

# CERTAIAN FRACTIONAL DIFFERENTIAL AND INTEGRAL TRANSFORM FORMULAS ASSOCIATED WITH THE PRODUCT OF GENERAL CLASS OF POLYNOMIAL, MULTIVARIABLE MITTAGE-LEFFLER FUNCTION AND GENERALIZED LAURICELLA'S HYPERGEOMETRIC FUNCTION

**Harendra Singh<sup>1</sup> and Prvindra Kumar<sup>2</sup>**

<sup>1</sup>Department of Mathematics, M.M.H College, Ghaziabad, India

<sup>2</sup>Department of Mathematics, D.J College, Baraut, India

\*Corresponding author email: [prvindradsj@gmail.com](mailto:prvindradsj@gmail.com)

**Abstract:** In this paper, first we establish two theorems and several corollaries by applying various fractional differential operators involving the product of Srivastava polynomial, multivariable Mittag-Leffler function and generalized Lauricella's hypergeometric function and express the result in terms of generalized Lauricella's hypergeometric function. Next, we establish the certain composition formulas by employing some integral transforms like Beta transform, Laplace transform, and Verma transform on the result of these two theorems.

**Keywords and phrases:** Generalized fractional differential operators, extended beta functions, generalized Lauricella's hypergeometric function, Srivastava's polynomial, multivariable Mittag-Leffler function, Beta transform, Laplace transform, and Verma transform.

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## 1. Introduction and Preliminaries

In 1903, Mittag-Leffler [8] introduced the function  $E_\alpha(y)$  in the following manner:

$$E_\alpha(y) = \sum_{k=0}^{\infty} \frac{y^k}{\Gamma(\alpha k + 1)} \quad \text{where } \alpha, y \in C, \Re(\alpha) > 0 \quad (1)$$

In 1905, Wiman [20] generalized the function  $E_\alpha(y)$  and gave the function  $E_{\alpha,\beta}(y)$  in the following manner:

$$E_{\alpha,\beta}(y) = \sum_{k=0}^{\infty} \frac{y^k}{\Gamma(\alpha k + \beta)} \quad \text{where } \alpha, \beta, y \in C, \Re(\alpha) > 0, \Re(\beta) > 0 \quad (2)$$

In 1971, Prabhakar [11] generalized the function  $E_{\alpha,\beta}(y)$  and gave the function  $E_{\alpha,\beta}^\lambda(y)$  in the following manner:

$$E_{\alpha,\beta}^\lambda(y) = \sum_{k=0}^{\infty} \frac{(\lambda)_k}{\Gamma(\alpha k + \beta)} \frac{y^k}{k!} \quad (3)$$

where  $\alpha, \beta, \lambda, y \in C, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\lambda) > 0$

Saxena et al. [15] gave the multivariable analogue of multivariable Mittag-leffler function in the following manner:

$$E_{\mu_j, c}^{\lambda_j}(y_1, \dots, y_l) = E_{\mu_1, \dots, \mu_l, c}^{\lambda_1, \dots, \lambda_l}(y_1, \dots, y_l) = \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(\lambda_1)_{k_1}, \dots, (\lambda_l)_{k_l}}{\Gamma(c + \sum_{j=1}^l \mu_j k_j)} \frac{y_1^{k_1}}{k_1!} \dots \frac{y_l^{k_l}}{k_l!} \quad (4)$$

where  $c, \lambda_j, \mu_j \in C, \Re(\mu_j) > 0, \forall j = 1, 2, \dots, l$

For the sake of convenience, let  $F_{\mu_j, \nu}^{\lambda_j} = \frac{(\lambda_1)_{k_1}, \dots, (\lambda_l)_{k_l}}{\Gamma(\nu + \sum_{j=1}^l \mu_j k_j)}$  (5)

Srivastava [18] introduced the general class of polynomials in the following manner:

$$S_N^M(y) = \sum_{k=0}^{[N/M]} \frac{(-N)_{Mk}}{k!} A_{N,k} y^k \quad N = 0, 1, 2, \dots \quad (6)$$

where  $M$  is an arbitrary positive integer and the coefficients  $A_{N,k} (N, k \geq 0)$  are arbitrary constants, real or complex and  $(\lambda)_N$  is the pochhammer symbol.

Recently Cetinkaya et al. [1] defined the generalized beta function in the following manner:

$$B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} {}_1F_1\left(\gamma; \delta; -\frac{p_1}{t^\tau} - \frac{p_2}{(1-t)^\nu}\right) dt \quad (7)$$

where  $\min\{\Re(p_1), \Re(p_2)\} \geq 0, \min\{\Re(x), \Re(y)\} > 0, \min\{\Re(\gamma), \Re(\delta), \Re(\tau), \Re(\nu)\} > 0$

when  $\tau = \nu = 1$ , the generalized beta function in equation (7) reduces to the following generalized beta function defined by Goswami et al. [4]

$$B_{p_1, p_2}^{(\gamma, \delta)}(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} {}_1F_1\left(\gamma; \delta; -\frac{p_1}{t} - \frac{p_2}{(1-t)}\right) dt \quad (8)$$

where  $\min\{\Re(p_1), \Re(p_2)\} \geq 0, \min\{\Re(x), \Re(y)\} > 0, \min\{\Re(\gamma), \Re(\delta)\} > 0$

when  $\tau = \nu = 1$  and  $p_1 = p_2$ , the generalized beta function in equation (7) reduces to the following generalized beta function defined by Ozergin et al. [9]

$$B_{p_1}^{(\gamma, \delta)}(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} {}_1F_1\left(\gamma; \delta; -\frac{p_1}{t(1-t)}\right) dt \quad (9)$$

where  $\Re(p_1) \geq 0$ ,  $\min\{\Re(x), \Re(y)\} > 0$ ,  $\min\{\Re(\gamma), \Re(\delta)\} > 0$

when  $\gamma = \delta = \tau = \nu = 1$ , the generalized beta function in equation (7) reduces to the following generalized beta function defined by J. Choi et al. [3]

$$B_{p_1, p_2}(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} \exp\left(-\frac{p_1}{t} - \frac{p_2}{1-t}\right) dt \quad (10)$$

where  $\min\{\Re(p_1), \Re(p_2)\} \geq 0$ ,  $\min\{\Re(x), \Re(y)\} > 0$

when  $\gamma = \delta = \tau = \nu = 1$  and  $p_1 = p_2$ , the generalized beta function in equation (7) reduces to the following generalized beta function defined by M.A. Chaudhry et al. [2]

$$B_{p_1}(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} \exp\left(-\frac{p_1}{t(1-t)}\right) dt \quad (11)$$

where  $\Re(p_1) \geq 0$ ,  $\min\{\Re(x), \Re(y)\} > 0$

when  $p_1 = p_2 = 0$  the generalized beta function in equation (7) reduces to the following classical beta function [19]

$$B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt \quad (12)$$

where  $\min\{\Re(x), \Re(y)\} > 0$

Recently Cetinkaya et al. [1] defined the generalized Lauricella's hypergeometric function  $F_D^3$  in the following manner:

$$F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1, y_2, y_3; n_1, n_2) = \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3} B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)}{(b)_{m_1+m_2+m_3} B(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)} \frac{y_1^{m_1} y_2^{m_2} y_3^{m_3}}{m_1! m_2! m_3!} \quad (13)$$

where  $|y_1| < 1, |y_2| < 1, |y_3| < 1$

## 2. Generalized Fractional Integral Operators

The generalized fractional integral operators (called the Marichev-Saigo-Maeda operators) including the Saigo operators and involving the Appell's function  $F_3(\cdot)$  of the third kind as the kernel, introduced by Marichev (see [7], [13]) are defined in the following manner:

