \right] \quad (21)$$

$$M_2 = k \left[1 + \frac{n'_2}{2k^2 v_2} + \frac{1}{2k^2 n'_2 v_2 \alpha_2} \left(\vec{k} \cdot \vec{V}_A \right)^2 + \frac{1}{2k^2 Q} \varepsilon \right] \quad (22)$$

It is obvious that the expansion resorted to here is due to reasons of mathematical tractability. This enabled us to analyze the stability of the system. Substituting the values of M_1 and M_2 , given by Eqs. (21) and (22), in Eq. (20), we obtained the dispersion relation as a ninth degree polynomial:

$$\sum_{i=0}^{i=9} E_i n^i = 0 \quad (23)$$

where the coefficients E_i ($i = 0-9$) are complicated expressions involving the wave number k and the parameters $\alpha_1, \alpha_2, U_{x1}, U_{y1}, U_{x2}, U_{y2}, H_x, H_y, v_1, v_2, \varepsilon$ and Q characterizing, respectively, the effects of density, streaming velocity, the magnetic field, the viscosity, the porosity, and the permeability of the porous medium of the fluids. These coefficients are not given here.

Numerical calculations: The dispersion relation given by Eq. (23) is quite complicated, particularly as the coefficient E_i values involve several parameters. It is thus not feasible to analyze the dispersion relation analytically. We therefore solved it numerically, for different values of the parameters, for an unstable arrangement of superposed fluids, i.e. a top-heavy configuration and the same. We were interested in the qualitative behavior of the various parameters on the instability of the configuration. Therefore, the dispersion relation, Eq. (23), was numerically solved to ascertain the values of the growth rate against the wave number for various values of one parameter, taking fixed values of other parameters. The dispersion relation was first non-dimensionalized by measuring n and the parameters in terms of \sqrt{g} .

For an unstable system, we must have $\alpha_1 < \alpha_2$ ($\alpha_1 + \alpha_2 = 1$). One can, therefore, take any set of values for $\alpha_1 < \alpha_2$ such that $\alpha_1 + \alpha_2 = 1$. In the present study, we took $\alpha_1 = 0.30$ and $\alpha_2 = 0.70$. For numerical calculations, we set $H_x = H_y = 1$. These calculations are presented in Figures 1-5. In Figure 1, we plotted the growth rate n (positive real values) against wave number k for porosity $\varepsilon = 0.1, 0.2, 0.3$, taking $v_1 = v_2 = 1$ and $U_{x1} = U_{y1} = U_{x2} = U_{y2} = 1$. We noticed that as the value of ε increased, the growth rate decreased for the same wave number. The effect of porosity on the instability of the system was consequently stabilizing. Among several researchers, Vaghela and Chhajlani (1988), Allah (2002), and Kumar and Lal (2005) have shown that the effect of porosity on the instability of a superposed fluid is stabilizing. The results obtained here thus agree with the findings of the earlier authors.

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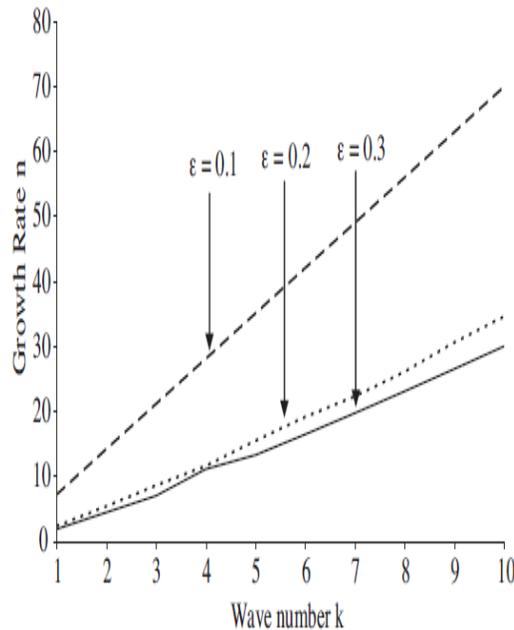


Figure 1. Plot of growth rate against wave number for porosity $\varepsilon = 0.1, 0.2, 0.3$.

