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FINITE INTEGRAL FORMULAS INVOLVING MULTIVARIABLE MITTAGE-LEFFLER FUNCTION AND MODIFIED I-FUNCTION

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Abstract: In literature, a lot of remarkable definite and indefinite integrals, whose integrand include various special functions have been given. In this paper, first we establish two integral formulas involving, multivariable Mittage–Leffler function and modified I-function of two variables. In the next section we shall give four special cases, out of which two are for I-function of Shantha Kumari et al. and other two are for modified H-function of Prasad and Prasad respectively.

Keywords and phrases: Multivariable Mittage–Leffler function, modified I-function, I-function, modified H-function, finite integral formula.

Mathematics subject classification (2010): Primary: 26A33, 33C60, 33C99; secondary: 44A20.

1. Introduction and Preliminaries

In 1903, Mittage-Leffler [3] introduced the function $E_\alpha(y)$ in the following manner:

$$E_\alpha(y)=\sum_{k=0}^{\infty}\frac{y^k}{\Gamma(\alpha k+1)} \tag{1}$$

where $\alpha,\ y\in C,\ \Re(\alpha)>0$

In 1905, Wiman [8] generalized the function $E_\alpha(y)$ and gave the function $E_{\alpha,\beta}(y)$ in the following manner:

$$E_{\alpha,\beta}(y)=\sum_{k=0}^{\infty}\frac{y^k}{\Gamma(\alpha k+\beta)} \tag{2}$$

where $\alpha,\ \beta,\ y\in C,\ \Re(\alpha)>0,\ \Re(\beta)>0$

In 1971, Prabhakar [4] generalized the function $E_{\alpha,\beta}(y)$ and gave the function $E^{\lambda}_{\alpha,\beta}(y)$ in the following manner:

$$E^{\lambda}_{\alpha,\beta}(y)=\sum_{k=0}^{\infty}\frac{(\lambda)_k}{\Gamma(\alpha k+\beta)}\frac{y^k}{k!} \tag{3}$$

where $\alpha,\ \beta,\ \lambda,\ y\in C,\ \Re(\alpha)>0,\ \Re(\beta)>0,\ \Re(\lambda)>0$

Saxena et al. [6] gave the multivariable analogue of multivariable Mittage-Leffler function in the following manner:

$$E^{\lambda_i}_{\delta_i,\chi}(y_1,\cdots,y_r)=E^{\lambda_1,\cdots,\lambda_r}_{\delta_1,\cdots,\delta_r,\chi}(y_1,\cdots,y_r) = \sum_{k_1,\cdots,k_r=0}^{\infty}\frac{(\lambda_1)_{k_1},\cdots,(\lambda_r)_{k_r}}{\Gamma(\chi+\sum_{i=1}^{r}\delta_i k_i)}\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_r^{k_r}}{k_r!} \tag{4}$$

where $\chi,\ \lambda_i,\ \delta_i \in C,\ \Re(\mu_i)>0,\ \forall i=1,2,\cdots,r$

For the sake of convenience, let $F^{\lambda_i}_{\delta_i,\chi}=\frac{(\lambda_1)_{k_1},\cdots,(\lambda_r)_{k_r}}{\Gamma(\chi+\sum_{i=1}^{r}\delta_i k_i)}$

In 1979, Prasad and Prasad [5] introduced modified H-function of two variables and in 2012, Shantha Kumari et al. [1] defined I-function of two variables. In this paper we are defining modified I-function of two variables, which is the generalization of both the modified H-function of two variables and I-function of two variables, in the following manner:

$$I[z_1,.z_r]=I^{m,n:m_1,n_1:m_2,n_2:m_3,n_3}_{p,q:p_1,q_1:p_2,q_2:p_3,q_3}$$

$$\left[\begin{matrix} z_1 \\ z_2 \end{matrix}\middle| \begin{matrix} (a_j;\alpha_j,A_j;\xi_j)_{1,p}:(c_j;\gamma_j,C_j;\xi_j^{'})_{1,p_1}:(e_j,E_j;U_j)_{1,p_2};(g_j,G_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q}:(d_j;\delta_j,D_j;\eta_j^{'})_{1,q_1}:(f_j,F_j;V_j)_{1,q_q},(h_j,H_j;Q_j)_{1,q_3} \end{matrix}\right]$$

$$=\frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}\psi(s_1,s_2)\theta_1(s_1)\theta_2(s_2)z_1^{s_1}z_2^{s_2}ds_1ds_2 \tag{5}$$

where

$$\psi(s_1,s_2)=\frac{\prod_{j=1}^{m}\Gamma^{\eta_j}(b_j-\beta_j s_1-B_j s_2)\prod_{j=1}^{n}\Gamma^{\xi_j}(1-a_j+\alpha_j s_1+A_j s_2)}{\prod_{j=1}^{q}\Gamma^{\eta_j}(1-b_j+\beta_j s_1+B_j s_2)\prod_{j=n+1}^{p}\Gamma^{\xi_j}(a_j-\alpha_j s_1-A_j s_2)}$$

$$\times\frac{\prod_{j=1}^{m_1}\Gamma^{\eta_j^{'}}(d_j-\delta_j s_1+D_j s_2)\prod_{j=1}^{n_1}\Gamma^{\xi_j^{'}}(1-c_j+\gamma_j s_1-C_j s_2)}{\prod_{j=1}^{q_1}\Gamma^{\eta_j^{'}}(1-d_j+\delta_j s_1-D_j s_2)\prod_{j=n+1}^{p_1}\Gamma^{\xi_j^{'}}(c_j-\gamma_j s_1+C_j s_2)} \tag{6}$$

$$\theta_1(s_1)=\frac{\prod_{j=1}^{m_2}\Gamma^{U_j}(f_j-F_j s_1)\prod_{j=1}^{n_2}\Gamma^{V_j}(1-e_j+E_j s_1)}{\prod_{j=m_2+1}^{q_2}\Gamma^{U_j}(1-f_j+F_j s_1)\prod_{j=n_2+1}^{p_2}\Gamma^{V_j}(e_j-E_j s_1)} \tag{7}$$

$$\theta_2(s_2)=\frac{\prod_{j=1}^{m_3}\Gamma^{P_j}(h_j-H_js_2)\prod_{j=1}^{n_3}\Gamma^{Q_j}(1-g_j+G_js_2)}{\prod_{j=m_3+1}^{q_3}\Gamma^{P_j}(1-h_j+H_js_2)\prod_{j=n_3+1}^{p_3}\Gamma^{Q_j}(g_j-G_js_2)} \tag{8}$$