**Definition:** Let  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta \in C$  and  $x > 0$ , then for  $\Re(\eta) > 0$  we have (see [4], [10])

$$(I_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} f)(x) = \frac{x^{-\alpha_1}}{\Gamma(\eta)} \int_0^x (x-t)^{\eta-1} t^{-\alpha_2} F_3 \left( \alpha_1, \alpha_2, \beta_1, \beta_2; \eta; 1-\frac{t}{x}, 1-\frac{x}{t} \right) f(t) dt \quad (14)$$

and

$$(I_{x,\infty}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} f)(x) = \frac{x^{-\alpha_2}}{\Gamma(\eta)} \int_x^\infty (t-x)^{\eta-1} t^{-\alpha_1} F_3 \left( \alpha_1, \alpha_2, \beta_1, \beta_2; \eta; 1-\frac{x}{t}, 1-\frac{t}{x} \right) f(t) dt \quad (15)$$

the corresponding Marichev-Saigo-Maeda fractional differential operators are given as follows (see [7], [13])

**Definition:** Let  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta \in C$  and  $x > 0$ , then for  $\Re(\eta) > 0$  we have (see [4], [10])

$$\begin{aligned} (D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} f)(x) &= (I_{0,x}^{-\alpha_2, -\alpha_1, -\beta_2, -\beta_1, -\eta} f)(x) = \left( \frac{d}{dx} \right)^m (I_{0,x}^{-\alpha_2, -\alpha_1, -\beta_2+m, -\beta_1, -\eta+m} f)(x) \\ &= \frac{1}{\Gamma(m-\eta)} \left( \frac{d}{dx} \right)^m (x)^{\alpha_2} \int_0^x (x-t)^{m-\eta-1} t^{\alpha_1} \\ &\quad \times F_3 \left( -\alpha_2, -\alpha_1, m-\beta_2, -\beta_1; m-\eta; 1-\frac{t}{x}, 1-\frac{x}{t} \right) f(t) dt \end{aligned} \quad (16)$$

and

$$\begin{aligned} (D_{x,\infty}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} f)(x) &= (I_{x,\infty}^{-\alpha_2, -\alpha_1, -\beta_2, -\beta_1, -\eta} f)(x) = \left( -\frac{d}{dx} \right)^m (I_{x,\infty}^{-\alpha_2, -\alpha_1, -\beta_2, -\beta_1+m, -\eta+m} f)(x) \\ &= \frac{1}{\Gamma(m-\eta)} \left( -\frac{d}{dx} \right)^m (x)^{\alpha_1} \int_x^\infty (t-x)^{m-\eta-1} t^{\alpha_2} \\ &\quad \times F_3 \left( -\alpha_2, -\alpha_1, -\beta_2, m-\beta_1; m-\eta; 1-\frac{x}{t}, 1-\frac{t}{x} \right) f(t) dt \end{aligned} \quad (17)$$

where  $m = [\Re(\eta)] + 1$  and  $[x]$  denotes the greatest integer function.

If we put  $\alpha_2 = \beta_2 = 0$ ,  $\beta_1 = -\eta$ ,  $\alpha_1 = \alpha_1 + \beta_1$  and  $\eta = \alpha_1$  then the above equations (16) and (17) reduce to the following Saigo fractional differential operators (see [12])

$$(D_{0,x}^{\alpha_1, \beta_1, \eta} f)(x) = (I_{0,x}^{-\alpha_1, -\beta_1, \alpha_1 + \eta} f)(x) = \left( \frac{d}{dx} \right)^m (I_{0,x}^{-\alpha_1 + m, -\beta_1 - m, \alpha_1 + \eta - m} f)(x) \quad (18)$$

and

$$(D_{x,\infty}^{\alpha_1, \beta_1, \eta} f)(x) = (I_{x,\infty}^{-\alpha_1, -\beta_1, \alpha_1 + \eta} f)(x) = \left( -\frac{d}{dx} \right)^m (I_{x,\infty}^{-\alpha_1 + m, -\beta_1 - m, \alpha_1 + \eta} f)(x) \quad (19)$$

where  $\Re(\alpha_1) > 0$ ,  $m = [\Re(\alpha_1)] + 1$  and  $[x]$  denotes the greatest integer function.

If we put  $\beta_1 = 0$ , then the Saigo fractional differential operators given in the equations (18) and (19) reduce to the following Erdelyi-Kober fractional differential operators (see [5])

$$\begin{aligned} (D_{0,x}^{\alpha_1, \eta} f)(x) &= \left( \frac{d}{dx} \right)^m \frac{x^{-\eta}}{\Gamma(m - \alpha_1)} \int_0^x t^{\alpha_1 + \eta} (x - t)^{m - \alpha_1 - 1} f(t) dt \\ &= \left( \frac{d}{dx} \right)^m (I_{0,x}^{-\alpha_1 + m, -\alpha_1, \alpha_1 + \eta - m} f)(x) \end{aligned} \quad (20)$$

and

$$\begin{aligned} (D_{x,\infty}^{\alpha_1, \eta} f)(x) &= \left( -\frac{d}{dx} \right)^m \frac{x^{\alpha_1 + \eta}}{\Gamma(m - \alpha_1)} \int_x^\infty t^{-\eta} (t - x)^{m - \alpha_1 - 1} f(t) dt \\ &= \left( -\frac{d}{dx} \right)^m (I_{x,\infty}^{-\alpha_1 + m, -\alpha_1, \alpha_1 + \eta} f)(x) \end{aligned} \quad (21)$$

If we put  $\beta_1 = -\alpha_1$ , then the Saigo fractional differential operators given in equations (18) and (19) reduce to the following Riemann-Liouville and Weyl fractional differential operators respectively (see [10])

$$(D_{0,x}^{\alpha_1} f)(x) = \left( \frac{d}{dx} \right)^m \frac{1}{\Gamma(m - \alpha_1)} \int_0^x (x - t)^{m - \alpha_1 - 1} f(t) dt = \left( \frac{d}{dx} \right)^m (I_{0,x}^{-\alpha_1 + m} f)(x) \quad (22)$$

and

$$(D_{x,\infty}^{\alpha_1} f)(x) = \left( -\frac{d}{dx} \right)^m \frac{1}{\Gamma(m - \alpha_1)} \int_x^\infty (t - x)^{m - \alpha_1 - 1} f(t) dt = \left( -\frac{d}{dx} \right)^m (I_{x,\infty}^{-\alpha_1 + m} f)(x) \quad (23)$$

Now, we are recalling the following lemma which gives the power function formulas [12], [13] of the above discussed Marichev-Saigo-Maeda fractional differential operators. These power function formulas are required for our present study.