The effect of kinematic viscosity on the system's stability is presented in Figures 2 and 3, in which we again plotted growth rate n versus wave number k for varying values of viscosity (ν_1 and ν_2) for $\varepsilon = 0.1$, $U_{x1} = U_{y1} = U_{x2} = U_{y2} = 1$. From Figures 2 and 3, we notice that the kinematic viscosity had a stabilizing influence on the instability of the system, as the increase in viscosity tended to decrease the value of n for the same k . Several other authors have examined the effect of viscosity on the stability of different hydrodynamic and hydro-magnetic systems. For non-streaming superposed fluids, Bhatia (1974), El-Sayed (2003), and Kumar et al. (2007) have pointed out the stabilizing character of viscosity on the stability of the system. The result obtained in the present chapter is thus in agreement with the observations of earlier researchers.

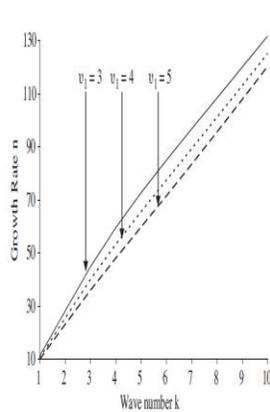


Figure 2. Plot of growth rate against wave numbers for variation of viscosity $\nu_1 = 3, 4, 5$.

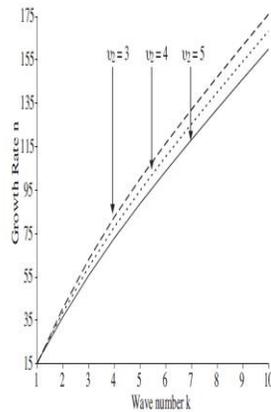


Figure 3. Plot of growth rate against wave numbers for variation of viscosity $\nu_2 = 3, 4, 5$.

The effect of streaming velocity on the system's stability is presented in Figures 4 and 5, in which growth rate n is plotted versus wave number k for varying values of streaming velocity for $\nu_1 = \nu_2 = 1$, $\varepsilon = 0.1$, and $U_{y1} = U_{y2} = 1$. We found that the more the values of streaming velocity increased for a fixed wave

number, the larger the values of the growth rate were. The streaming velocity thus had a destabilizing influence on the system. The effect of streaming motion on the stability of a superposed fluid has been investigated by several researchers in the past, such as Allah (1998), Meignin et al. (2001), Watson et al. (2004), and Bhatia and Mathur (2006). For nonporous fluid media, they all found that the streaming motion had a destabilizing influence on the system. The result obtained in the present chapter for the effect of streaming motion in porous fluids thus agrees with earlier findings.

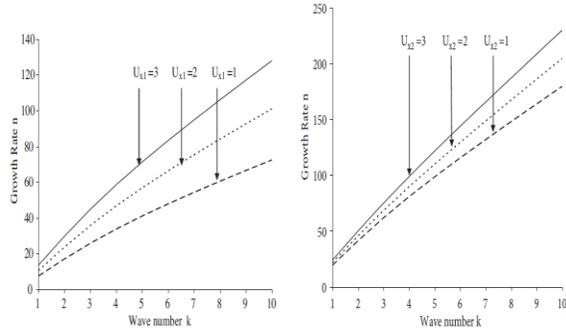


Figure 4. Plot of growth rate against wave numbers for stream velocity $U_{s1} = 1, 2, 3$.

Figure 5. Plot of Growth rate against wave numbers for Stream Velocity $U_{s2} = 1, 2, 3$.

Conclusion :- The numerical calculations presented above, although carried out for a few representative values of the physical parameters involved, reveal the tendencies of the nature of the physical effects on the instability of the superposed porous streaming fluids. we can say that both viscosity and porosity suppress the instability of streaming superposed fluids. The streaming motion tends to further destabilize the unstable arrangement of the superposed fluids.

(i) Unstable case: For the potentially unstable configuration $\rho_2 > \rho_1$, it is evident from Eq.(19) that one of the values of n is positive which means that the perturbations grow with time and so the system is unstable.

(ii) Stable case: For the potentially stable configuration $\rho_2 < \rho_1$, Eq.(19) yields that both the values of n are either real, negative or complex conjugates with negative real parts implying stability of the system.

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