Here, the variables z_1 and z_2 are non-zero real or complex numbers and an empty product is interpreted as unity $m,n,m_1,n_1,m_2,n_2,m_3,n_3,p,q,p_1,q_1,p_2,q_2,p_3,q_3$ are all non-negative integers such that $0\le n\le p,\ 0\le m\le q,\ 0\le n_1\le p_1,\ 0\le m_1\le q_1,\ 0\le n_2\le p_2$ $0\le m_2\le q_2, 0\le n_3\le p_3, 0\le m_3\le q_3$ and $\alpha_j,\beta_j,\gamma_j,\delta_j,A_j,B_j,C_j,D_j,E_j,F_j,G_j$ and H_j are all positive. $a_j,b_j,c_j,d_j,e_j,f_j,g_j,h_j$ are all complex numbers. The integration path L_1 in the complex s_1 plane runs from $\sigma_1-i\infty$ to $\sigma_1+i\infty$ so that all the poles of $\Gamma^{U_j}(f_j-F_js_1)$ for $(j=1,2,\cdots,m_2)$ lie to the right of L_1 while all the poles of $\Gamma^{V_j}(1-e_j-E_js_1)$ for $(j=1,2,\cdots,n_2)$, $\Gamma^{\xi_j}(1-a_j+\alpha_js_1-A_js_2)$ for $(j=1,2,\cdots,n)$ and $\Gamma^{\xi_j'}(1-c_j+\gamma_js_1-C_js_2)$ for $(j=1,2,\cdots,n_1)$ lie to the left of L_1. The integration path L_2 in the complex s_2 plane runs from $\sigma_2-i\infty$ to $\sigma_2+i\infty$ so that all the poles of $\Gamma^{P_j}(h_j-H_js_2)$ for $(j=1,2,\cdots,m_3)$ lie to the left of L_2 while all the poles of $\Gamma^{Q_j}(1-g_j+G_js_2)$ for $(j=1,2,\cdots,n_3)$, $\Gamma^{\xi_j}(1-a_j+\alpha_js_1-A_js_2)$ for $(j=1,2,\cdots,n)$ and $\Gamma^{\xi_j'}(1-c_j+\gamma_js_1-C_js_2)$ for $(j=1,2,\cdots,n_1)$ lie to the left of L_2.

The function $I[z_1,.z_r]$ defined by the equation (1) is an analytic function of z_1 and z_2 if

$$V_1=\sum_{j=1}^{p}\xi_j\alpha_j+\sum_{j=1}^{p_1}\xi_j'\gamma_j+\sum_{j=1}^{p_2}U_jE_j-\sum_{j=1}^{q}\eta_j\beta_j-\sum_{j=1}^{q_1}\eta_j'\delta_j-\sum_{j=1}^{q_2}V_jF_j<0 \tag{9}$$

$$V_2=\sum_{j=1}^{p}\xi_jA_j+\sum_{j=1}^{q_1}\eta_j'D_j+\sum_{j=1}^{p_3}P_jG_j-\sum_{j=1}^{q}\eta_jB_j-\sum_{j=1}^{p_1}\xi_j'C_j-\sum_{j=1}^{q_3}Q_jH_j<0 \tag{10}$$

exist.

The double integral defined in the equation (5) converges absolutely if

$|\arg z_1|<\frac{1}{2}\pi\Omega_1$ and $|\arg z_2|<\frac{1}{2}\pi\Omega_2$ where

$$\Omega_1=\sum_{j=1}^{n}\xi_j\alpha_j-\sum_{j=n+1}^{p}\xi_j\alpha_j+\sum_{j=1}^{m}\eta_j\beta_j-\sum_{j=m+1}^{q}\eta_j\beta_j+\sum_{j=1}^{n_1}\xi_j'\gamma_j-\sum_{j=n_1+1}^{p_1}\xi_j'\gamma_j+\sum_{j=1}^{m_1}\eta_j'\delta_j$$

$$-\sum_{m_1+1}^{q_1}\eta_j^{'}\delta_j+\sum_{j=1}^{n_2}U_jE_j-\sum_{j=n_2+1}^{p_2}U_jE_j+\sum_{j=1}^{m_2}V_jF_j-\sum_{j=m_2+1}^{q_2}V_jF_j>0 \tag{11}$$

and

$$\Omega_2=\sum_{j=1}^{n}\xi_jA_j-\sum_{j=n+1}^{p}\xi_jA_j+\sum_{j=1}^{m}\eta_jB_j-\sum_{j=m+1}^{q}\eta_jB_j+\sum_{j=1}^{n_1}\xi_j^{'}C_j-\sum_{j=n_1+1}^{p_1}\xi_j^{'}C_j+\sum_{j=1}^{m_2}\eta_j^{'}D_j$$

$$-\sum_{m_1+1}^{q_2}\eta_j^{'}D_j+\sum_{j=1}^{n_3}P_jG_j-\sum_{j=n_3+1}^{p_3}P_jG_j+\sum_{j=1}^{m_3}Q_jH_j-\sum_{j=m_3+1}^{q_3}Q_jH_j>0 \tag{12}$$