**Lemma:** Let  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta \in \mathbb{C}$  and  $x > 0$ . Then the following formulas hold (see [12], [13])

(a) Let  $\Re(\eta) > 0$  and  $\Re(\rho) > \max\{0, \Re(\eta - \alpha_1 - \alpha_2 - \beta_2), \Re(\beta_1 - \alpha_1)\}$ , then

$$(D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} t^{\rho-1})(x) = \frac{\Gamma(\rho)\Gamma(\rho - \eta + \alpha_1 + \alpha_2 + \beta_2)\Gamma(\rho - \beta_1 + \alpha_1)}{\Gamma(\rho - \eta + \alpha_1 + \alpha_2)\Gamma(\rho - \eta + \alpha_1 + \beta_2)\Gamma(\rho - \beta_1)} x^{\rho - \eta + \alpha_1 + \alpha_2 - 1} \quad (24)$$

(b) Let  $\Re(\eta) > 0$  and  $\Re(\rho) < 1 + \min\{\Re(\beta_2), \Re(\eta - \alpha_1 - \alpha_2), \Re(\eta - \alpha_2 - \beta_1)\}$ , then

$$(D_{x,\infty}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} t^{\rho-1})(x) = \frac{\Gamma(1 + \beta_2 - \rho)\Gamma(1 - \alpha_1 - \alpha_2 + \eta - \rho)\Gamma(1 - \alpha_2 - \beta_1 + \eta - \rho)}{\Gamma(1 - \rho)\Gamma(1 - \alpha_1 - \alpha_2 - \beta_1 + \eta - \rho)\Gamma(1 - \alpha_2 + \beta_2 - \rho)} x^{\rho - \eta + \alpha_1 + \alpha_2 - 1} \quad (25)$$

### 3. Fractional Integration of the Product of Srivastava Polynomial, Multivariable Mittag-Leffler Function and Extended Hypergeometric Function

In this section, we shall give some fractional derivative formulas involving the product of Srivastava's polynomial, multivariable Mittag-Leffler function and generalized Lauricella's hypergeometric function by using the fractional differential operators.

**Theorem 1:** Let  $x > 0$ ,  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta, \rho, \lambda_j, \mu_j, c \in \mathbb{C}$ ,  $d, d_j \in \mathbb{R}^+$  where  $j = 1, 2, \dots, l$ ;  $|t| \leq 1$  be such that  $\Re(\eta) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$$\Re(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3) > \max\{0, \Re(\eta - \alpha_1 - \alpha_2 - \beta_2), \Re(\beta_1 - \alpha_1)\},$$

then the following fractional derivative formula holds true:

$$\begin{aligned} & \left\{ D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M(\sigma t^d) E_{\mu_j, c}^{\lambda_j}(\sigma_1 t^{d_1}, \dots, \sigma_l t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1 t, y_2 t, y_3 t; n_1, n_2) \right) \right\} (x) \\ &= x^{\rho + \alpha_1 + \alpha_2 - \eta - 1} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)} \\ & \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j}(\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\ & \times {}_3F_{D,3}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{array}{l} a, \rho + dk + \sum_{j=1}^l d_j k_j, \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2, \\ b, \rho + dk + \sum_{j=1}^l d_j k_j - \beta_1, \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2, \end{array} \right. \end{aligned}$$

$$\left. \begin{aligned} & \rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1, a_1, a_2, a_3; \\ & \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2,; \end{aligned} \right\} y_1 x, y_2 x, y_3 x \quad (26)$$

**Proof:** let  $\Delta$  denote the left hand side of the equation (26). Then using equations (4), (6) and (13) in the equation (26) and changing the order of integration and summation, which is valid under the given conditions, we have

$$\begin{aligned} \Delta &= \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} \sigma^k \frac{\sigma_1^{k_1}}{k_1!} \dots \frac{\sigma_l^{k_l}}{k_l!} \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{(b)_{m_1+m_2+m_3}} \\ &\quad \times \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)}{B(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)} \frac{y_1^{m_1} y_2^{m_2} y_3^{m_3}}{m_1! m_2! m_3!} \\ &\quad \times \left( D_{0, x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} t^{\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3 - 1} \right) (x) \end{aligned} \quad (27)$$

now using the equation (24), then the above equation (27) reduces to

$$\begin{aligned} \Delta &= \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} \sigma^k \frac{\sigma_1^{k_1}}{k_1!} \dots \frac{\sigma_l^{k_l}}{k_l!} \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{(b)_{m_1+m_2+m_3}} \\ &\quad \times \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)}{B(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)} \frac{y_1^{m_1} y_2^{m_2} y_3^{m_3}}{m_1! m_2! m_3!} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + m_1 + m_2 + m_3)} \\ &\quad \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2 + m_1 + m_2 + m_3) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1 + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + m_1 + m_2 + m_3) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2 + m_1 + m_2 + m_3)} \\ &\quad \times x^{\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3 + \alpha_1 + \alpha_2 - \eta - 1} \end{aligned} \quad (28)$$

now using the result  $\Gamma(\lambda + n) = (\lambda)_n \Gamma(\lambda)$ , the above equation (28) reduces to

$$= x^{\rho + \alpha_1 + \alpha_2 - \eta - 1} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)}$$

$$\begin{aligned}
& \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\
& \times \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (\rho + dk + \sum_{j=1}^l d_j k_j)_{m_1+m_2+m_3} (\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)_{m_1+m_2+m_3}}{(b)_{m_1+m_2+m_3} (\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1)_{m_1+m_2+m_3} \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)_{m_1+m_2+m_3}} \\
& \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)_{m_1+m_2+m_3}} \\
& \times \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a - n_1 + m_1 + m_2 + m_3, b - a + n_2) (xy_1)^{m_1} (xy_2)^{m_2} (xy_3)^{m_3}}{B(a - n_1 + m_1 + m_2 + m_3, b - a + n_2) m_1! m_2! m_3!} \tag{29}
\end{aligned}$$

the above equation (29) in view of the equation (13) gives the required result (26).

**Theorem 2:** Let  $x > 0$ ,  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta, \rho, \lambda_j, \mu_j, c \in \mathbb{C}$ ,  $d, d_j \in \mathbb{R}^+$  where  $j = 1, 2, \dots, l$ ;  $|1/t| \leq 1$  be such that  $\Re(\eta) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$$\Re(\rho + dk + \sum_{j=1}^l d_j k_j - m_1 - m_2 - m_3) < 1 + \min\{\Re(\beta_2), \Re(\eta - \alpha_1 - \alpha_2), \Re(\eta - \alpha_2 - \beta_1)\},$$

then the following fractional derivative formula holds true:

$$\begin{aligned}
& \left\{ D_{x, \infty}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 t^{d_1}, \dots, \sigma_l t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1/t, y_2/t, y_3/t; n_1, n_2) \right) \right\} (x) \\
& = x^{\rho + \alpha_1 + \alpha_2 - \eta - 1} \frac{\Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j + \beta_2) \Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2)}{\Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j) \Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2 - \beta_1)} \\
& \times \frac{\Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_2 - \beta_1)^{[N/M]} \sum_{k=0}^{\infty} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!}}{\Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j - \alpha_2 + \beta_2)} \\
& \times {}_3F_{D,3}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{matrix} a, 1 - \rho - dk - \sum_{j=1}^l d_j k_j + \beta_2, 1 - \rho - dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2, \\ b, 1 - \rho - dk - \sum_{j=1}^l d_j k_j, 1 - \rho - dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2 - \beta_1, \end{matrix} \right]
\end{aligned}$$

$$\left. \begin{aligned} &1 - \rho - dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_2 - \beta_1, a_1, a_2, a_3; \frac{y_1}{x}, \frac{y_2}{x}, \frac{y_3}{x} \\ &1 - \rho - dk - \sum_{j=1}^l d_j k_j - \alpha_2 + \beta_2, ; \end{aligned} \right] (30)$$

#### 4. Special Cases

Now we shall give some special cases by taking suitable values of parameters  $\alpha_1, \alpha_2, \beta_1, \beta_2$  and  $\eta$ . If put  $\alpha_2 = \beta_2 = 0$ ,  $\beta_1 = -\eta$ ,  $\alpha_1 = \alpha_1 + \beta_1$  and  $\eta = \alpha_1$  in the theorem 1 and theorem 2 we shall get following fractional derivative formulas for the Saigo fractional differential operators.