For simplicity, we shall use the following notations:

$$U=m_1,n_1:m_2,n_2:m_3,n_3 \tag{13}$$
$$V=p_1,q_1:p_2,q_2:p_3,q_3 \tag{14}$$
$$A_1=(a_j;\alpha_j,A_j;\xi_j)_{1,p} \tag{15}$$
$$A_2=(c_j;\gamma_j,C_j;\xi_j^{'})_{1,p_1} \tag{16}$$
$$A_3=(e_j,E_j;U_j)_{1,p_2} \tag{17}$$
$$A_4=(g_j;G_j;P_j)_{1,p_3} \tag{18}$$
$$B_1=(b_j;\beta_j,B_j;\eta_j)_{1,q} \tag{19}$$
$$B_2=(d_j;\delta_j,D_j;\eta_j^{'})_{1,q_1} \tag{20}$$
$$B_3=(f_j,F_j;V_j)_{1,q_3} \tag{21}$$
$$B_4=(h_j,H_j;Q_j)_{1,q_3} \tag{22}$$

2. Required Formula

Yury A. Brychkov [9]

$$\int_0^{\pi/2}\cos^{\mu}x\cos(ax)J_v(b\cos x)=\frac{2^{-\mu-2v-1}\pi b^v\Gamma(\mu+v+1)}{\Gamma(v+1)\Gamma\left(\frac{\mu+v-a}{2}+1\right)\Gamma\left(\frac{\mu+v+a}{2}+1\right)}$$

$$\times{}_2F_3\left(\begin{matrix}\frac{\mu+v+1}{2},\frac{\mu+v}{2}+1;-\frac{b^2}{4}\\ v+1,\frac{\mu+v-a}{2}+1,\frac{\mu+v+a}{2}+1\end{matrix}\right) \tag{23}$$

where $\Re(\mu+v)>-1$

3. Main Integrals

Now we are giving two theorems in which we will get main integrals. After that we will give four corollaries out of which two for the I-function of Shantha Kumari et al. [1] and two for modified H-function of Prasad and Prasad [5] respectively.

Theorem 1: Let $\chi,\ \lambda_i,\ \delta_i \in C,\ \Re(\mu_i)>0,\ \forall i=1,2,\cdots,l$ and $\mu,\rho_1,\rho_2,\sigma_1,\ldots,\sigma_l \in R^+$, $\Re(\mu)>0$. Also let

$$\Re(\mu)+(\rho_1+\rho_2)\min_{1\le j\le m_2}\Re\left(V_j\frac{f_j}{F_j}\right)>-1,\ \ \Re(\mu)+(\rho_1+\rho_2)\min_{1\le j\le m_3}\Re\left(Q_j\frac{h_j}{H_j}\right)>-1.$$

Further, let $\left|\arg(z_i\cos^{\rho_i})\right|<\frac{1}{2}\Omega_i\pi\ (i=1,2)$ with Ω_i the same as in the equations (11) and (12), then

$$\int_0^{\pi/2}\cos^{\mu}x\cos(ax)J_v(b\cos x)E^{\lambda_i}_{\delta_i,\chi}(y_1\cos^{\sigma_1}x,\ldots,y_l\cos^{\sigma_l}x)I^{m,n:U}_{p,q:V}\left[\begin{matrix}z_1\cos^{\rho_1}x\\ z_2\cos^{\rho_2}x\end{matrix}\right]dx$$

$$=2^{-\mu-2v-1}\pi b^{v}\sum_{r=0}^{\infty}\sum_{k,\ldots k_l}^{\infty}F^{\lambda_i}_{\delta_i,\chi}\left(-\frac{b^2}{4}\right)^{r}\prod_{i=1}^{l}\frac{1}{k!}\left(\frac{y_i}{2^{\sigma_i}}\right)^{k_i}I^{m,n+3:U}_{p+3,q+4:V}\left[\begin{matrix}z_12^{-\rho_1}\\ z_22^{-\rho_2}\end{matrix}\left|\begin{matrix}A_1,X_1,X_2,X_3:A_2:A_3;A_4\\ B_1,Y_1,Y_2,Y_3,Y_4:B_2:B_3;B_4\end{matrix}\right.\right] \quad (24)$$

where

$$X_1=\left(-\mu-\sum_{i=1}^{l}\sigma_ik_i-v;\ \rho_1,\rho_2;\ 1\right) \quad (25)$$

$$X_2=\left(-\frac{1}{2}-\frac{\mu}{2}-\frac{\sum_{i=1}^{l}\sigma_ik_i}{2}-\frac{v}{2}-r;\ \frac{\rho_1}{2},\frac{\rho_2}{2};\ 1\right) \quad (26)$$

$$X_3=\left(-\frac{\mu}{2}-\frac{\sum_{i=1}^{l}\sigma_ik_i}{2}-\frac{v}{2}-r;\ \frac{\rho_1}{2},\frac{\rho_2}{2};\ 1\right) \quad (27)$$

$$Y_1 = \left(-\frac{1}{2} - \frac{\mu}{2} - \frac{\sum_{i=1}^{l} \sigma_i k_i}{2} - \frac{v}{2};\ \frac{\rho_1}{2}, \frac{\rho_2}{2};\ 1 \right) \tag{28}$$

$$Y_2 = \left(-\frac{\mu}{2} - \frac{\sum_{i=1}^{l} \sigma_i k_i}{2} - \frac{v}{2};\ \frac{\rho_1}{2}, \frac{\rho_2}{2};\ 1 \right) \tag{29}$$

$$Y_3 = \left(-\frac{\mu}{2} - \frac{\sum_{i=1}^{l} \sigma_i k_i}{2} - \frac{v}{2} + \frac{a}{2} - r;\ \frac{\rho_1}{2}, \frac{\rho_2}{2};\ 1 \right) \tag{30}$$

$$Y_4 = \left(-\frac{\mu}{2} - \frac{\sum_{i=1}^{l} \sigma_i k_i}{2} - \frac{v}{2} - \frac{a}{2} - r;\ \frac{\rho_1}{2}, \frac{\rho_2}{2};\ 1 \right) \tag{31}$$

Proof: We denote the left hand side of the equation (24) by Δ. Then we express multivariable Mittage-Leffler function in the series form with the help of equations (4) and expressing the modified I-function of two variables in terms of Mellin-Barnes contour integral with the help of equation (5), we obtain

$$\Delta = \sum_{k\ ,\ldots k_l}^{\infty} F_{\delta_i,\chi}^{\lambda_i} \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_l^{k_l}}{k_l!} \int_0^{\pi/2} (\cos x)^{\mu + k_1\sigma_1 + \cdots + k_l\sigma_l} \cos(ax) J_v(b\cos x) \times$$

$$\times \left[\frac{1}{(2\pi i)^2} \int_L \int_{L_2} \psi(s_1, s_2) \theta_1(s_1) \theta_2(s_2) (\cos x)^{\rho_1 s_1} z_1^{s_1} (\cos x)^{\rho_2 s_2} ds_1 ds_2 \right] dx \tag{32}$$

now changing the order of integration which is permissible under the given set of conditions, we get

$$\Delta = \sum_{k\ ,\ldots k_l}^{\infty} F_{\delta_i,\chi}^{\lambda_i} \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_l^{k_l}}{k_l!} \frac{1}{(2\pi i)^2} \int_{L_1} \int_{L_2} \psi(s_1, s_2) \theta_1(s_1) \theta_2(s_2) z_1^{s_1} z_2^{s_2} \times$$

$$\times \left[\int_0^{\pi/2} (\cos x)^{\mu + k_1\sigma_1 + \cdots + k_l\sigma_l + \rho_1 s_1 + \rho_2 s_2} \cos(ax) J_v(b\cos x) dx \right] ds_1 ds_2 \tag{33}$$

now evaluating the inner x-integral with the help of integral equation (23), we get

$$\int_0^{\pi/2} \cos^{\mu + \sum_{i=1}^{l} k_i\sigma_i + \rho_1 s_1 + \rho_2 s_2} x \cos(ax) J_v(b\cos x) =$$

$$= \frac{2^{-\mu-\sum_{i=1}^{l}k_i\sigma_i-\rho_1 s_1-\rho_2 s_2-2v-1}\pi b^v\Gamma(\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+1)}{\Gamma(v+1)\Gamma\left(\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v-a}{2}+1\right)\Gamma\left(\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+a}{2}+1\right)}$$

$$\times {}_2F_3\left(\frac{\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+1}{2},\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v}{2}+1;-\frac{b^2}{4}}{v+1,\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v-a}{2}+1,\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+a}{2}+1}\right) \tag{34}$$

now putting the equation (35) in the equation (34), we get

$$\Delta = \sum_{k\ ,\dots k_l}^{\infty} F_{\delta_i,\chi}^{\lambda_i}\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_l^{k_l}}{k_l!}\frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}\psi(s_1,s_2)\theta_1(s_1)\theta_2(s_2)z_1^{s_1}z_2^{s_2}\times$$

$$\times\frac{2^{-\mu-\sum_{i=1}^{l}k_i\sigma_i-\rho_1 s_1-\rho_2 s_2-2v-1}\pi b^v\Gamma(\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+1)}{\Gamma(v+1)\Gamma\left(\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v-a}{2}+1\right)\Gamma\left(\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+a}{2}+1\right)}$$

$$\times {}_2F_3\left(\frac{\frac{\mu+\sum_{i=1}^{l}m_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+1}{2},\frac{\mu+\sum_{i=1}^{l}m_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v}{2}+1;-\frac{b^2}{4}}{v+1,\frac{\mu+\sum_{i=1}^{l}m_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v-a}{2}+1,\frac{\mu+\sum_{i=1}^{l}m_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+a}{2}+1}\right)ds_1 ds_s \tag{35}$$

$$\Delta = \sum_{k\ ,\dots k_l}^{\infty} F_{\delta_i,\chi}^{\lambda_i}\frac{y_1^{k_1}}{k_1!}\cdots\frac{y_l^{k_l}}{k_l!}\frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}\psi(s_1,s_2)\theta_1(s_1)\theta_2(s_2)z_1^{s_1}z_2^{s_2}\times$$

$$\times\frac{2^{-\mu-\sum_{i=1}^{l}k_i\sigma_i-\rho_1 s_1-\rho_2 s_2-2v-1}\pi b^v\Gamma(\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+1)}{\Gamma(v+1)\Gamma\left(\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v-a}{2}+1\right)\Gamma\left(\frac{\mu+\sum_{i=1}^{l}k_i\sigma_i+\rho_1 s_1+\rho_2 s_2+v+a}{2}+1\right)}$$

$$\times\sum_{r=0}^{\infty}\frac{\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+\rho_1 s_1+\rho_2 s_2+v+1}{2}\right)_r\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+\rho_1 s_1+\rho_2 s_2+v}{2}+1\right)_r}{(v+1)_r\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+\rho_1 s_1+\rho_2 s_2+v-a}{2}+1\right)_r\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+\rho_1 s_1+\rho_2 s_2+v+a}{2}+1\right)_r}\frac{1}{r!}\left(-\frac{b^2}{4}\right)^r ds_1 ds_2 \tag{36}$$