**Corollary 1:** Let  $x > 0$ ,  $\alpha_1, \beta_1, \eta, \rho, \lambda_j, \mu_j, c \in C$ ,  $d, d_j \in R^+$  where  $j = 1, 2, \dots, l$ ;  $|t| \leq 1$  be such that  $\Re(\alpha_1) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$\Re(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3) > -\min\{0, \Re(\alpha_1 + \beta_1 + \eta)\}$ , then the following fractional derivative formula holds true:

$$\begin{aligned} &\left\{ D_{0,x}^{\alpha_1, \beta_1, \eta} \left( t^{\rho-1} S_N^M(\sigma t^d) E_{\mu_j, c}^{\lambda_j}(\sigma_1 t^{d_1}, \dots, \sigma_l t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1 t, y_2 t, y_3 t; n_1, n_2) \right) \right\} (x) \\ &= x^{\rho + \beta_1 - 1} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j + \eta + \alpha_1 + \beta_1)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j + \beta_1) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j + \eta)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j}(\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\ &\times {}_2F_{D,2}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{array}{c} a, \rho + dk + \sum_{j=1}^l d_j k_j, \rho + dk + \sum_{j=1}^l d_j k_j + \eta + \alpha_1 + \beta_1, a_1, a_2, a_3; \\ b, \rho + dk + \sum_{j=1}^l d_j k_j + \beta_1, \rho + dk + \sum_{j=1}^l d_j k_j + \eta; \end{array} \begin{array}{c} y_1 x, y_2 x, y_3 x \end{array} \right] (31) \end{aligned}$$

**Corollary 2:** Let  $x > 0$ ,  $\alpha_1, \beta_1, \eta, \rho, \lambda_j, \mu_j, c \in C$ ,  $d, d_j \in R^+$  where  $j = 1, 2, \dots, l$ ;  $|1/t| \leq 1$  be such that  $\Re(\alpha_1) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$\Re(\rho + dk + \sum_{j=1}^l d_j k_j - m_1 - m_2 - m_3) < 1 + \min\{\Re(-\beta_1 - m), \Re(\eta + \alpha)\}$ , then the following fractional derivative formula holds true:

$$\begin{aligned} &\left\{ D_{x,\infty}^{\alpha_1, \beta_1, \eta} \left( t^{\rho-1} S_N^M(\sigma t^d) E_{\mu_j, c}^{\lambda_j}(\sigma_1 t^{d_1}, \dots, \sigma_l t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1/t, y_2/t, y_3/t; n_1, n_2) \right) \right\} (x) \\ &= x^{\rho + \beta_1 - 1} \frac{\Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j + \alpha_1 + \eta)}{\Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j) \Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j + \eta - \beta_1)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j}(\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \end{aligned}$$

$$\times_2 F_{D,2}^{3(\gamma,\delta,\tau,v,p_1,p_2)} \left[ \begin{array}{c} a, 1-\rho-dk - \sum_{j=1}^l d_j k_j - \beta_1, 1-\rho-dk - \sum_{j=1}^l d_j k_j + \alpha_1 + \eta, a_1, a_2, a_3; \\ b, 1-\rho-dk - \sum_{j=1}^l d_j k_j, 1-\rho-dk - \sum_{j=1}^l d_j k_j - \beta_1 + \eta; \end{array} \frac{y_1}{x}, \frac{y_2}{x}, \frac{y_3}{x} \right] \quad (32)$$

If we put  $\beta_1 = 0$  in the equations (30) and (31), then we shall get the following fractional derivative formulas for the Erdelyi-Kober fractional differential operators.

**Corollary 3:** Let  $x > 0$ ,  $\alpha_1, \eta, \rho, \lambda_j, \mu_j, c \in C$ ,  $d, d_j \in R^+$  where  $j = 1, 2, \dots, l$ ;  $|t| \leq 1$  be such that  $\Re(\alpha_1) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$\Re(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3) > -\Re(\eta + \alpha_1)$ , then the following fractional derivative formula holds true:

$$\left\{ D_{0,x}^{\alpha_1, \eta} \left( t^{\rho-1} S_N^M (\sigma t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 t^{d_1}, \dots, \sigma_l t^{d_l}) F_D^{3(\gamma, \delta, \tau, v, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1 t, y_2 t, y_3 t; n_1, n_2) \right) \right\} (x)$$

$$= x^{\rho-1} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j + \alpha_1 + \eta)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j + \eta)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!}$$

$$\times_1 F_{D,1}^{3(\gamma, \delta, \tau, v, p_1, p_2)} \left[ \begin{array}{c} a, \rho + dk + \sum_{j=1}^l d_j k_j + \alpha_1 + \eta, a_1, a_2, a_3; \\ b, \rho + dk + \sum_{j=1}^l d_j k_j + \eta; \end{array} y_1 x, y_2 x, y_3 x \right] \quad (33)$$

**Corollary 4:** Let  $x > 0$ ,  $\alpha_1, \eta, \rho, \lambda_j, \mu_j, c \in C$ ,  $d, d_j \in R^+$  where  $j = 1, 2, \dots, l$ ;  $|1/t| \leq 1$  be such that  $\Re(\alpha_1) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$\Re(\rho + dk + \sum_{j=1}^l d_j k_j - m_1 - m_2 - m_3) < 1 + \Re(\alpha_1 + \eta) - m$ , then the following fractional derivative formula holds true:

$$\left\{ D_{x, \infty}^{\alpha_1, \eta} \left( t^{\rho-1} S_N^M (\sigma t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 t^{d_1}, \dots, \sigma_l t^{d_l}) F_D^{3(\gamma, \delta, \tau, v, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1/t, y_2/t, y_3/t; n_1, n_2) \right) \right\} (x)$$

$$= x^{\rho-1} \frac{\Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \alpha_1 + \eta)}{\Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!}$$

$$\times_1 F_{D,1}^{3(\gamma, \delta, \tau, v, p_1, p_2)} \left[ \begin{array}{c} a, 1-\rho-dk - \sum_{j=1}^l d_j k_j + \alpha_1 + \eta, a_1, a_2, a_3; \\ b, 1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta; \end{array} \frac{y_1}{x}, \frac{y_2}{x}, \frac{y_3}{x} \right] \quad (34)$$

If we  $\beta_1 = -\alpha_1$  in the equations (30) and (31), then we shall get the following fractional derivative formulas for the Riemann-Liouville and Weyl fractional differential operators respectively.