now using the relation $(a)_r=\frac{\Gamma(a+r)}{\Gamma(a)}$, we get

$$\Delta=2^{-\mu-2v-1}\pi b^v\sum_{r=0}^{\infty}\sum_{k,\ldots k_l}^{\infty}F_{\delta_i,\chi}^{\lambda_i}\frac{1}{k_1!}\left(\frac{y_1}{2^{\sigma_1}}\right)^{k_1}\cdots\frac{1}{k_l!}\left(\frac{y_l}{2^{\sigma_l}}\right)^{k_l}\frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}\psi(s_1,s_2)\theta_1(s_1)\theta_2(s_2)z_1^{s_1}z_2^{s_2}$$

$$\times\frac{\Gamma(\mu+\sum_{i=1}^{l}\sigma_i k_i+v+1+\rho_1 s_1+\rho_2 s_2)}{\Gamma\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+v+1+\rho_1 s_1+\rho_2 s_2}{2}\right)}\frac{\Gamma\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+v+1+\rho_1 s_1+\rho_2 s_2}{2}+r\right)}{\Gamma\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+v+\rho_1 s_1+\rho_2 s_2}{2}+1\right)}$$

$$\times\frac{\Gamma\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+v+\rho_1 s_1+\rho_2 s_2}{2}+1+r\right)}{\Gamma\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+v-a+\rho_1 s_1+\rho_2 s_2}{2}+r\right)}\frac{1}{\Gamma\left(\frac{\mu+\sum_{i=1}^{l}\sigma_i k_i+v+a+\rho_1 s_1+\rho_2 s_2}{2}+r\right)}ds_1 ds_2 \tag{37}$$

now interpreting the above equation (37) with the help of equation (5), we get required result (24).

Theorem 2: Let $\chi,\lambda_i,\delta_i\in C,\ \Re(\mu_i)>0,\ \forall i=1,2,\cdots,l$ and $\mu,\ \rho_1,\ \rho_2,\ \sigma_1,\ldots,\sigma_l\in R^+$, $\Re(\mu)>0.$ Also let

$$\Re(\mu)+(\rho_1+\rho_2)\min_{1\le j\le m_2}\Re\left(V_j\frac{f_j}{F_j}\right)>-1,\ \Re(\mu)+(\rho_1+\rho_2)\min_{1\le j\le m_3}\Re\left(Q_j\frac{h_j}{H_j}\right)>-1.$$

Further, let $\left|\arg(z_i\cos^{\rho_i})\right|<\frac{1}{2}\Omega_i\pi\ (i=1,2)$ with Ω_i the same as in the equations (12) and (13), then

$$\int_0^{\pi/2}\cos^{\mu}x\cos(ax)J_v(b\cos x)E_{\delta_i,\chi}^{\lambda_i}(y_1\cos^{-\sigma_1}x,\ldots,y_l\cos^{-\sigma_l}x)I_{p,q:V}^{m,n:U}\left[\begin{matrix}z_1\cos^{\rho_1}x\\ z_2\cos^{\rho_2}x\end{matrix}\right]dx$$

$$=2^{-\mu-2v-1}\pi b^{v}\sum_{r=0}^{\infty}\sum_{k,\ldots k_l}^{\infty}F_{\delta_i,\chi}^{\lambda_i}\left(-\frac{b^2}{4}\right)^r\prod_{i=1}^{l}\frac{1}{k!}\left(\frac{y_i}{2^{\sigma_i}}\right)^{k_i}I_{p+3,q+4:V}^{m,n+3:U}\left[\begin{matrix}z_1 2^{-\rho_1}\\ z_2 2^{-\rho_2}\end{matrix}\middle|\begin{matrix}A_1,X_4,X_5,X_6:A_2:A_3;A_4\\ B_1,Y_4,Y_5,Y_6,Y_7:B_2:B_3;B_4\end{matrix}\right] \quad (38)$$

where

$$X_4=(-\mu+\textstyle\sum_{i=1}^{l}\sigma_i k_i-v;\ \rho_1,\rho_2;\ 1) \quad (39)$$

$$X_5=\left(-\frac{1}{2}-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}-r;\ \frac{\rho_1}{2},\frac{\rho_2}{2};\ 1\right) \quad (40)$$

$$X_6=\left(-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}-r;\ \frac{\rho_1}{2},\frac{\rho_2}{2};\ 1\right) \quad (41)$$

$$Y_5=\left(-\frac{1}{2}-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2};\ \frac{\rho_1}{2},\frac{\rho_2}{2};\ 1\right), \quad (42)$$

$$Y_6=\left(-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2};\ \frac{\rho_1}{2},\frac{\rho_2}{2};\ 1\right) \quad (43)$$

$$Y_7=\left(-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}+\frac{a}{2}-r;\ \frac{\rho_1}{2},\frac{\rho_2}{2};\ 1\right) \quad (44)$$

$$Y_8=\left(-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}-\frac{a}{2}-r;\ \frac{\rho_1}{2},\frac{\rho_2}{2};\ 1\right) \quad (45)$$

4. Special Cases

In this section, by changing parameters, we shall give four special cases of theorem 1 and theorem 2. By taking $m,m_1,n_1,p_1,q_1=0$ we shall obtain two corollaries for the I-function of two variables given by Shantha Kumari et al. [1].

Corollary 1: Let $\chi,\ \lambda_i,\ \delta_i\in C,\ \Re(\mu_i)>0,\ \forall i=1,2,\cdots,l$ and $\mu,\rho_1,\rho_2,\sigma_1,\ldots,\sigma_l\in R^{+}$, $\Re(\mu)>0.$ Also let

$$\Re(\mu)+(\rho_1+\rho_2)\min_{1\le j\le m_2}\Re\left(V_j\frac{f_j}{F_j}\right)>-1,\quad \Re(\mu)+(\rho_1+\rho_2)\min_{1\le j\le m_3}\Re\left(Q_j\frac{h_j}{H_j}\right)>0.$$

Further, let $\left|\arg(z_i\cos^{\rho_i})\right|<\frac{1}{2}\Omega_i\pi\ (i=1,2)$ with Ω_i as given below in the equations (46) and (47).

$$\Omega_1=\sum_{j=1}^{n}\xi_j\alpha_j-\sum_{j=n+1}^{p}\xi_j\alpha_j-\sum_{j=1}^{q}\eta_j\beta_j+\sum_{j=1}^{n_2}U_jE_j-\sum_{j=n_2+1}^{p_2}U_jE_j+\sum_{j=1}^{m_2}V_jF_j-\sum_{j=m_2+1}^{q_2}V_jF_j>0 \tag{46}$$

and

$$\Omega_2=\sum_{j=1}^{n}\xi_jA_j-\sum_{j=n+1}^{p}\xi_jA_j-\sum_{j=1}^{q}\eta_jB_j+\sum_{j=1}^{n_3}P_jG_j-\sum_{j=n_3+1}^{p_3}P_jG_j+\sum_{j=1}^{m_3}Q_jH_j-\sum_{j=m_3+1}^{q_3}Q_jH_j>0. \tag{47}$$

Then

$$\int_0^{\pi/2}\cos^{\mu}x\cos(ax)J_v(b\cos x)E_{\delta_i,\chi}^{\lambda_i}(y_1\cos^{\sigma_1}x,\ldots,y_l\cos^{\sigma_l}x)I_{p,q:p_2,q_2:p_3,q_3}^{0,n:m_2,n_2:m_3,n_3}\left[\begin{matrix}z_1\cos^{\rho_1}x\\ z_2\cos^{\rho_2}x\end{matrix}\right]dx$$

$$=2^{-\mu-2v-1}\pi b^{v}\sum_{r=0}^{\infty}\sum_{k,\ldots k_l}^{\infty}F_{\delta_i,\chi}^{\lambda_i}\left(-\frac{b^2}{4}\right)^{r}\prod_{i=1}^{l}\frac{1}{k!}\left(\frac{y_i}{2^{\sigma_i}}\right)^{k_i}I_{p+3,q+4:p_2,q_2;p_3,q_3}^{0,n+3:m_2,n_2;m_3,n_3}\left[\begin{matrix}z_12^{-\rho_1}\\ z_22^{-\rho_2}\end{matrix}\left|\begin{matrix}A_1,X_1,X_2,X_3:A_3;A_4\\ B_1,Y_1,Y_2,Y_3,Y_4:B_3;B_4\end{matrix}\right.\right] \tag{48}$$

where $A_1,A_3,A_4,B_1,B_3,B_4,X_1,X_2,X_3,Y_1,Y_2,Y_3,Y_4$ are already given above in the equations (15), (17), (18), (19), (21), (22), (25), (26), (27), (28), (29), (30) and (31) respectively.

Corollary 2: Let $\chi,\ \lambda_i,\ \delta_i\in C,\Re(\mu_i)>0,\ \forall i=1,2,\cdots,l$ and $\mu,\rho_1,\rho_2,\sigma_1,\ldots,\sigma_l\in R^{+}$, $\Re(\mu)>0$. Also let

$$\Re(\mu)+(\rho_1+\rho_2)\min_{1\le j\le m_2}\Re\left(V_j\frac{f_j}{F_j}\right)>-1,\ \Re(\mu)+(\rho_1+\rho_2)\min_{1\le j\le m_3}\Re\left(Q_j\frac{h_j}{H_j}\right)>-1.$$

Further, let $\left|\arg(z_i\cos^{\rho_i})\right|<\frac{1}{2}\Omega_i\pi\ (i=1,2)$ with Ω_i as given below in the equations (49) and (50).

$$\Omega_1=\sum_{j=1}^{n}\xi_j\alpha_j-\sum_{j=n+1}^{p}\xi_j\alpha_j-\sum_{j=1}^{q}\eta_j\beta_j+\sum_{j=1}^{n_2}U_jE_j-\sum_{j=n_2+1}^{p_2}U_jE_j+\sum_{j=1}^{m_2}V_jF_j-\sum_{j=m_2+1}^{q_2}V_jF_j>0 \tag{49}$$

and

$$\Omega_2=\sum_{j=1}^{n}\xi_jA_j-\sum_{j=n+1}^{p}\xi_jA_j-\sum_{j=1}^{q}\eta_jB_j+\sum_{j=1}^{n_3}P_jG_j-\sum_{j=n_3+1}^{p_3}P_jG_j+\sum_{j=1}^{m_3}Q_jH_j-\sum_{j=m_3+1}^{q_3}Q_jH_j>0. \tag{50}$$

Then

$$\int_0^{\pi/2} \cos^{\mu} x \cos(ax) J_v(b\cos x) E_{\delta_i,\chi}^{\lambda_i}(y_1 \cos^{-\sigma_1} x,\ldots, y_l \cos^{-\sigma_l} x) I_{p,q:p_2,q_2:p_3,q_3}^{0,n:m_2,n_2:m_3,n_3}\left[\begin{matrix} z_1 \cos^{\rho_1} x \\ z_2 \cos^{\rho_2} x \end{matrix}\right] dx$$

$$= 2^{-\mu-2v-1} \pi b^v \sum_{r=0}^{\infty} \sum_{k,\ldots,k_l}^{\infty} F_{\delta_i,\chi}^{\lambda_i}\left(-\frac{b^2}{4}\right)^r \prod_{i=1}^{l} \frac{1}{k!}\left(\frac{y_i}{2^{\sigma_i}}\right)^{k_i} I_{p+3,q+4:p_2,q_2:p_3,q_3}^{0,n+3:m_2,n_2:m_3,n_3}\left[\begin{matrix} z_1 2^{-\rho_1} \\ z_2 2^{-\rho_2} \end{matrix}\middle| \begin{matrix} A_1, X_4, X_5, X_6 : A_3; A_4 \\ B_1, Y_4, Y_5, Y_6, Y_7 : B_3; B_4 \end{matrix}\right] \quad (51)$$

where $A_1, A_3, A_4, B_1, B_3, B_4, X_4, X_5, X_6, Y_5, Y_6, Y_7, Y_8$ are already given above in the equations (15), (17), (18), (19), (21), (22), (39), (40), (41), (42), (43), (44) and (45) respectively.