**Corollary 5:** Let  $x > 0$ ,  $\alpha_1, \rho, \lambda_j, \mu_j, c \in \mathbb{C}$ ,  $d, d_j \in \mathbb{R}^+$  where  $j = 1, 2, \dots, l$ ;  $|t| \leq 1$  be such that  $\Re(\alpha_1) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$\Re(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3) > \Re(\alpha_1) > 0$ , then the following fractional derivative formula holds true:

$$\begin{aligned} & \left\{ D_{0,x}^{\alpha_1} \left( t^{\rho-1} S_N^M (\sigma t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 t^{d_1}, \dots, \sigma_l t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1 t, y_2 t, y_3 t; n_1, n_2) \right) \right\} (x) \\ &= x^{\rho-1} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \alpha_1)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\ & \times_1 F_{D,1}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{array}{c} a, \rho + dk + \sum_{j=1}^l d_j k_j, a_1, a_2, a_3; \\ b, \rho + dk + \sum_{j=1}^l d_j k_j - \alpha_1; \end{array} \begin{array}{c} y_1 x, y_2 x, y_3 x \end{array} \right] \end{aligned} \quad (35)$$

**Corollary 6:** Let  $x > 0$ ,  $\alpha_1, \rho, \lambda_j, \mu_j, c \in \mathbb{C}$ ,  $d, d_j \in \mathbb{R}^+$  where  $j = 1, 2, \dots, l$ ;  $|1/t| \leq 1$  be such that  $\Re(\alpha_1) > 0$ ,  $\Re(\mu_j) > 0$ . Further let

$\Re(\rho + dk + \sum_{j=1}^l d_j k_j - m_1 - m_2 - m_3) < 1 + \Re(\alpha_1) - m$ , then the following fractional derivative formula holds true:

$$\begin{aligned} & \left\{ D_{x,\infty}^{\alpha_1} \left( t^{\rho-1} S_N^M (\sigma t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 t^{d_1}, \dots, \sigma_l t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1/t, y_2/t, y_3/t; n_1, n_2) \right) \right\} (x) \\ &= x^{\rho-1} \frac{\Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j + \alpha_1)}{\Gamma(1 - \rho - dk - \sum_{j=1}^l d_j k_j)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\ & \times_1 F_{D,1}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{array}{c} a, 1 - \rho - dk - \sum_{j=1}^l d_j k_j + \alpha_1, a_1, a_2, a_3; \\ b, 1 - \rho - dk - \sum_{j=1}^l d_j k_j; \end{array} \begin{array}{c} \frac{y_1}{x}, \frac{y_2}{x}, \frac{y_3}{x} \end{array} \right] \end{aligned} \quad (36)$$

## 5. Integral Transform of the Product of Srivastava Polynomial, Multivariable Mittag-Leffler Function and Extended Hypergeometric Function

In this section we shall obtain integral transforms like, Beta transform, Laplace transform and Verma transform involving the results obtained in the previous section.

### 5.1 Beta Transform

**Definition:** The beta transform of a function  $f(z)$  is defined as (see [15])

$$B\{f(z); s, p\} = \int_0^1 z^{s-1} (1-z)^{p-1} f(z) dz \quad (37)$$

**Theorem 3:** Let  $x > 0$ ,  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta, \rho, \lambda_j, \mu_j, c \in C$ ,  $d, d_j \in R^+$  where  $j = 1, 2, \dots, l$ ;  $|t| \leq 1$  be such that  $\Re(\eta) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$$\Re(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3) > \max\{0, \Re(\eta - \alpha_1 - \alpha_2 - \beta_2), \Re(\beta_1 - \alpha_1)\},$$

then the following fractional derivative formula holds true:

$$\begin{aligned} & B\left\{D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left(t^{\rho-1} S_N^M (\sigma z^\theta t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1 z t, y_2 z t, y_3 z t; n_1, n_2)\right)(x) : s, p\right\} \\ &= x^{\rho + \alpha_1 + \alpha_2 - \eta - 1} \Gamma(\rho) \frac{\Gamma(s + \theta k + \sum_{j=1}^l \theta_j k_j)}{\Gamma(s + p + \theta k + \sum_{j=1}^l \theta_j k_j)} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)} \\ &\quad \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\ &\quad \times {}_4F_{D,4}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{array}{l} a, \rho + dk + \sum_{j=1}^l d_j k_j, \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2, \\ b, \rho + dk + \sum_{j=1}^l d_j k_j - \beta_1, \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2, \\ \rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1, s + \theta k + \sum_{j=1}^l \theta_j k_j, a_1, a_2, a_3; \\ \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_2 + \beta_2, s + p + \theta k + \sum_{j=1}^l \theta_j k_j; \end{array} \right. \left. \begin{array}{l} y_1 x, y_2 x, y_3 x \end{array} \right] \quad (38) \end{aligned}$$

**Proof:** To prove the equation(38) using the definition (37) of Beta transform, we get

$$B\left\{D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left(t^{\rho-1} S_N^M (\sigma z^\theta t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1 z t, y_2 z t, y_3 z t; n_1, n_2)\right)(x) : s, p\right\}$$

$$\begin{aligned}
&= \int_0^1 z^{s-1} (1-z)^{p-1} \left\{ D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^\lambda) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) \right. \right. \\
&\quad \left. \left. \times F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1 z t, y_2 z t, y_3 z t; n_1, n_2) \right) (x) \right\} dz \quad (39)
\end{aligned}$$

now using the equation (24), then the above equation (39) reduces to

$$\begin{aligned}
&= \int_0^1 z^{s+\theta k + \sum_{j=1}^l \theta_j k_j + m_1 + m_2 + m_3 - 1} (1-z)^{p-1} \left\{ x^{\rho + \alpha_1 + \alpha_2 - \eta - 1} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \right. \\
&\quad \times \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{(b)_{m_1+m_2+m_3}} \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)}{B(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)} \\
&\quad \times \frac{(y_1 x)^{m_1}}{m_1!} \frac{(y_2 x)^{m_2}}{m_2!} \frac{(y_3 x)^{m_3}}{m_3!} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2 + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + m_1 + m_2 + m_3)} \\
&\quad \left. \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + m_1 + m_2 + m_3)} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1 + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2 + m_1 + m_2 + m_3)} \right\} dz \quad (40)
\end{aligned}$$

after interchanging the order of integration and summation and using  $\Gamma(\lambda + n) = (\lambda)_n \Gamma(\lambda)$ , we get

$$\begin{aligned}
&= x^{\rho + \alpha_1 + \alpha_2 - \eta - 1} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 - \beta_2)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)} \\
&\quad \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\
&\quad \times \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (\rho + dk + \sum_{j=1}^l d_j k_j)_{m_1+m_2+m_3} (\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)_{m_1+m_2+m_3}}{(b)_{m_1+m_2+m_3} (\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1)_{m_1+m_2+m_3} \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)_{m_1+m_2+m_3}} \\
&\quad \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)_{m_1+m_2+m_3}} \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)}{B(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)}
\end{aligned}$$

$$\times \frac{(xy_1)^{m_1}}{m_1!} \frac{(xy_2)^{m_2}}{m_2!} \frac{(xy_3)^{m_3}}{m_3!} \int_0^1 z^{s+\theta k + \sum_{j=1}^l \theta_j k_j + m_1 + m_2 + m_3 - 1} (1-z)^{p-1} dz \quad (41)$$

now evaluating the z-integral and after a little simplification, we get

$$\begin{aligned} &= x^{\rho+\alpha_1+\alpha_2-\eta-1} \Gamma(p) \frac{\Gamma(s+\theta k + \sum_{j=1}^l \theta_j k_j)}{\Gamma(s+p+\theta k + \sum_{j=1}^l \theta_j k_j)} \frac{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)} \\ &\times \frac{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\ &\times \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (\rho+dk + \sum_{j=1}^l d_j k_j)_{m_1+m_2+m_3} (\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)_{m_1+m_2+m_3}}{(b)_{m_1+m_2+m_3} (\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1)_{m_1+m_2+m_3} \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)_{m_1+m_2+m_3}} \\ &\times \frac{(\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)_{m_1+m_2+m_3} (s+\theta k + \sum_{j=1}^l \theta_j k_j)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)_{m_1+m_2+m_3} (s+p+\theta k + \sum_{j=1}^l \theta_j k_j)_{m_1+m_2+m_3}} \\ &\times \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a-n_1+m_1+m_2+m_3, b-a+n_2)}{B(a-n_1+m_1+m_2+m_3, b-a+n_2)} \frac{(xy_1)^{m_1}}{m_1!} \frac{(xy_2)^{m_2}}{m_2!} \frac{(xy_3)^{m_3}}{m_3!} \end{aligned} \quad (42)$$

the above equation (42), in view of the equation (13) gives the required result (38).

**Theorem 4:** Let  $x > 0$ ,  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta, \rho, \lambda_j, \mu_j, c \in \mathbb{C}$ ,  $d, d_j \in \mathbb{R}^+$  where  $j = 1, 2, \dots, l$ ;  $|1/t| \leq 1$  be such that  $\Re(\eta) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$$\Re(\rho + dk + \sum_{j=1}^l d_j k_j - m_1 - m_2 - m_3) < 1 + \min \{ \Re(\beta_2), \Re(\eta - \alpha_1 - \alpha_2), \Re(\eta - \alpha_2 - \beta_1) \},$$

then the following fractional derivative formula holds true:

$$\begin{aligned} &B \left\{ D_{x, \infty}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1/t, y_2/t, y_3/t; n_1, n_2) \right) (x) : s, p \right\} \\ &= x^{\rho+\alpha_1+\alpha_2-\eta-1} \Gamma(p) \frac{\Gamma(s+\theta k + \sum_{j=1}^l \theta_j k_j)}{\Gamma(s+p+\theta k + \sum_{j=1}^l \theta_j k_j)} \frac{\Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \beta_2) \Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2)}{\Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j) \Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2 - \beta_1)} \end{aligned}$$

$$\begin{aligned}
& \times \frac{\Gamma(1-\rho-dk-\sum_{j=1}^l d_j k_j + \eta - \alpha_2 - \beta_1)}{\Gamma(1-\rho-dk-\sum_{j=1}^l d_j k_j - \alpha_2 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\
& \times {}_4F_{D,4}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{array}{l} a, 1-\rho-dk-\sum_{j=1}^l d_j k_j + \beta_2, 1-\rho-dk-\sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2, \\ b, 1-\rho-dk-\sum_{j=1}^l d_j k_j, 1-\rho-dk-\sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2 - \beta_1, \\ 1-\rho-dk-\sum_{j=1}^l d_j k_j + \eta - \alpha_2 - \beta_1, s + \theta k + \sum_{j=1}^l \theta_j k_j, a_1, a_2, a_3; \frac{y_1}{x}, \frac{y_2}{x}, \frac{y_3}{x} \\ 1-\rho-dk-\sum_{j=1}^l d_j k_j - \alpha_2 + \beta_2, s + p + \theta k + \sum_{j=1}^l \theta_j k_j; \end{array} \right] \quad (43)
\end{aligned}$$

## 5.2 Laplace Transform

**Definition:** The Laplace transform of a function  $f(z)$  is defined as (see [13])

$$L\{f(z)\} = \int_0^{\infty} e^{-qz} f(z) dz \quad (44)$$

**Theorem 5:** Let  $x > 0$ ,  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta, \rho, \lambda_j, \mu_j, c \in \mathbb{C}$ ,  $d, d_j \in \mathbb{R}^+$  where  $j = 1, 2, \dots, l$ ;  $|t| \leq 1$  be such that  $\Re(\eta) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$$\Re(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3) > \max\{0, \Re(\eta - \alpha_1 - \alpha_2 - \beta_2), \Re(\beta_1 - \alpha_1)\},$$

then the following fractional derivative formula holds true:

$$\begin{aligned}
& L \left\{ D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1 z t, y_2 z t, y_3 z t; n_1, n_2) \right) (x) \right\} \\
& = \frac{x^{\rho + \alpha_1 + \alpha_2 - \eta - 1}}{q} \frac{\Gamma(1 + \theta k + \sum_{j=1}^l \theta_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)} \\
& \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} \left( \frac{\sigma x^d}{q^\theta} \right)^k \prod_{j=1}^l \frac{1}{k_j!} \left( \frac{\sigma_j x^{d_j}}{q^{\theta_j}} \right)^{k_j} \\
& \times {}_4F_{D,3}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{array}{l} a, \rho + dk + \sum_{j=1}^l d_j k_j, \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2, \\ b, \rho + dk + \sum_{j=1}^l d_j k_j - \beta_1, \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2, \end{array} \right]
\end{aligned}$$

$$\left[ \begin{array}{c} \rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1, 1 + \theta k + \sum_{j=1}^l \theta_j k_j, a_1, a_2, a_3; \frac{xy_1}{q}, \frac{xy_2}{q}, \frac{xy_3}{q} \\ \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2; \end{array} \right] \quad (45)$$

**Proof:** To prove the equation (45) using the definition (44) of Laplace transform, we get

$$\begin{aligned} & L \left\{ D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1 z t, y_2 z t, y_3 z t; n_1, n_2) \right) (x) \right\} \\ &= \int_0^\infty e^{-qz} \left\{ D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^\lambda) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) \right. \right. \\ & \quad \left. \left. \times F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1 z t, y_2 z t, y_3 z t; n_1, n_2) \right) (x) \right\} dz \end{aligned} \quad (46)$$

now using the equation (24), then the above equation (46) reduces to

$$\begin{aligned} &= \int_0^\infty e^{-qz} z^{\theta k + \sum_{j=1}^l \theta_j k_j + m_1 + m_2 + m_3} \left\{ x^{\rho + \alpha_1 + \alpha_2 - \eta - 1} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^\infty \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \right. \\ & \quad \times \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \sum_{m_1, m_2, m_3=0}^\infty \frac{(a)_{m_1 + m_2 + m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{(b)_{m_1 + m_2 + m_3}} \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)} (a - n_1 + m_1 + m_2 + m_3, b - a + n_2)}{B(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)} \\ & \quad \times \frac{(y_1 x)^{m_1}}{m_1!} \frac{(y_2 x)^{m_2}}{m_2!} \frac{(y_3 x)^{m_3}}{m_3!} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2 + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + m_1 + m_2 + m_3)} \\ & \quad \left. \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + m_1 + m_2 + m_3)} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1 + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2 + m_1 + m_2 + m_3)} \right\} dz \end{aligned} \quad (47)$$

after interchanging the order of integration and summation and using  $\Gamma(\lambda + n) = (\lambda)_n \Gamma(\lambda)$ , we get

$$\begin{aligned} &= x^{\rho + \alpha_1 + \alpha_2 - \eta - 1} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)} \\ & \quad \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^\infty \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \end{aligned}$$

$$\begin{aligned}
& \times \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (\rho+dk+\sum_{j=1}^l d_j k_j)_{m_1+m_2+m_3} (\rho+dk+\sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)_{m_1+m_2+m_3}}{(b)_{m_1+m_2+m_3} (\rho+dk+\sum_{j=1}^l d_j k_j - \beta_1)_{m_1+m_2+m_3} \Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)_{m_1+m_2+m_3}} \\
& \times \frac{\Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3} B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a-n_1+m_1+m_2+m_3, b-a+n_2)}{\Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)_{m_1+m_2+m_3} B(a-n_1+m_1+m_2+m_3, b-a+n_2)} \\
& \times \frac{(xy_1)^{m_1}}{m_1!} \frac{(xy_2)^{m_2}}{m_2!} \frac{(xy_3)^{m_3}}{m_3!} \int_0^{\infty} e^{-qz} z^{\theta k + \sum_{j=1}^l \theta_j k_j + m_1 + m_2 + m_3} dz
\end{aligned} \tag{48}$$