Now by taking $\xi_j, \xi_j', \eta_j, \eta_j', U_j, V_j, P_j, Q_j = 1$ we shall obtain two corollaries for the modified H-function of two variables given by Prasad and Prasad [5].

Corollary 3: Let $\chi,\ \lambda_i,\ \delta_i \in C,\ \Re(\mu_i) > 0,\ \forall i = 1,2,\cdots,l$ and $\mu, \rho_1, \rho_2, \sigma_1, \ldots, \sigma_l \in R^+$, $\Re(\mu) > 0$. Also let

$$\Re(\mu) + (\rho_1 + \rho_2) \min_{1\le j\le m_2} \Re\left(\frac{f_j}{F_j}\right) > -1,\quad \Re(\mu) + (\rho_1 + \rho_2) \min_{1\le j\le m_3} \Re\left(\frac{h_j}{H_j}\right) > -1.$$

Further, let $\left|\arg(z_i \cos^{\rho_i})\right| < \frac{1}{2}\Omega_i \pi\ (i = 1,2)$ with Ω_i as given in the equations (52) and (53).

$$\Omega_1 = \sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j + \sum_{j=1}^{n_1} \gamma_j - \sum_{j=n_1+1}^{p_1} \gamma_j + \sum_{j=1}^{m_1} \delta_j$$

$$-\sum_{m_1+1}^{q_1} \delta_j + \sum_{j=1}^{n_2} E_j - \sum_{j=n_2+1}^{p_2} E_j + \sum_{j=1}^{m_2} F_j - \sum_{j=m_2+1}^{q_2} F_j > 0 \quad (52)$$

and

$$\Omega_2 = \sum_{j=1}^{n} A_j - \sum_{j=n+1}^{p} A_j + \sum_{j=1}^{m} B_j - \sum_{j=m+1}^{q} B_j + \sum_{j=1}^{n_1} C_j - \sum_{j=n_1+1}^{p_1} C_j + \sum_{j=1}^{m_2} D_j$$

$$-\sum_{m_1+1}^{q_2} D_j + \sum_{j=1}^{n_3} G_j - \sum_{j=n_3+1}^{p_3} G_j + \sum_{j=1}^{m_3} H_j - \sum_{j=m_3+1}^{q_3} H_j > 0. \quad (53)$$

Then

$$\int_0^{\pi/2} \cos^{\mu} x \cos(ax) J_v(b\cos x) E_{\delta_i,\chi}^{\lambda_i}(y_1 \cos^{\sigma_1} x,\ldots, y_l \cos^{\sigma_l} x) H_{p,q:V}^{m,n:U}\left[\begin{matrix} z_1 \cos^{\rho_1} x \\ z_2 \cos^{\rho_2} x \end{matrix}\right] dx$$

$$= 2^{-\mu-2v-1}\pi b^{v}\sum_{r=0}^{\infty}\sum_{k,\ldots k_l}^{\infty} F_{\delta_i,\chi}^{\lambda_i}\left(-\frac{b^2}{4}\right)^{r}\prod_{i=1}^{l}\frac{1}{k!}\left(\frac{y_i}{2^{\sigma_i}}\right)^{k_i} H_{p+3,q+4:V}^{m,n+3:U}\left[\begin{array}{c|c} z_1 2^{-\rho_1} & A_5, X_7, X_8, X_9 : A_6 : A_7; A_8 \\ z_2 2^{-\rho_2} & B_5, Y_9, Y_{10}, Y_{11}, Y_{12} : B_6 : B_7; B_8 \end{array}\right] \quad (54)$$

where

$$A_5 = (a_j;\alpha_j, A_j)_{1,p}, A_6 = (c_j;\gamma_j, C_j)_{1,p_1}, \ A_7 = (e_j, E_j)_{1,p_2}, \ A_8 = (g_j;G_j)_{1,p_3}, \quad (55)$$

$$B_5 = (b_j;\beta_j, B_j)_{1,q}, \ B_6 = (d_j;\delta_j, D_j)_{1,q_1}, \ B_7 = (f_j, F_j)_{1,q_3}, \ B_8 = (h_j, H_j)_{1,q_3} \quad (56)$$

$$X_7 = (-\mu - \textstyle\sum_{i=1}^{l}\sigma_i k_i - v; \rho_1, \rho_2) \quad (57)$$

$$X_8 = \left(-\frac{1}{2}-\frac{\mu}{2}-\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}-r; \frac{\rho_1}{2}, \frac{\rho_2}{2}\right) \quad (58)$$

$$X_9 = \left(-\frac{\mu}{2}-\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}-r; \frac{\rho_1}{2}, \frac{\rho_2}{2}\right) \quad (59)$$

$$Y_9 = \left(-\frac{1}{2}-\frac{\mu}{2}-\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}; \frac{\rho_1}{2}, \frac{\rho_2}{2}\right) \quad (60)$$

$$Y_{10} = \left(-\frac{\mu}{2}-\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}; \frac{\rho_1}{2}, \frac{\rho_2}{2}\right) \quad (61)$$

$$Y_{11} = \left(-\frac{\mu}{2}-\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}+\frac{a}{2}-r; \frac{\rho_1}{2}, \frac{\rho_2}{2}\right) \quad (62)$$

$$Y_{12} = \left(-\frac{\mu}{2}-\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}-\frac{a}{2}-r; \frac{\rho_1}{2}, \frac{\rho_2}{2}\right) \quad (63)$$