now evaluating the z-integral and after a little simplification, we get

$$\begin{aligned}
& = \frac{x^{\rho-\alpha_1-\alpha_2+\eta-1} \Gamma(1+\theta k + \sum_{j=1}^l \theta_j k_j) \Gamma(\rho+dk+\sum_{j=1}^l d_j k_j) \Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{q \Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)} \\
& \times \frac{\Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} \left( \frac{\sigma x^d}{q^\theta} \right)^k \prod_{j=1}^l \frac{1}{k_j!} \left( \frac{\sigma_j x^{d_j}}{q^{\theta_j}} \right)^{k_j} \\
& \times \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (\rho+dk+\sum_{j=1}^l d_j k_j)_{m_1+m_2+m_3} (\rho+dk+\sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)_{m_1+m_2+m_3}}{(b)_{m_1+m_2+m_3} (\rho+dk+\sum_{j=1}^l d_j k_j - \beta_1)_{m_1+m_2+m_3} \Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)_{m_1+m_2+m_3}} \\
& \times \frac{\Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)_{m_1+m_2+m_3} (1+\theta k + \sum_{j=1}^l \theta_j k_j)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{\Gamma(\rho+dk+\sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)_{m_1+m_2+m_3}} \\
& \times \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a-n_1+m_1+m_2+m_3, b-a+n_2)}{B(a-n_1+m_1+m_2+m_3, b-a+n_2)} \frac{1}{m_1!} \left( \frac{xy_1}{q} \right)^{m_1} \frac{1}{m_2!} \left( \frac{xy_2}{q} \right)^{m_2} \frac{1}{m_3!} \left( \frac{xy_3}{q} \right)^{m_3}
\end{aligned} \tag{49}$$

the above equation (49), in view of the equation (13) gives the required result (45).

**Theorem 6:** Let  $x > 0$ ,  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta, \rho, \lambda_j, \mu_j, c \in \mathbb{C}$ ,  $d, d_j \in \mathbb{R}^+$  where  $j = 1, 2, \dots, l$ ;  $|1/t| \leq 1$  be such that  $\Re(\eta) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$\Re(\rho+dk+\sum_{j=1}^l d_j k_j - m_1 - m_2 - m_3) < 1 + \min\{\Re(\beta_2), \Re(\eta - \alpha_1 - \alpha_2), \Re(\eta - \alpha_2 - \beta_1)\}$ , then the following fractional derivative formula holds true:

$$\begin{aligned}
& L \left\{ D_{x,\infty}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1/t, y_2/t, y_3/t; n_1, n_2) \right) (x) \right\} \\
&= \frac{x^{\rho+\alpha_1+\alpha_2-\eta-1} \Gamma(1+\theta k + \sum_{j=1}^l \theta_j k_j) \Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \beta_2) \Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2)}{q} \\
&\quad \frac{\Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_2 - \beta_1)}{\Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j - \alpha_2 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} \left( \frac{\sigma x^d}{q^\theta} \right) \prod_{j=1}^l \frac{1}{k_j!} \left( \frac{\sigma_j x^{d_j}}{q^{\theta_j}} \right)^{k_j} \\
&\quad \times {}_4F_{D,3}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{matrix} a, 1-\rho-dk - \sum_{j=1}^l d_j k_j + \beta_2, 1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2, \\ b, 1-\rho-dk - \sum_{j=1}^l d_j k_j, 1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2 - \beta_1, \\ 1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_2 - \beta_1, 1+\theta k + \sum_{j=1}^l \theta_j k_j, a_1, a_2, a_3; \\ 1-\rho-dk - \sum_{j=1}^l d_j k_j - \alpha_2 + \beta_2; \end{matrix} \frac{y_1}{qx}, \frac{y_2}{qx}, \frac{y_3}{qx} \right] \quad (50)
\end{aligned}$$

### 5.3 Verma Transform

**Definition:** Verma transform of a function  $f(z)$  is defined as (see [3])

$$V \{ f(z) \} = \int_0^\infty (sz)^{r-1} e^{-\frac{1}{2}sz} W_{\kappa, \omega}(sz) f(z) dz \quad (51)$$

where  $W_{\kappa, \omega}(z)$  represents Whittaker function.

**Theorem 7:** Let  $x > 0$ ,  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta, \rho, \lambda_j, \mu_j, c \in C$ ,  $d, d_j \in R^+$  where  $j = 1, 2, \dots, l$ ;  $|t| \leq 1$  be such that  $\Re(\eta) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$$\Re(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3) > \max \{ 0, \Re(\eta - \alpha_1 - \alpha_2 - \beta_2), \Re(\beta_1 - \alpha_1) \},$$

then the following fractional derivative formula holds true:

$$\begin{aligned}
& V \left\{ D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1 z t, y_2 z t, y_3 z t; n_1, n_2) \right) (x) \right\} \\
&= \frac{x^{\rho+\alpha_1+\alpha_2-\eta-1} \Gamma(\frac{1}{2} \pm \omega + \theta k + \sum_{j=1}^l \theta_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{s^r} \\
&\quad \frac{\Gamma(\frac{1}{2} \pm \omega + \theta k + \sum_{j=1}^l \theta_j k_j)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)}
\end{aligned}$$

$$\begin{aligned}
& \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} \left( \frac{\sigma x^d}{s^\theta} \right)^k \prod_{j=1}^l \frac{1}{k_j!} \left( \frac{\sigma_j x^{d_j}}{s^{\theta_j}} \right)^{k_j} \\
& \times {}_5F_{D,4}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{array}{l} a, \rho + dk + \sum_{j=1}^l d_j k_j, \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2, \\ b, \rho + dk + \sum_{j=1}^l d_j k_j - \beta_1, \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2, \\ \rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1, \frac{1}{2} \pm \omega + \theta k + \sum_{j=1}^l \theta_j k_j, a_1, a_2, a_3; \frac{xy_1}{s}, \frac{xy_2}{s}, \frac{xy_3}{s} \\ \rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2, 1 - \kappa + \theta k + \sum_{j=1}^l \theta_j k_j; \end{array} \right] \quad (52)
\end{aligned}$$

**Proof:** To prove the equation (52) using the definition (51) of Verma transform, we get

$$\begin{aligned}
& V \left\{ D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1 z t, y_2 z t, y_3 z t, n_1, n_2) \right) (x) \right\} \\
& = \int_0^\infty z^{r-1} e^{-\frac{1}{2}sz} W_{\kappa, \omega}(sz) \left\{ D_{0,x}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^\lambda) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) \right. \right. \\
& \quad \left. \left. \times F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} (a, a_1, a_2, a_3; b; y_1 z t, y_2 z t, y_3 z t, n_1, n_2) \right) (x) \right\} dz \quad (53)
\end{aligned}$$

Now using the equation (24), then the above equation (53) reduces to

$$\begin{aligned}
& = \int_0^\infty z^{r+\theta k + \sum_{j=1}^l \theta_j k_j + m_1 + m_2 + m_3 - 1} e^{-\frac{1}{2}sz} W_{\kappa, \omega}(sz) \left\{ x^{\rho + \alpha_1 + \alpha_2 - \eta - 1} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \right. \\
& \quad \times \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{(b)_{m_1+m_2+m_3}} \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)} (a - n_1 + m_1 + m_2 + m_3, b - a + n_2)}{B(a - n_1 + m_1 + m_2 + m_3, b - a + n_2)} \\
& \quad \times \frac{(y_1 x)^{m_1}}{m_1!} \frac{(y_2 x)^{m_2}}{m_2!} \frac{(y_3 x)^{m_3}}{m_3!} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2 + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + m_1 + m_2 + m_3)} \\
& \quad \left. \times \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + m_1 + m_2 + m_3)} \frac{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1 + m_1 + m_2 + m_3)}{\Gamma(\rho + dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2 + m_1 + m_2 + m_3)} \right\} dz \quad (54)
\end{aligned}$$

after interchanging the order of integration and summation and using  $\Gamma(\lambda + n) = (\lambda)_n \Gamma(\lambda)$ , we get

$$\begin{aligned}
&= x^{\rho+\alpha_1+\alpha_2-\eta-1} \frac{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)} \\
&\times \frac{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\
&\times \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (\rho+dk + \sum_{j=1}^l d_j k_j)_{m_1+m_2+m_3} (\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)_{m_1+m_2+m_3}}{(b)_{m_1+m_2+m_3} (\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1)_{m_1+m_2+m_3} \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)_{m_1+m_2+m_3}} \\
&\times \frac{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3} B_{\rho_1, \rho_2}^{(\gamma, \delta, \tau, \nu)}(a-n_1+m_1+m_2+m_3, b-a+n_2)}{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)_{m_1+m_2+m_3} B(a-n_1+m_1+m_2+m_3, b-a+n_2)} \\
&\times \frac{(xy_1)^{m_1}}{m_1!} \frac{(xy_2)^{m_2}}{m_2!} \frac{(xy_3)^{m_3}}{m_3!} \int_0^{\infty} z^{r+\theta k + \sum_{j=1}^l \theta_j k_j + m_1+m_2+m_3-1} e^{-\frac{1}{2}sz} W_{\kappa, \omega}(sz) dz
\end{aligned} \tag{55}$$

now evaluating the z-integral and after a little simplification, we get

$$\begin{aligned}
&= \frac{x^{\rho+\alpha_1+\alpha_2-\eta-1}}{s^r} \frac{\Gamma(\frac{1}{2} \pm \omega + \theta k + \sum_{j=1}^l \theta_j k_j) \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j) \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)}{\Gamma(1-\kappa + \theta k + \sum_{j=1}^l \theta_j k_j) \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1) \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)} \\
&\times \frac{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)}{\Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)} \sum_{k=0}^{[N/M]} \sum_{k_1, \dots, k_l=0}^{\infty} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} \left( \frac{\sigma x^d}{s^\theta} \right)^k \prod_{j=1}^l \frac{1}{k_j!} \left( \frac{\sigma_j x^{d_j}}{s^{\theta_j}} \right)^{k_j} \\
&\times \sum_{m_1, m_2, m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3} (\rho+dk + \sum_{j=1}^l d_j k_j)_{m_1+m_2+m_3} (\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2 + \beta_2)_{m_1+m_2+m_3}}{(b)_{m_1+m_2+m_3} (\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1)_{m_1+m_2+m_3} \Gamma(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \alpha_2)_{m_1+m_2+m_3}} \\
&\times \frac{(\rho+dk + \sum_{j=1}^l d_j k_j - \beta_1 + \alpha_1)_{m_1+m_2+m_3} (\frac{1}{2} \pm \omega + \theta k + \sum_{j=1}^l \theta_j k_j)_{m_1+m_2+m_3} (a_1)_{m_1} (a_2)_{m_2} (a_3)_{m_3}}{(\rho+dk + \sum_{j=1}^l d_j k_j - \eta + \alpha_1 + \beta_2)_{m_1+m_2+m_3} (1-\kappa + \theta k + \sum_{j=1}^l \theta_j k_j)_{m_1+m_2+m_3}}
\end{aligned}$$

$$\times \frac{B_{p_1, p_2}^{(\gamma, \delta, \tau, \nu)}(a-n_1+m_1+m_2+m_3, b-a+n_2)}{B(a-n_1+m_1+m_2+m_3, b-a+n_2)} \frac{1}{m_1!} \left(\frac{xy_1}{s}\right)^{m_1} \frac{1}{m_2!} \left(\frac{xy_2}{s}\right)^{m_2} \frac{1}{m_3!} \left(\frac{xy_3}{s}\right)^{m_3} \quad (56)$$

the above equation (56) in view of the equation (13) gives the required result (52).

**Theorem 8:** Let  $x > 0$ ,  $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta, \rho, \lambda_j, \mu_j, c \in C$ ,  $d, d_j \in R^+$  where  $j = 1, 2, \dots, l$ ;  $|1/t| \leq 1$  be such that  $\Re(\eta) > 0$ ,  $\Re(\mu_j) > 0$ . Further, let

$\Re(\rho + dk + \sum_{j=1}^l d_j k_j - m_1 - m_2 - m_3) < 1 + \min\{\Re(\beta_2), \Re(\eta - \alpha_1 - \alpha_2), \Re(\eta - \alpha_2 - \beta_1)\}$ , then the following fractional integral formula holds true:

$$\begin{aligned} & V \left\{ D_{x, \infty}^{\alpha_1, \alpha_2, \beta_1, \beta_2, \eta} \left( t^{\rho-1} S_N^M (\sigma z^\theta t^d) E_{\mu_j, c}^{\lambda_j} (\sigma_1 z^{\theta_1} t^{d_1}, \dots, \sigma_l z^{\theta_l} t^{d_l}) F_D^{3(\gamma, \delta, \tau, \nu, p_1, p_2)}(a, a_1, a_2, a_3; b; y_1/t, y_2/t, y_3/t; n_1, n_2) \right) (x) \right\} \\ &= \frac{x^{\rho+\alpha_1+\alpha_2-\eta-1}}{s^r} \frac{\Gamma(\frac{1}{2} \pm \omega + \theta k + \sum_{j=1}^l \theta_j k_j) \Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \beta_2) \Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2)}{\Gamma(1-\kappa + \theta k + \sum_{j=1}^l \theta_j k_j) \Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j) \Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2 - \beta_1)} \\ &\times \frac{\Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_2 - \beta_1)}{\Gamma(1-\rho-dk - \sum_{j=1}^l d_j k_j - \alpha_2 + \beta_2)} \sum_{k=0}^{\infty} \sum_{k_1, \dots, k_l=0}^{[N/M]} \frac{(-N)_{Mk}}{k!} A_{N,k} F_{\mu_j, c}^{\lambda_j} (\sigma x^d)^k \prod_{j=1}^l \frac{(\sigma_j x^{d_j})^{k_j}}{k_j!} \\ &\times_5 F_{D,4}^{3(\gamma, \delta, \tau, \nu, p_1, p_2)} \left[ \begin{matrix} a, 1-\rho-dk - \sum_{j=1}^l d_j k_j + \beta_2, 1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2, \\ b, 1-\rho-dk - \sum_{j=1}^l d_j k_j, 1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_1 - \alpha_2 - \beta_1, \end{matrix} \right. \\ &\left. \begin{matrix} 1-\rho-dk - \sum_{j=1}^l d_j k_j + \eta - \alpha_2 - \beta_1, \frac{1}{2} \pm \omega + \theta k + \sum_{j=1}^l \theta_j k_j, a_1, a_2, a_3; \\ 1-\rho-dk - \sum_{j=1}^l d_j k_j - \alpha_2 + \beta_2, 1-\kappa + \theta k + \sum_{j=1}^l \theta_j k_j; \end{matrix} \frac{y_1}{sx}, \frac{y_2}{sx}, \frac{y_3}{sx} \right] \quad (57) \end{aligned}$$

## 6. Conclusion

In a similar manner, by suitably specializing the parameters, several theorems and corollaries for various fractional differential and integral transform formulas involving the product of a number of simpler special functions, can be obtained, as derived in this paper.

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