Corollary 4: Let $\chi, \lambda_i, \delta_i \in C, \Re(\mu_i) > 0, \forall i = 1,2,\cdots,l$ and $\mu, \rho_1, \rho_2, \sigma_1, \ldots, \sigma_l \in R^{+}$, $\Re(\mu) > 0$. Also let

$$\Re(\mu) + (\rho_1+\rho_2)\min_{1\le j\le m_2}\Re\left(\frac{f_j}{F_j}\right) > -1, \ \ \Re(\mu) + (\rho_1+\rho_2)\min_{1\le j\le m_3}\Re\left(\frac{h_j}{H_j}\right) > -1.$$

Further, let $\left|\arg(z_i\cos^{\rho_i})\right| < \frac{1}{2}\Omega_i\pi \ (i=1,2)$ with Ω_i as given below in the equations (64) and (65).

$$\Omega_1 = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j + \sum_{j=1}^{n_1}\gamma_j - \sum_{j=n_1+1}^{p_1}\gamma_j + \sum_{j=1}^{m_1}\delta_j$$

$$-\sum_{m_1+1}^{q_1}\delta_j+\sum_{j=1}^{n_2}E_j-\sum_{j=n_2+1}^{p_2}E_j+\sum_{j=1}^{m_2}F_j-\sum_{j=m_2+1}^{q_2}F_j>0 \tag{64}$$

and

$$\Omega_2=\sum_{j=1}^{n}A_j-\sum_{j=n+1}^{p}A_j+\sum_{j=1}^{m}B_j-\sum_{j=m+1}^{q}B_j+\sum_{j=1}^{n_1}C_j-\sum_{j=n_1+1}^{p_1}C_j+\sum_{j=1}^{m_2}D_j$$

$$-\sum_{m_1+1}^{q_2}D_j+\sum_{j=1}^{n_3}G_j-\sum_{j=n_3+1}^{p_3}G_j+\sum_{j=1}^{m_3}H_j-\sum_{j=m_3+1}^{q_3}H_j>0. \tag{65}$$

Then

$$\int_0^{\pi/2}\cos^{\mu}x\cos(ax)J_v(b\cos x)E_{\delta_i,\chi}^{\lambda_i}(y_1\cos^{-\sigma_1}x,\ldots,y_l\cos^{-\sigma_l}x)H_{p,q:V}^{m,n:U}\left[\begin{matrix}z_1\cos^{\rho_1}x\\ z_2\cos^{\rho_2}x\end{matrix}\right]dx$$

$$=2^{-\mu-2v-1}\pi b^{v}\sum_{r=0}^{\infty}\sum_{k_1,\ldots,k_l}^{\infty}F_{\delta_i,\chi}^{\lambda_i}\left(-\frac{b^2}{4}\right)^r\prod_{i=1}^{l}\frac{1}{k!}\left(\frac{y_i}{2^{\sigma_i}}\right)^{k_i}H_{p+3,q+4:V}^{m,n+3:U}\left[\begin{matrix}z_1 2^{-\rho_1}\\ z_2 2^{-\rho_2}\end{matrix}\middle|\begin{matrix}A_5,X_{10},X_{11},X_{12}:A_6:A_7;A_8\\ B_5,Y_{13},Y_{14},Y_{15},Y_{16}:B_6:B_7;B_8\end{matrix}\right] \tag{66}$$

where

$$A_5=(a_j;\alpha_j,A_j)_{1,p},\ A_6=(c_j;\gamma_j,C_j)_{1,p_1},\ A_7=(e_j,E_j)_{1,p_2},\ A_8=(g_j;G_j)_{1,p_3}, \tag{67}$$

$$B_5=(b_j;\beta_j,B_j)_{1,q},\ B_6=(d_j;\delta_j,D_j)_{1,q_1},\ B_7=(f_j,F_j)_{1,q_3},\ B_8=(h_j,H_j)_{1,q_3} \tag{68}$$

$$X_{10}=(-\mu+\textstyle\sum_{i=1}^{l}\sigma_i k_i-v;\rho_1,\rho_2) \tag{69}$$

$$X_{11}=\left(-\frac{1}{2}-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}-r;\ \frac{\rho_1}{2},\frac{\rho_2}{2}\right) \tag{70}$$

$$X_{12}=\left(-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2}-r;\ \frac{\rho_1}{2},\frac{\rho_2}{2}\right) \tag{71}$$

$$Y_{13}=\left(-\frac{1}{2}-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2};\ \frac{\rho_1}{2},\frac{\rho_2}{2}\right), \tag{72}$$

$$Y_{14}=\left(-\frac{\mu}{2}+\frac{\sum_{i=1}^{l}\sigma_i k_i}{2}-\frac{v}{2};\ \frac{\rho_1}{2},\frac{\rho_2}{2}\right) \tag{73}$$

$$Y_{15} = \left(-\frac{\mu}{2} + \frac{\sum_{i=1}^{l} \sigma_i k_i}{2} - \frac{v}{2} + \frac{a}{2} - r; \frac{\rho_1}{2}, \frac{\rho_2}{2} \right) \tag{74}$$

$$Y_{16} = \left(-\frac{\mu}{2} + \frac{\sum_{i=1}^{l} \sigma_i k_i}{2} - \frac{v}{2} - \frac{a}{2} - r; \frac{\rho_1}{2}, \frac{\rho_2}{2} \right) \tag{75}$$

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